The Planck constant, *h*, and the redefinition of the SI

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Outline

1. The SI and the definition of the kg

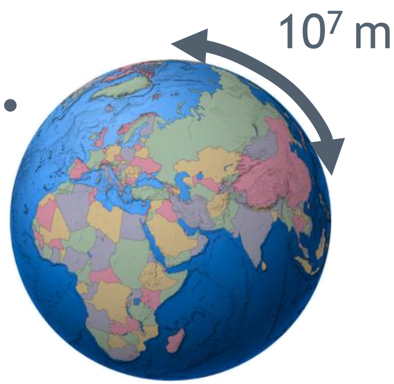
2. The principle of the Watt balance

3. The past, the present, & the future of the Ekg

A brief history of units

I. Based on man.

II. Based on Earth.

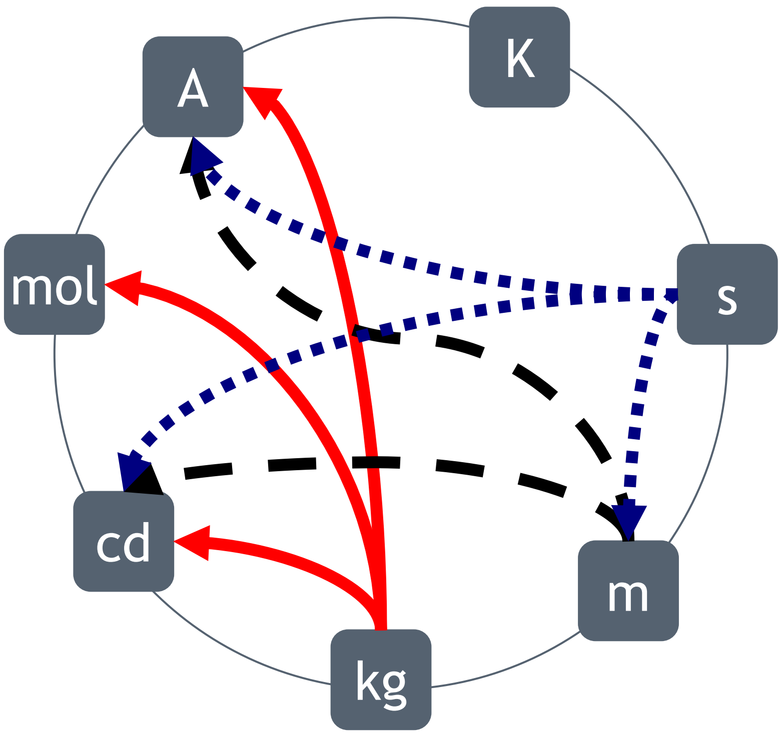


III. Based on fundamental constants.

c

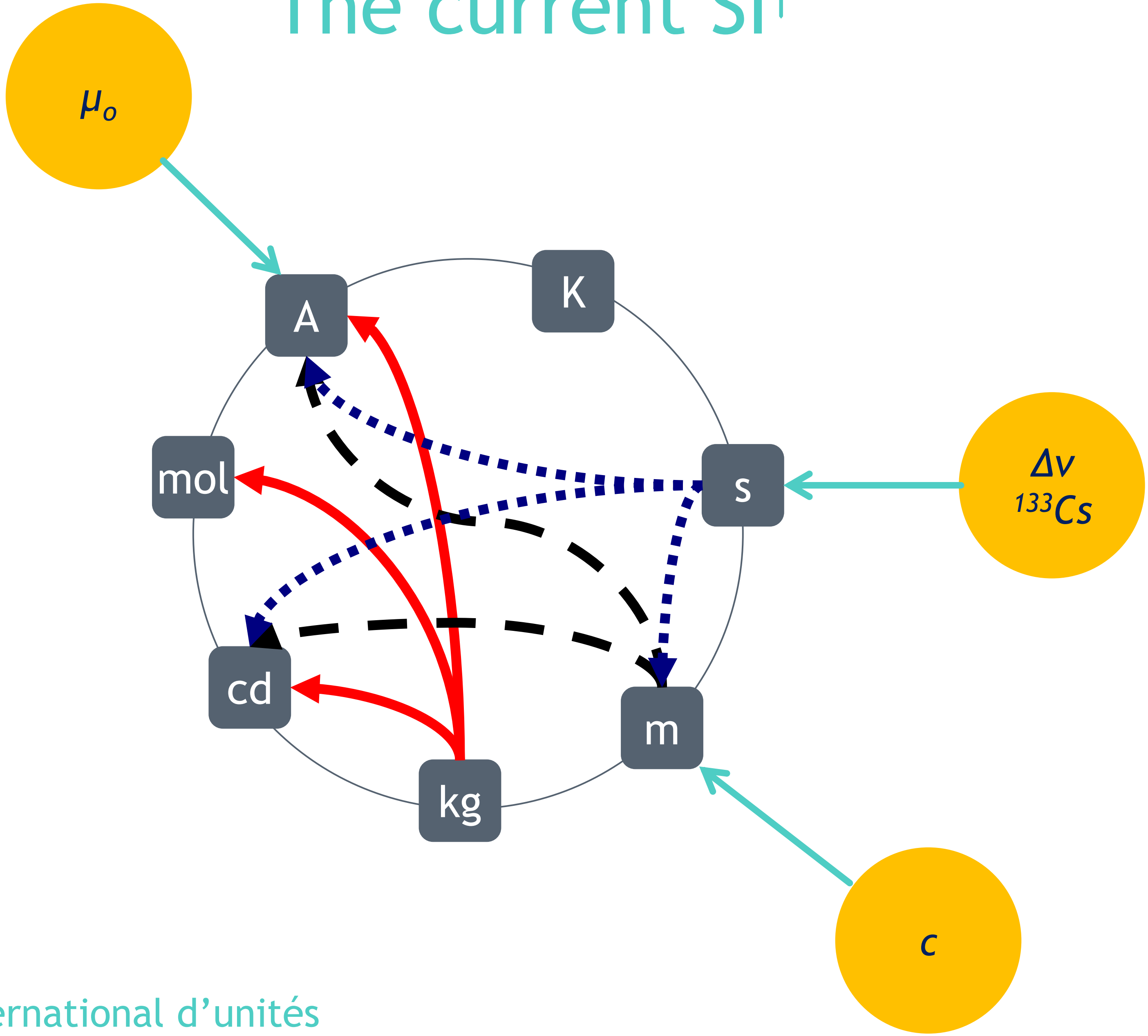
$\hbar$

The current SI†



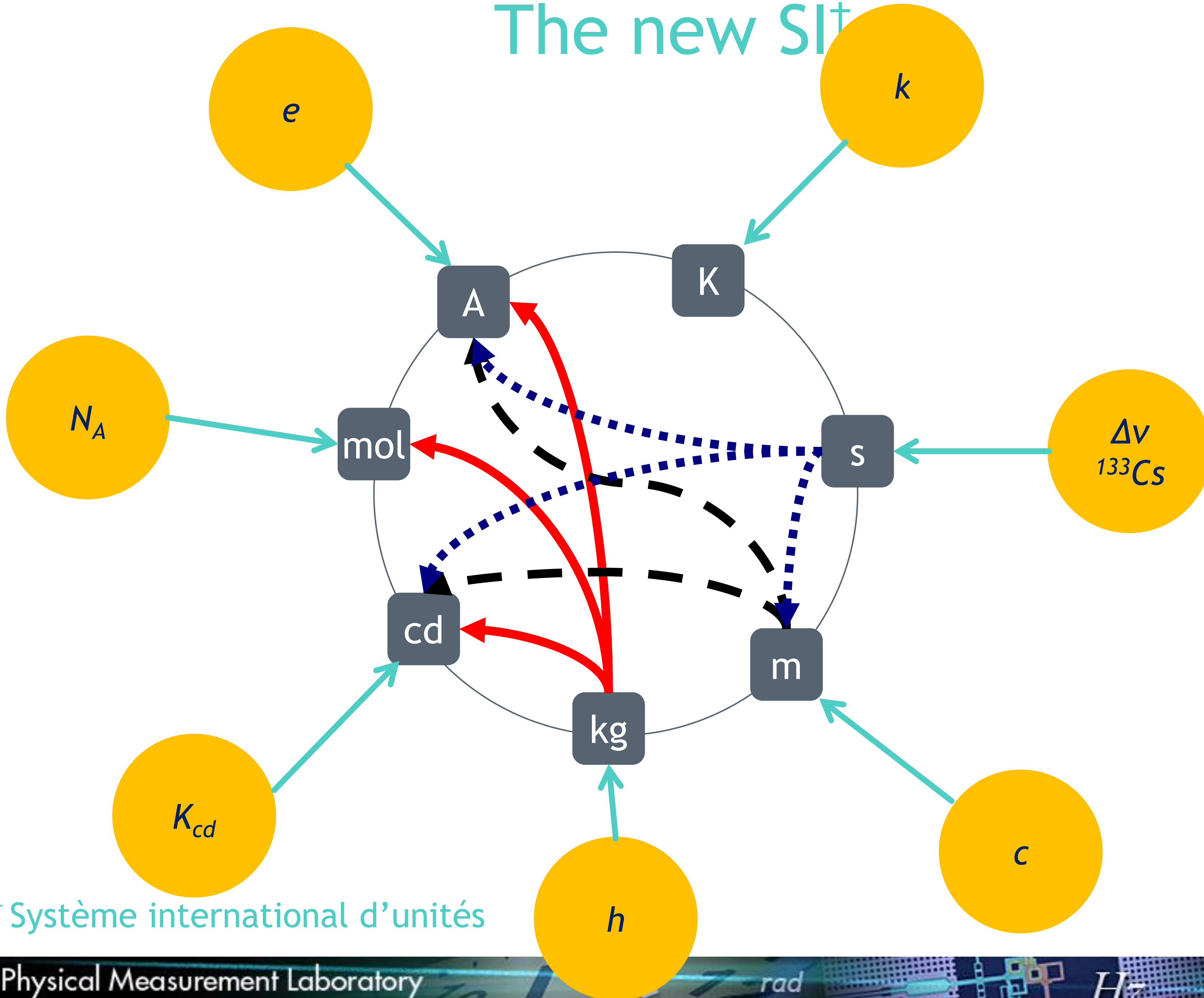
† Système international d'unités

The current SI†



† Système international d'unités

The new SI†



† Système international d'unités

Why fix it, if it is not broken?

The kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (1889)

The kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (1889)

CIPM 1989: The reference mass of the international prototype is that immediately after cleaning and washing by a specified method.

Cylindrical
height:39 mm, diameter: 39 mm
Alloy 90 % platinum and 10 % iridium

International Prototype:	IPK
Official Copies:	K1, 7, 8(41), 32, 43, 47
BIPM prototypes	
for special use:	25
for routine use:	9,31,67

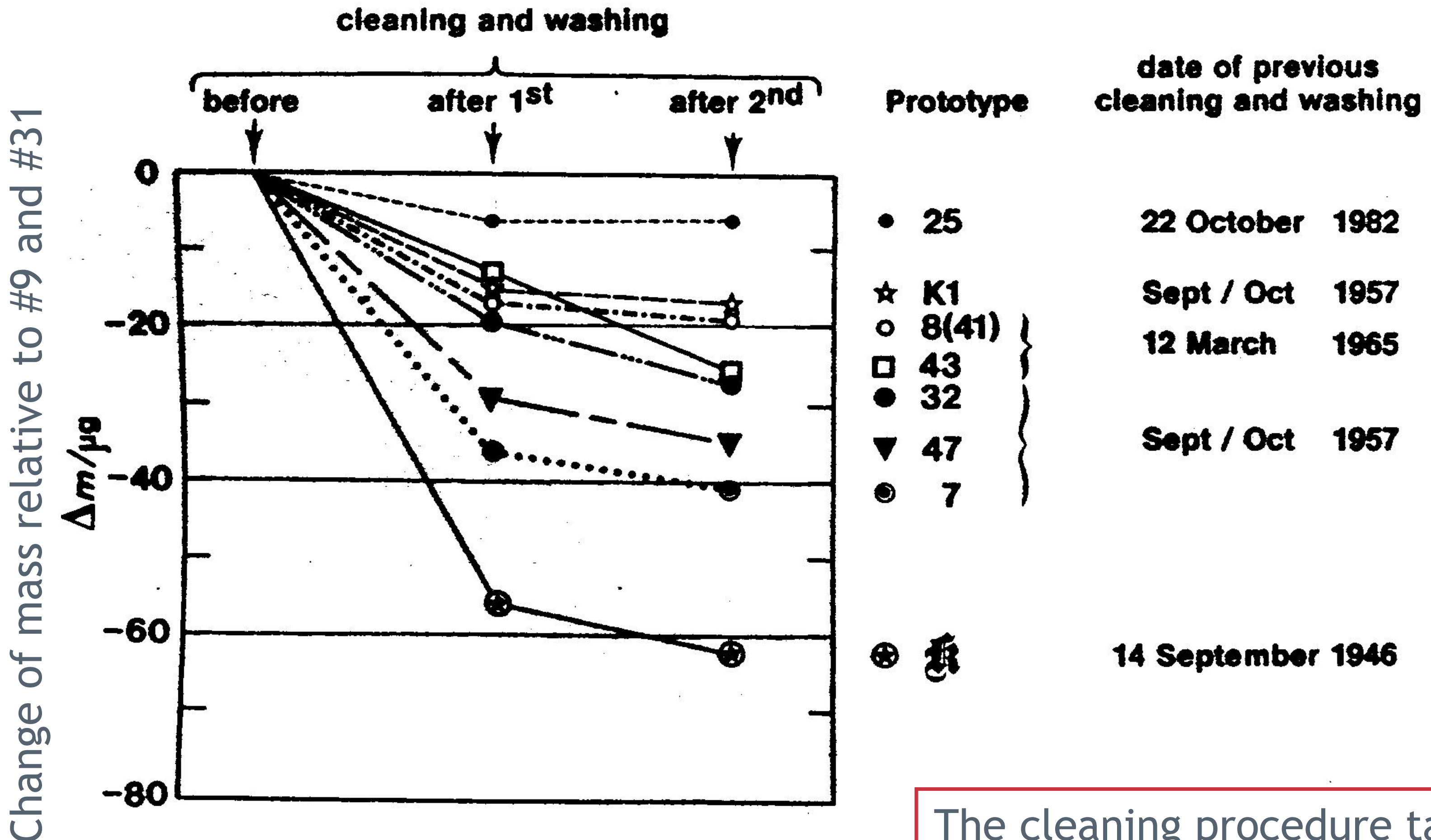
National prototypes:
12,21,5,2,16,36,6,20,23,37,
18,46, 35, 38, 24, 57, 39,
40, 50, 48, 44, 55, 49,53
56, 51, 54, 58, 68, 60,70,
65,69
} distributed by raffle

New prototypes	67,71,72,74,75,77-94
Other prototypes	34

USA:
K4 (1890) check standard
K20 (1890) national prototype
K79 (1996)
K85 (2003) watt balance
K92 (2008)

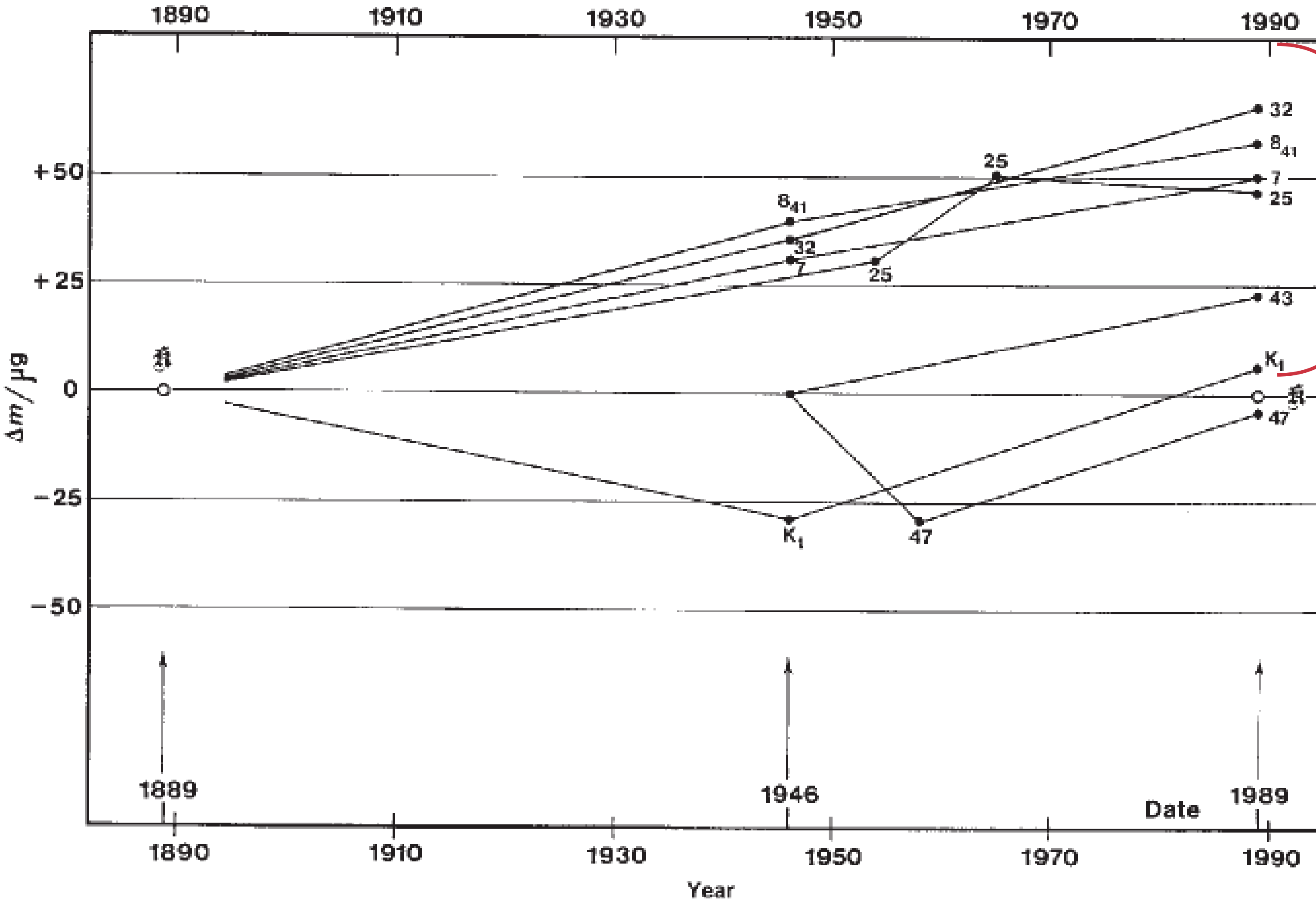
1889: -39 µg
1950: -19 µg
1990: -21 µg

Let's look at washing



The cleaning procedure takes 1 μg/year for each year since the last cleaning procedure occurred.

How stable is the kilogram?



The mass of 6 out of 7 has in-creased

Shortcomings

- Can be damaged or destroyed.
- Collects dirt from the ambient atmosphere.
- Cannot be used routinely for fear of wear.
- IPK changes by 50µg/100yrs relative to the ensemble of PtIr standards.
- The drift of the world wide ensembles of PtIr standards is unknown at a level of 1mg/100yrs.
- Can only be accessed at the BIPM.
- Cannot be communicated to extraterrestrial intelligence.

The kilogram influences other units

derived unit

base units

$$1\,\mathrm{V} = 1\,\frac{\mathrm{J}}{\mathrm{A\,s}} = 1\,\frac{\mathrm{kg\,m^2}}{\mathrm{A\,s^3}}$$

Two advances in metrology in “recent years”

1. Josephson Effect

Josephson Junction (JJ) array

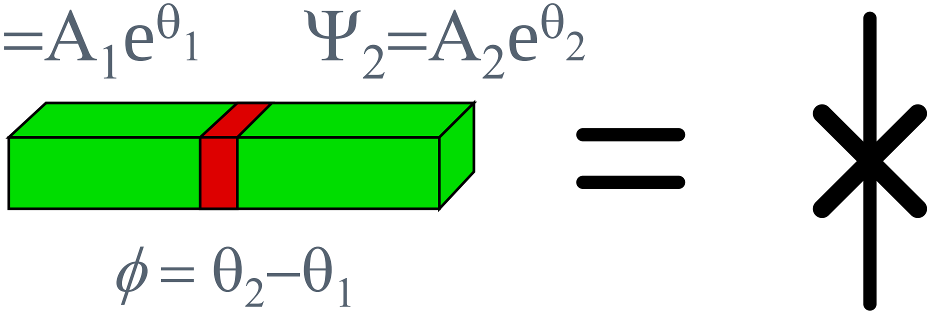
2. Integral Quantum Hall Effect

Quantum Hall Resistor

Josephson Junction

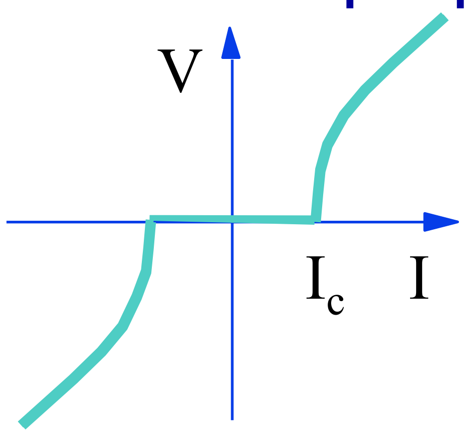
- Weak link between two superconductors

$\Psi_1 = A_1 e^{\theta_1}$ $\Psi_2 = A_2 e^{\theta_2}$

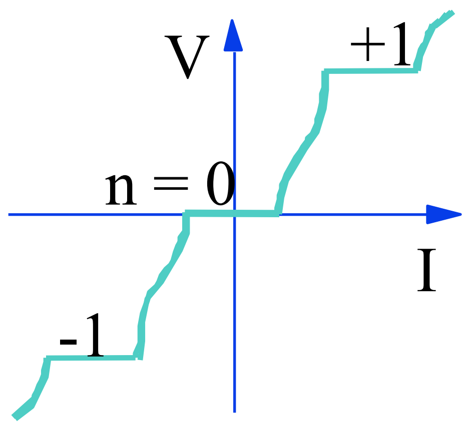


$\phi = \theta_2 - \theta_1$

- Electrical properties



DC only

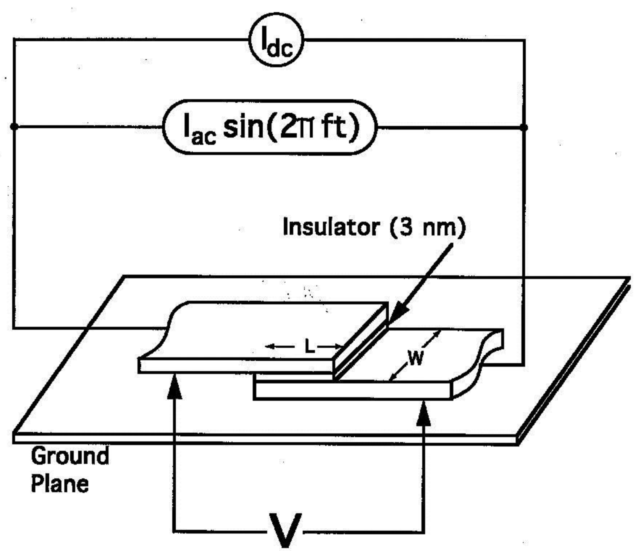


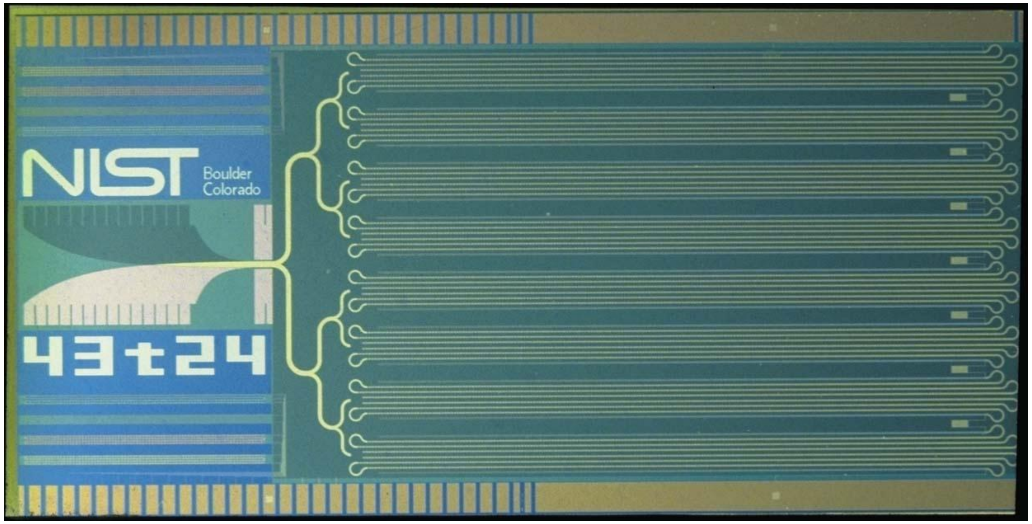
with AC

$$V = \frac{h}{2e}\frac{1}{2\pi}\frac{d\phi}{dt}$$

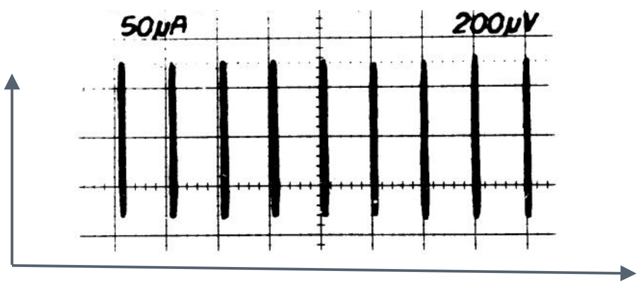
$$or, \quad V = \frac{h}{2e} f$$

JJ array





19 mm



Voltage

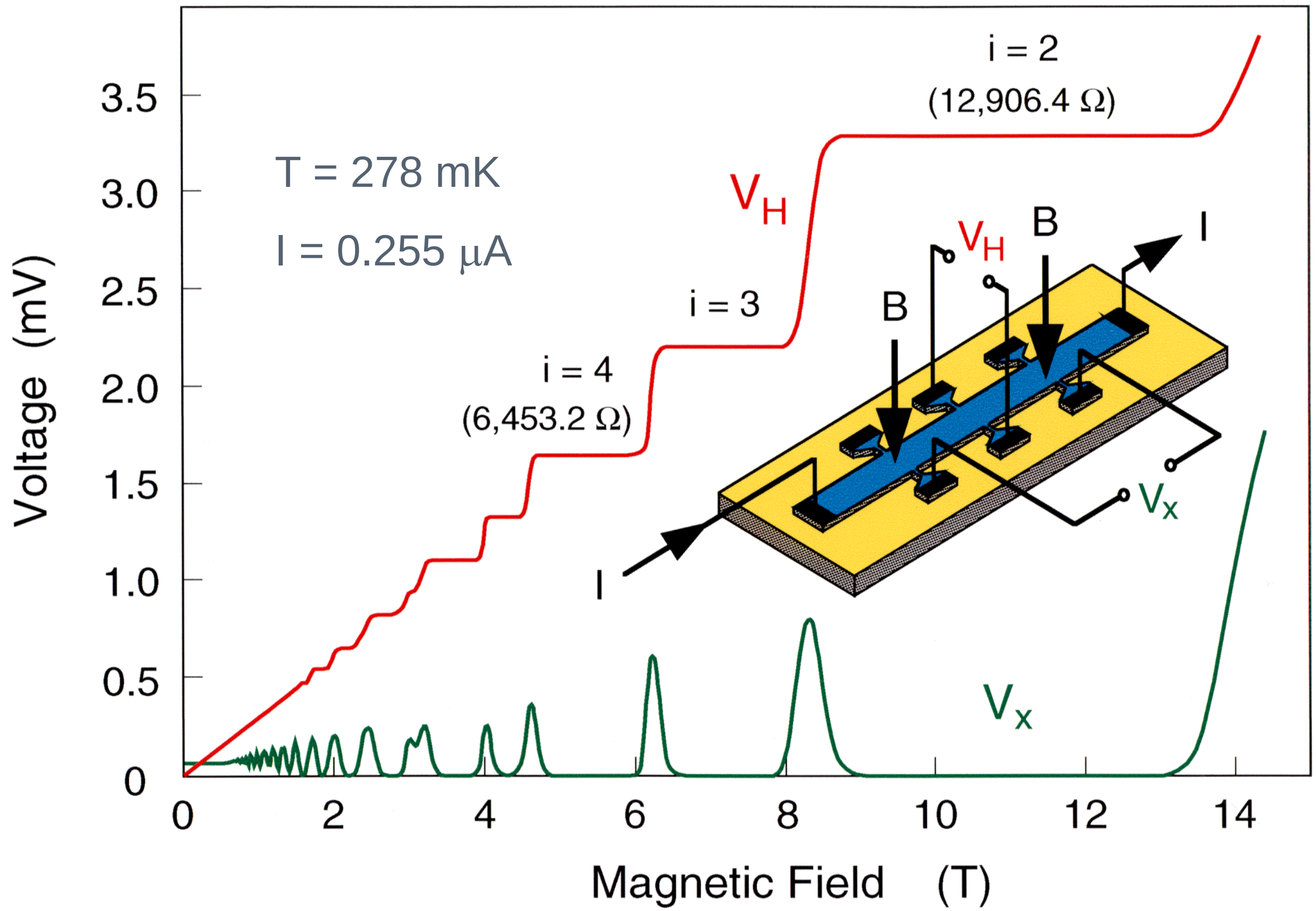
20208 Junctions
-14 to +14 Volts
typical: f=14..75 GHz

Quantum Hall Effect

$$R_H = \frac{V_H}{I} = \frac{B}{eN_s}$$

$$N_s = \left(\frac{eB}{h}\right) i$$

$$R_H = \left(\frac{h}{ie^2}\right)$$



Two new constants

- Josephson constant

$$K_J = \frac{2e}{h} =$$

$$(483597.870 \pm 0.011)\frac{\text{GHz}}{\text{V}}$$

- von Klitzing constant

$$R_K = \frac{h}{e^2} =$$

$$(25812.8074434 \pm 0.0000084)\,\Omega$$

$$K_{J-90} =$$

$$483597.9\,\frac{\text{GHz}}{\text{V}}$$

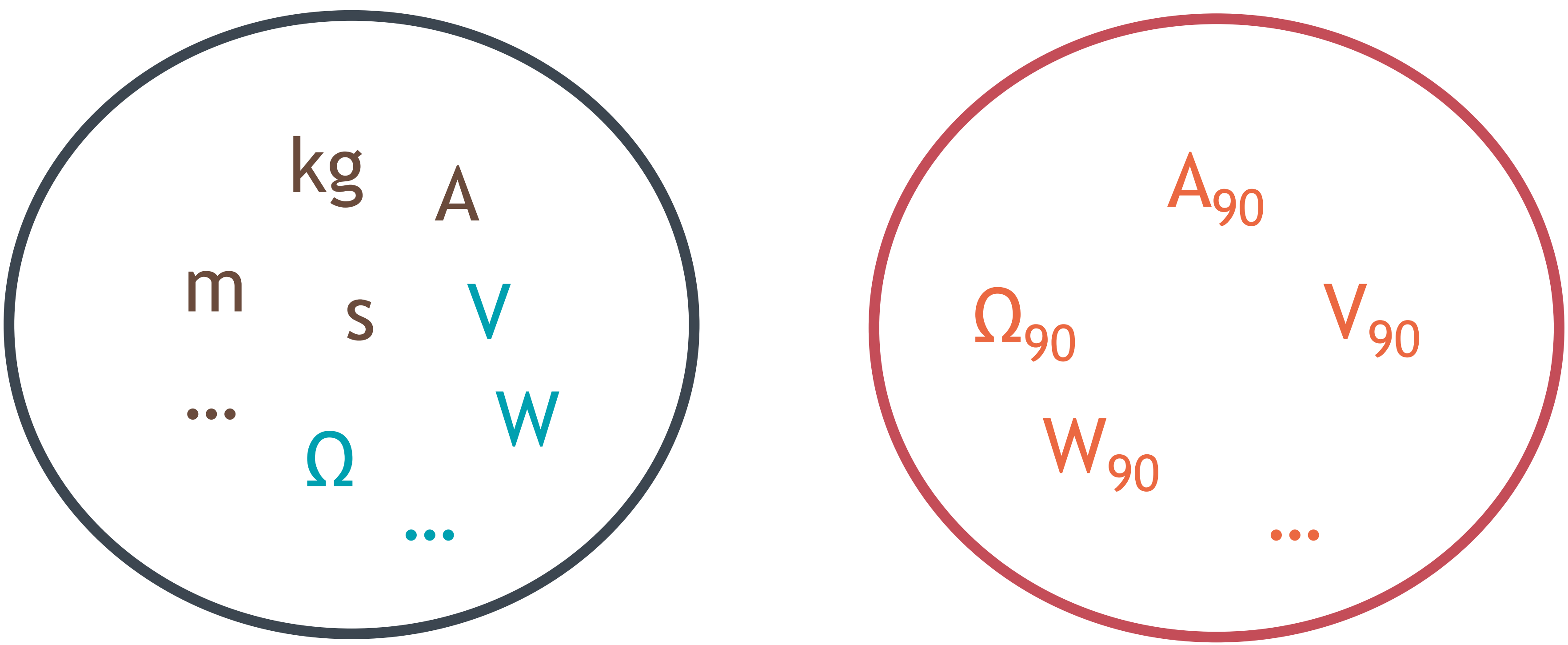
$$R_{K-90} =$$

$$25812.807\,\Omega$$

A schism in metrology

SI units
(base and derived)

conventional units



Difference between SI and 90 units

$$1\ \text{V} = (1 - 62.0 \times 10^{-9} \pm 22.7 \times 10^{-9})\ \text{V}_{90}$$

$$1\ \Omega = (1 - 17.2 \times 10^{-9} \pm 0.3 \times 10^{-9})\ \Omega_{90}$$

$$1\ \text{A} = (1 - 44.9 \times 10^{-9} \pm 22.7 \times 10^{-9})\ \text{A}_{90}$$

$$1\ \text{C} = (1 - 44.9 \times 10^{-9} \pm 22.7 \times 10^{-9})\ \text{C}_{90}$$

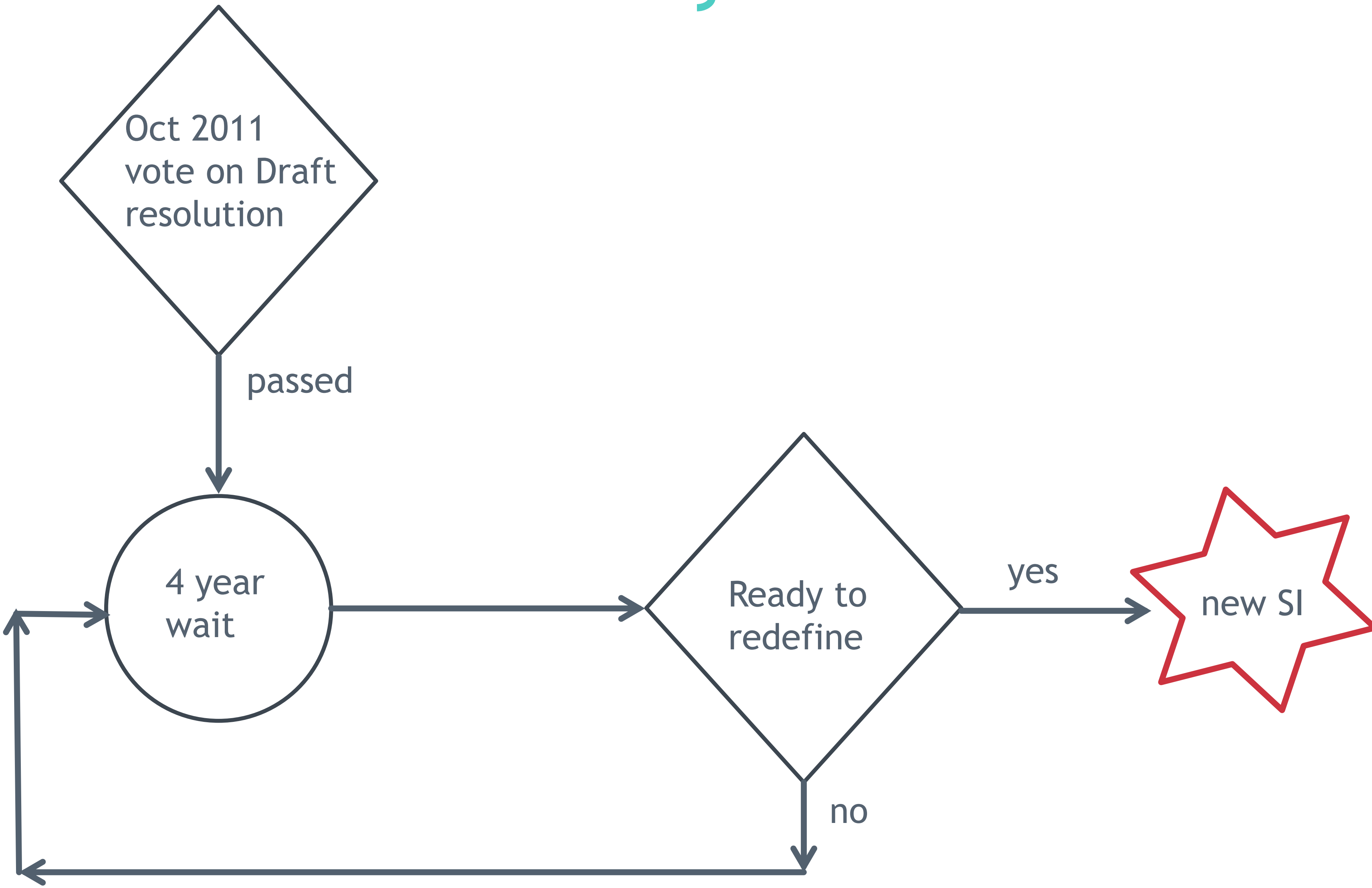
$$1\ \text{W} = (1 - 106.9 \times 10^{-9} \pm 45.5 \times 10^{-9})\ \text{W}_{90}$$

$$1\ \text{F} = (1 + 17.2 \times 10^{-9} \pm 0.3 \times 10^{-9})\ \text{F}_{90}$$

$$1\ \text{H} = (1 - 17.2 \times 10^{-9} \pm 0.3 \times 10^{-9})\ \text{H}_{90}$$

http://physics.nist.gov/cuu/Constants/index.html

Redefinition by committee



The current SI

m	The meter is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second
kg	The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.
s	The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.
A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per meter of length.
K	The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
mol	The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12; its symbol is “mol.”
cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 × 1012 hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

The "new" SI

Based on seven reference constants.

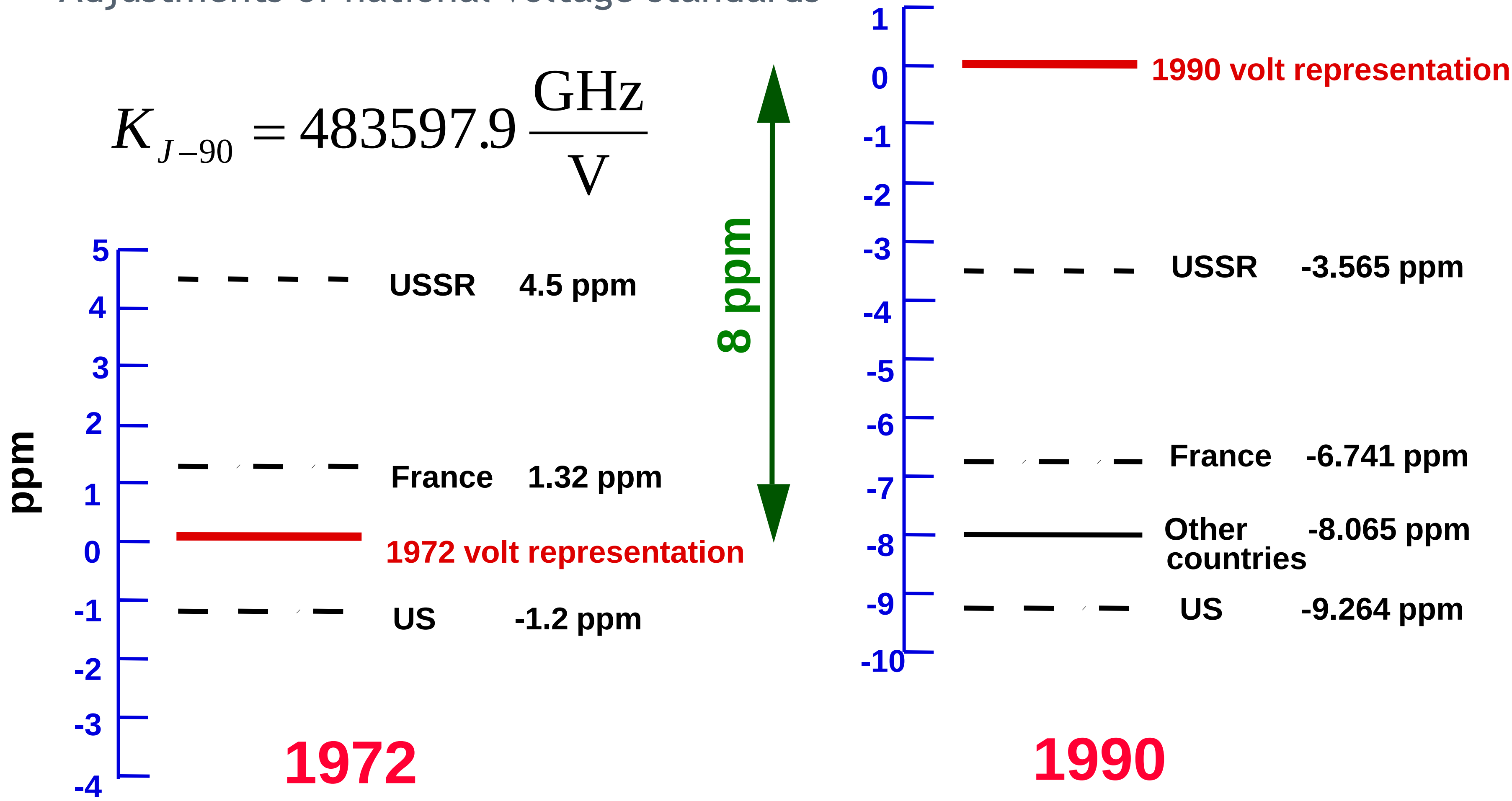
$\Delta\nu(^{133}Cs)$	ground state hfs	9.192 631 770	$\cdot 10^{9}$	s^{-1}
c	speed of light	2.99 792 458	$\cdot 10^{8}$	ms^{-1}
h	Planck's constant	6.626 069 57*	$\cdot 10^{-34}$	$kg\ m^{2}\ s^{-1}$
e	elementary charge	1.602 176 565	$\cdot 10^{-19}$	As
k	Boltzmann constant	1.380 6448	$\cdot 10^{-23}$	$kg\ m^{2}\ s^{-1}K^{-1}$
N_A	Avogadro constant	6.022 141 29	$\cdot 10^{23}$	mol^{-1}
K_{cd}	luminous efficacy	6.83	$\cdot 10^{2}$	$lm\ kg^{-1}\ m^{-2}\ s^{3}$

* The exact numerical value will be determined by CODATA at the time of the redefiniton. The above values are today's CODATA values.

Redefinitions do happen

Adjustments of national voltage standards

$$K_{J-90} = 483597.9 \frac{\text{GHz}}{\text{V}}$$



B.Taylor and T.Witt, "New International Electrical Reference Standards Based on the Josephson and Quantum Hall Effects," Metrologia, 26, 47-62 (1989)

Outline

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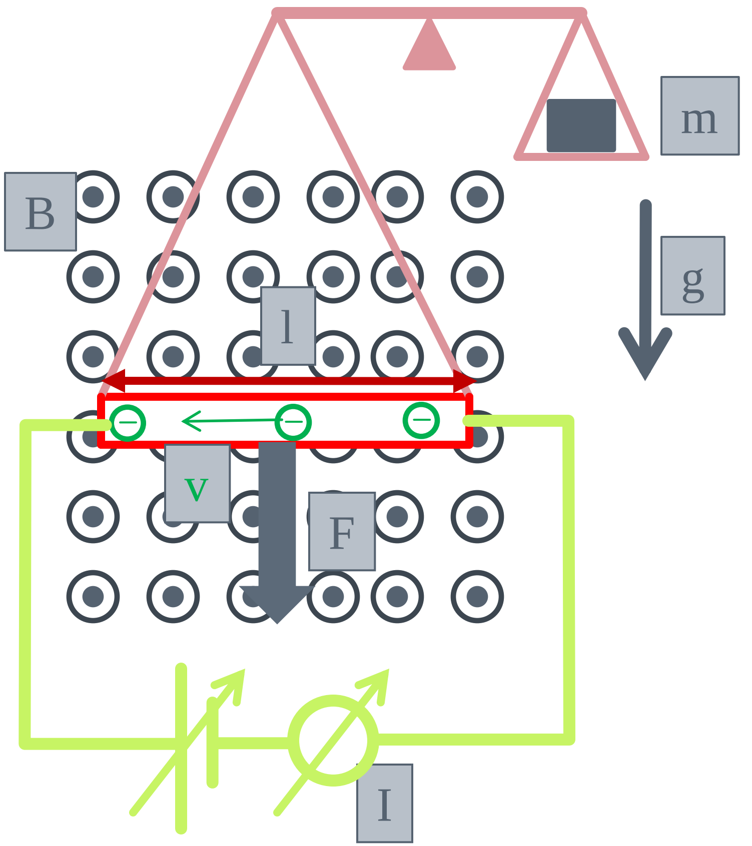
2. The principle of the Watt balance

3. The past, the present, & the future of the Ekg

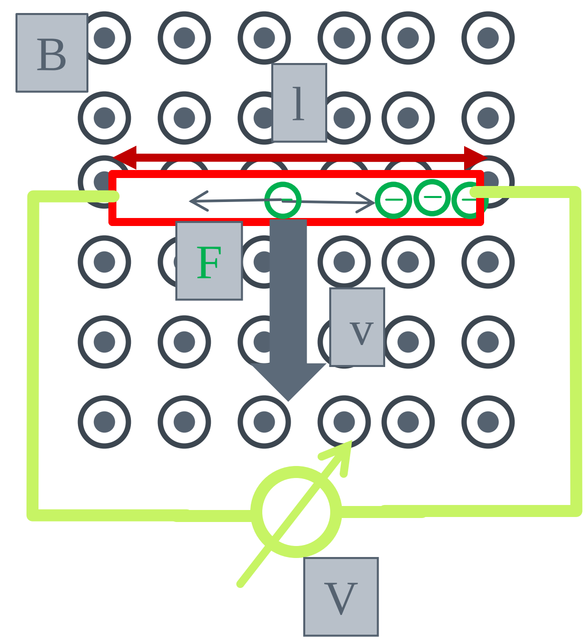
The principle of the Watt balance

force mode



$$mg = -NevB = IlB \;\Rightarrow\; lB = \frac{mg}{I}$$

velocity mode



$$evB = eE = e\frac{V}{l} \;\Rightarrow\; lB = \frac{V}{v}$$

The Watt equation

$$mg\ v = V\ I$$

The Watt equation

$$mg\,v = V\,I$$

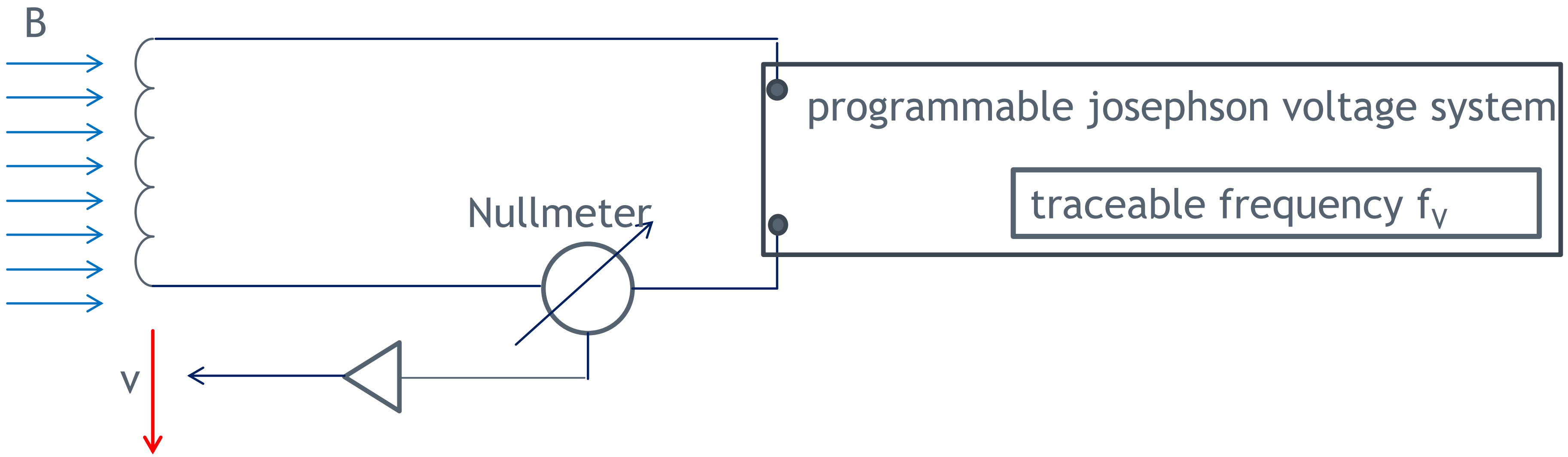
force mode

velocity mode

Virtual power !

Connection to Planck's constant

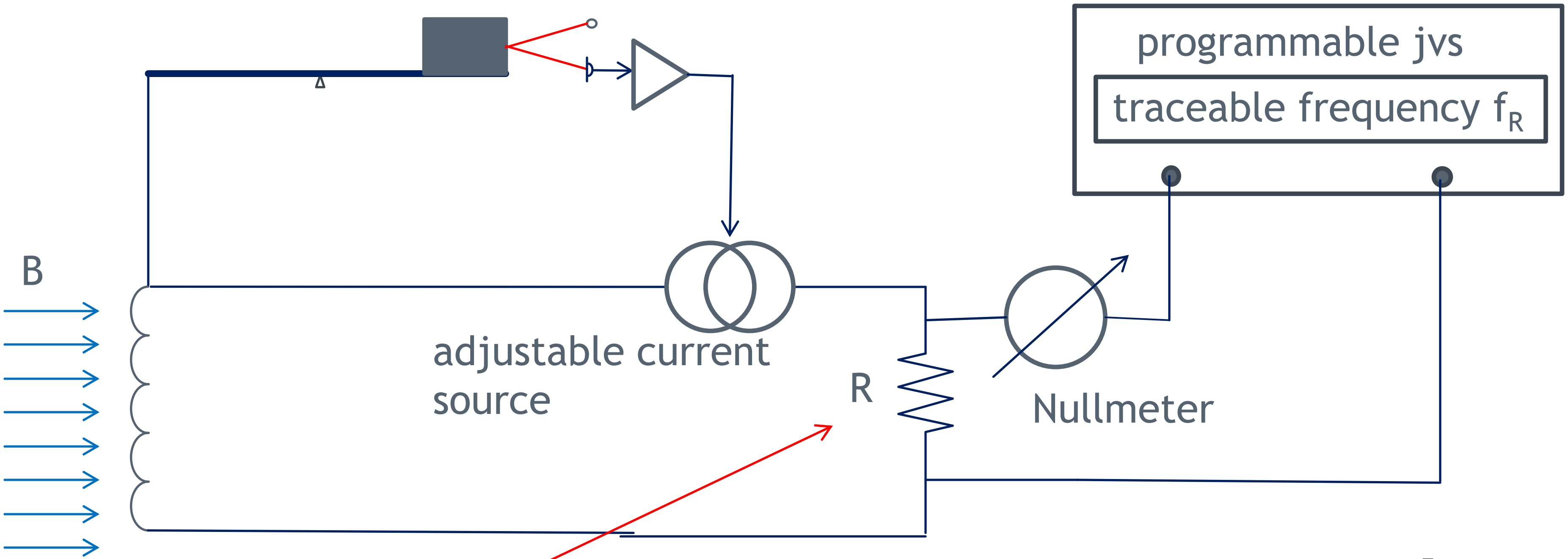
velocity mode



$$V = c_V f_V \frac{h}{2e} \qquad \frac{h}{2e} = 2.066..\frac{\mu V}{GHz}$$

Connection to Planck's constant

force mode



R is calibrated against a Quantum Hall Resistor R_K

$$R = c_R R_K = c_R \frac{h}{e^2}$$

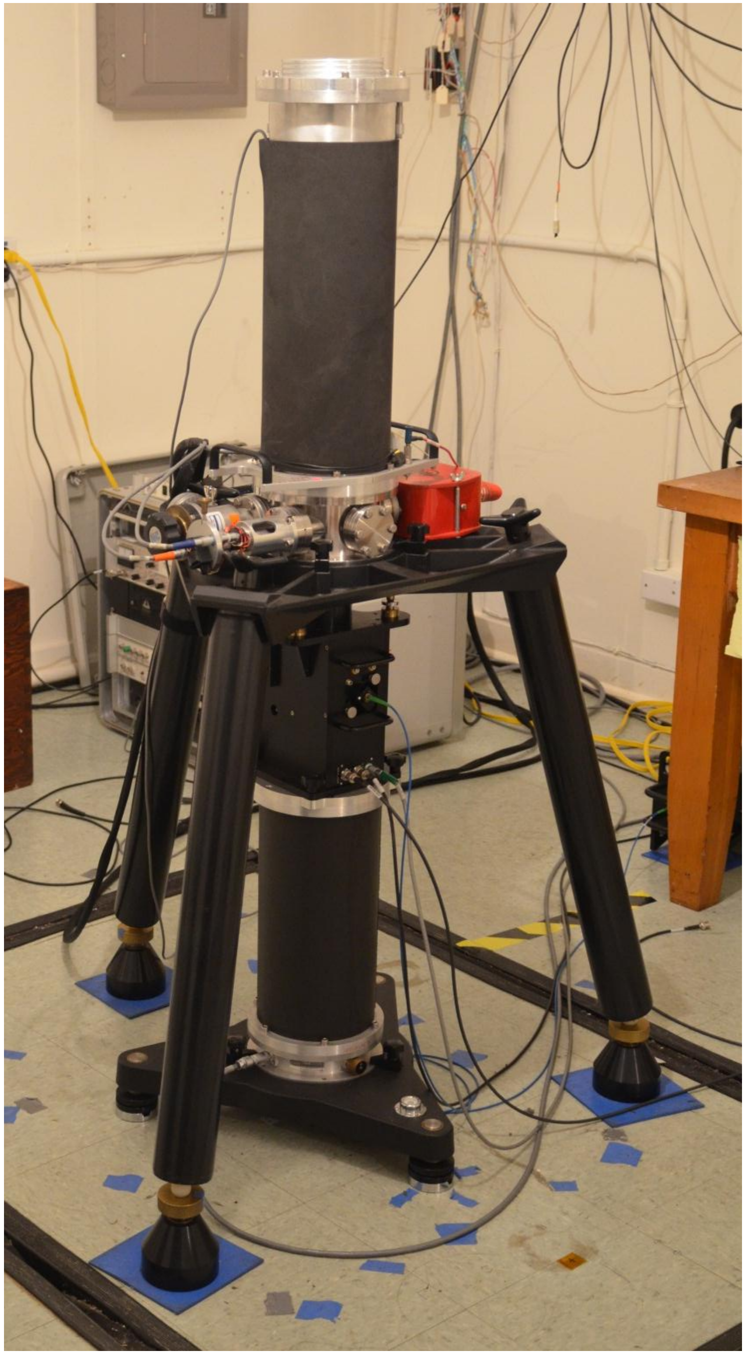
$$I = \frac{V_R}{R} = \frac{c_{RV} f_R \frac{h}{2e}}{c_R \frac{h}{e^2}}$$

Combining the two modes

$$V\,I = V\frac{V_R}{R} = \frac{c_V\, f_V\, h}{2e}\,\frac{c_{RV}\, f_R\, \frac{h}{2e}}{c_R\, \frac{h}{e^2}} = \frac{c_V\, c_{RV}}{c_R\, 4}\, f_R\, f_V\, h$$

$$m = \frac{c_V\, c_{RV}}{4c_R}\,\frac{f_v\, f_{VR}}{gv}\, h$$

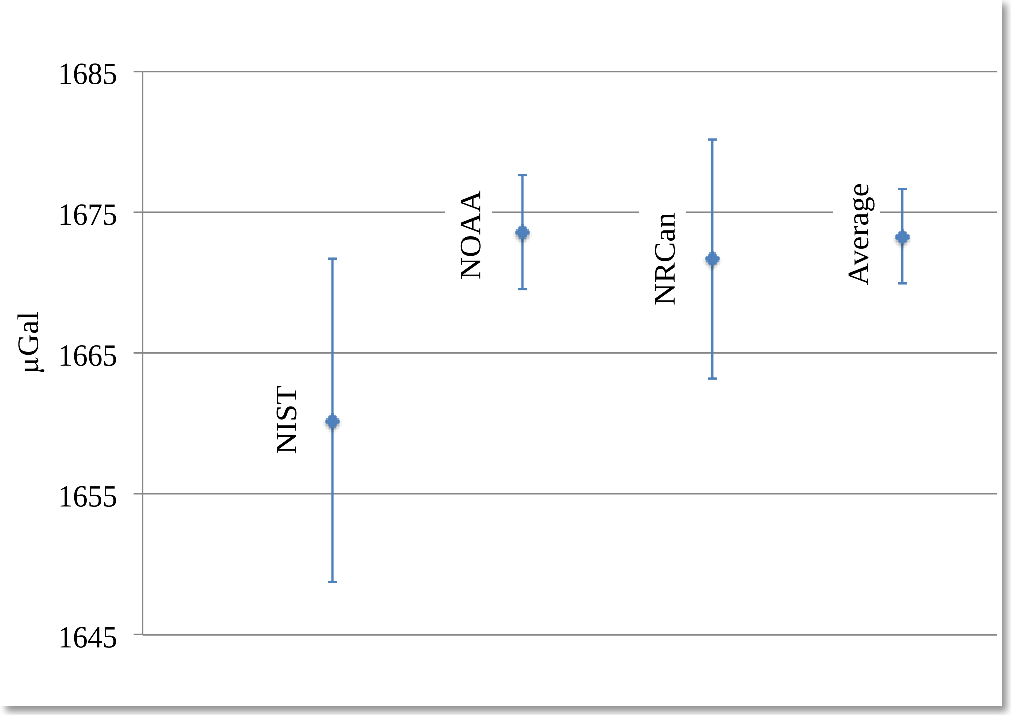
Mass can be defined in terms of Planck's constant.



Gravity

Absolute gravimeter,
can measure
g to 1 ppb

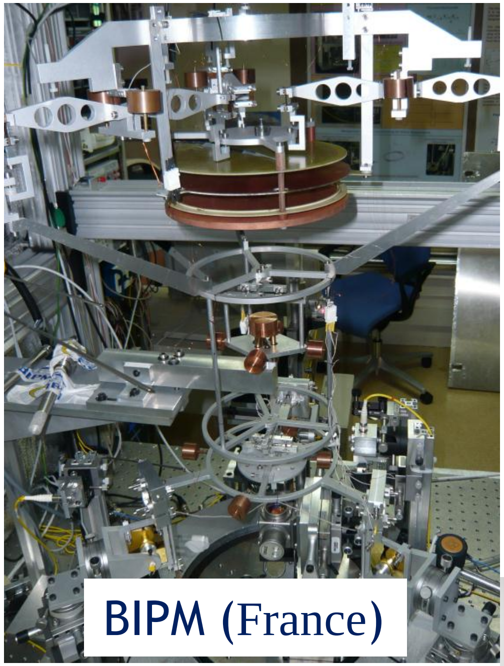
Gravity

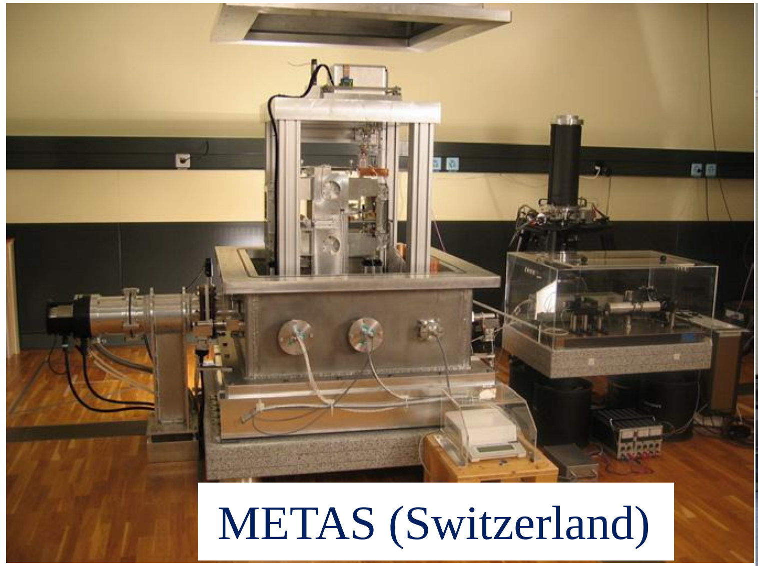


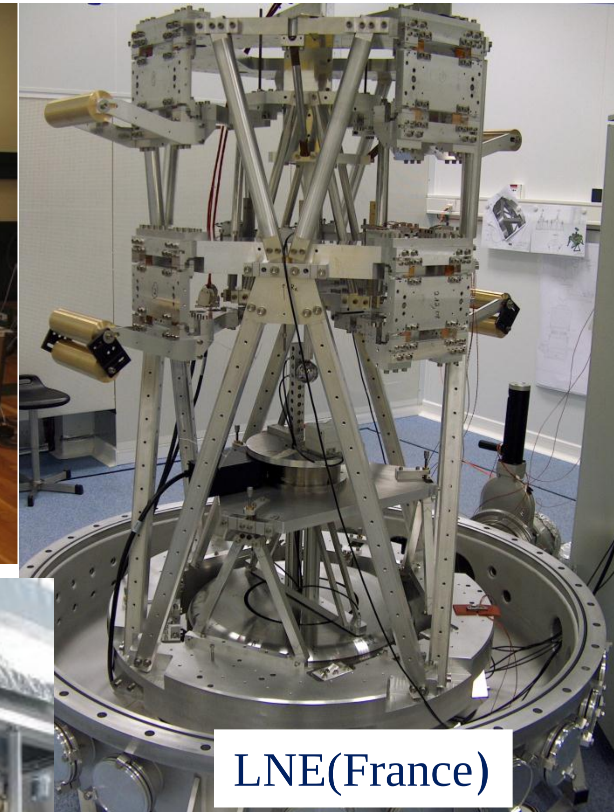
2012 North American Watt Balance Absolute Gravity Comparison (NAWBAG-2012)

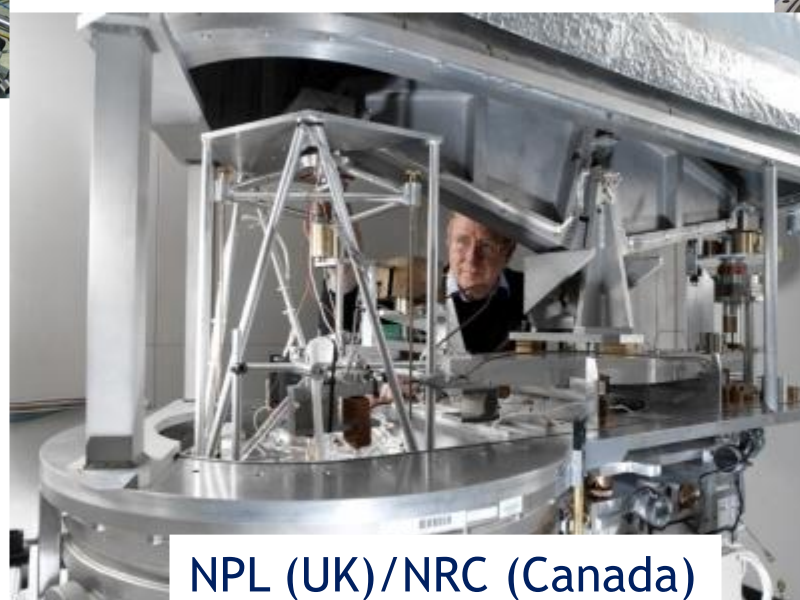
Feb6-Feb10 2012

Worldwide Watt Balances (WWB)


BIPM (France)


METAS (Switzerland)


LNE(France)


NPL (UK)/NRC (Canada)

NIM (China)
MSL (New Zealand)
KRISS (South Korea)
PTB (Germany) ?

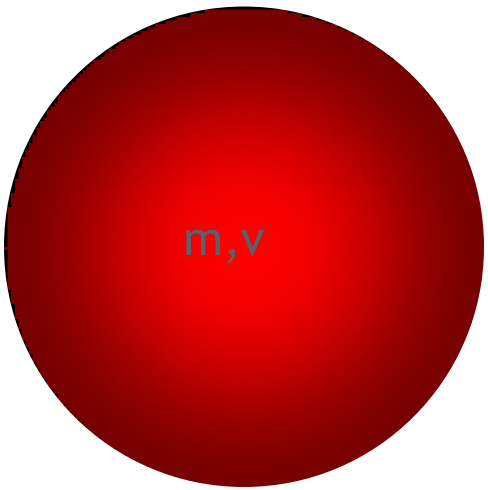
A digression… … …

- There is another way to measure *h*
- The International Avogadro Project

PTB, Germany
NMIJ, Japan
NMI, Australia
METAS, Switzerland
NIST, US
INRIM, Italy
BIPM
IRMM, Belgium

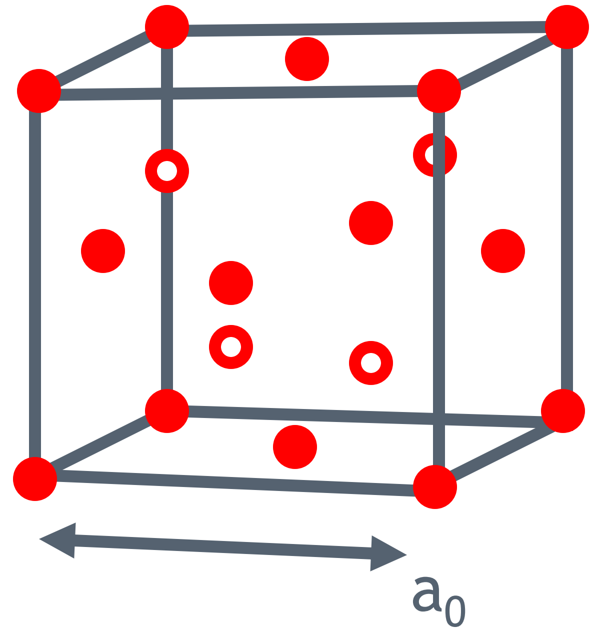
Si sphere (Avogadro project)

A single crystal of Si



$$\rho = \frac{m}{V} \qquad V_{mol} = \frac{M_{mol}}{\rho}$$

Unit cell (8 atoms per cell)



$$V_{uc} = a_0^3 = 8^{3/2} d_{220}^3$$

$$N_A = 8\frac{V_{mol}}{V_{uc}} = 8\frac{M_{mol} V}{m 8^{3/2} d_{220}^3}$$

Measurements needed

physics and chemistry of bulk and surface

isotopic abundance & mass spectroscopy

volume determination

$$N_A = 8\frac{V_{mol}}{V_{uc}} = 8\frac{M_{mol}V}{m8^{3/2}d_{220}^{3}}$$

XRD

mass metrology

Measurements needed

- physics and chemistry of bulk and surface
- isotopic abundance & mass spectroscopy
- volume determination

$$N_A = 8\frac{V_{mol}}{V_{uc}} = 8\frac{M_{mol}V}{m8^{3/2}d_{220}^3}$$

- mass metrology
- XRD

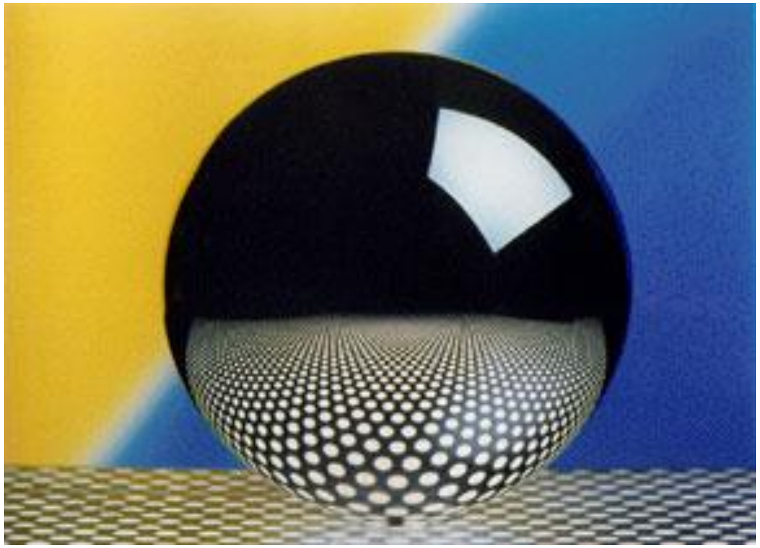
Measurement completed in 2003

Largest contribution to the uncertainty:
measurement of the isotopic abundance

Solution: Use enriched Si.

Pictures

5kg ^{28}Si crystal grown
by the float zoning method



Si sphere

Connection between *h* and N_A

$$R_\infty = \frac{m_e c \alpha^2}{2h}$$

Connection between *h* and N_A

$$R_\infty = \frac{m_e c \alpha^2}{2h} = \frac{m_e c \alpha^2 \overbrace{M_p / N_A}^{m_p}}{2h m_p}$$

Connection between *h* and N_A

$$R_\infty = \frac{m_e c\alpha^2}{2h} = \frac{m_e c\alpha^2 \overbrace{M_p / N_A}^{m_p}}{2hm_p} = \frac{M_P c\alpha^2}{2h(m_p / m_e)N_A}$$

Connection between *h* and N_A

$5 \cdot 10^{-12}$

$8.9 \cdot 10^{-11}$

fixed

$3.2 \cdot 10^{-10}$

$$R_\infty = \frac{M_P c \alpha^2}{2h(m_p / m_e) N_A}$$

$4.1 \cdot 10^{-10}$

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The birth of the Watt balance

A MEASUREMENT OF THE GYROMAGNETIC RATIO OF THE PROTON BY THE STRONG FIELD METHOD

B. P. Kibble

Division of Electrical Science

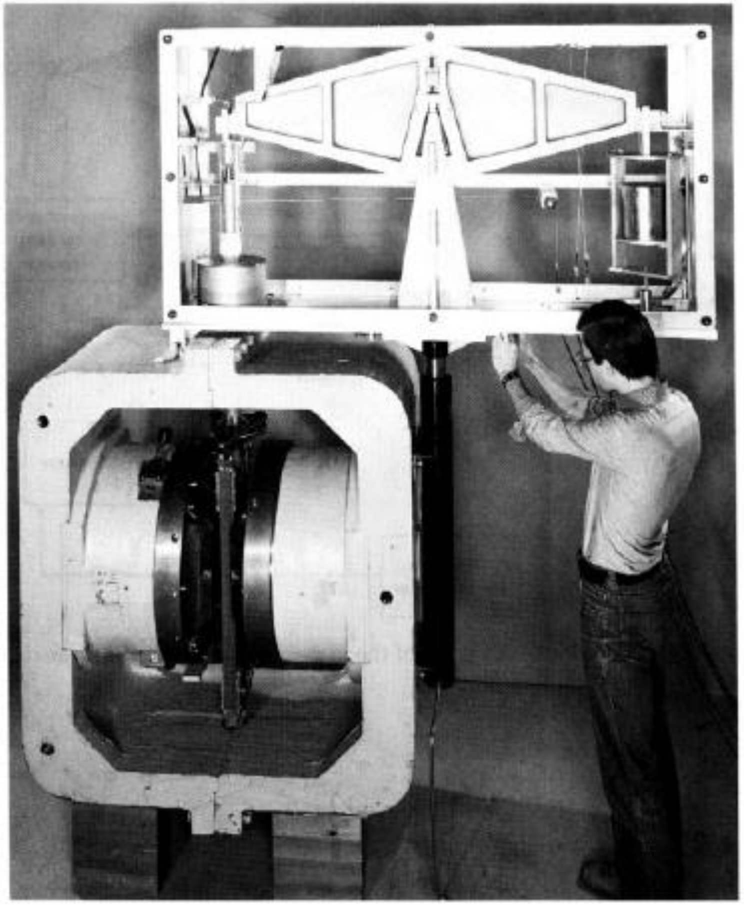
National Physical Laboratory, Teddington, England

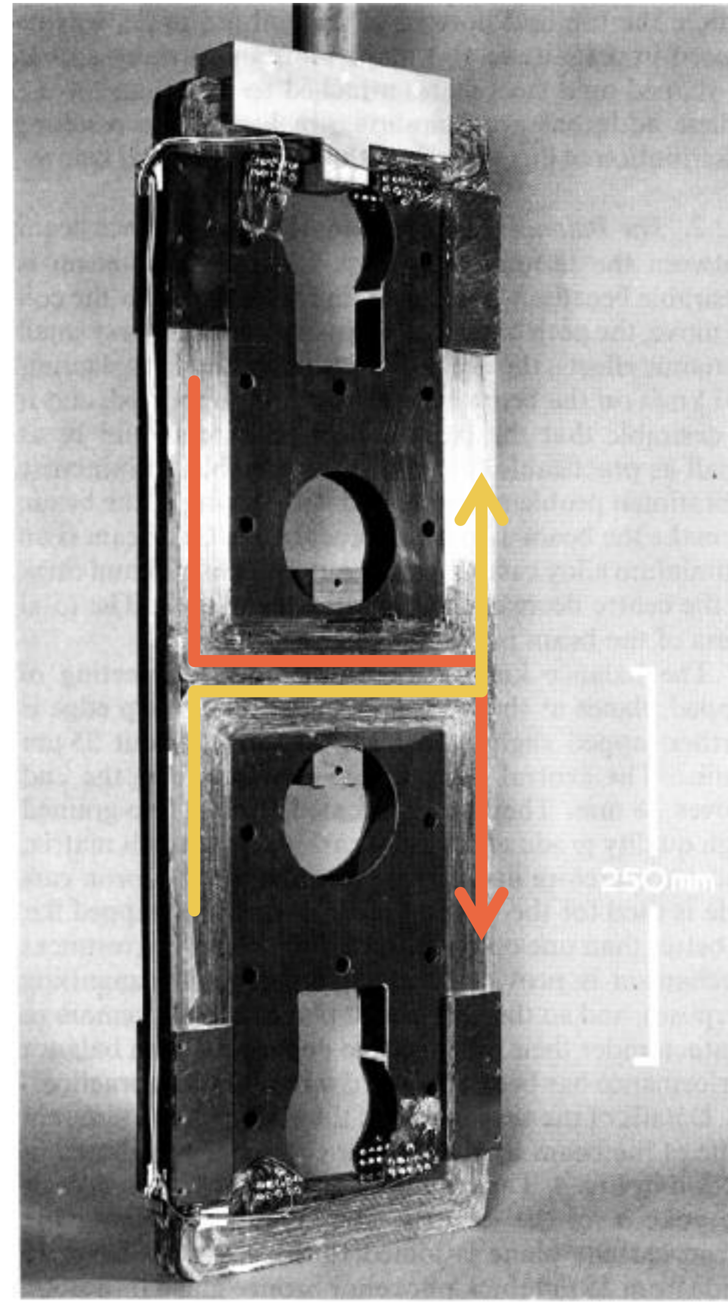
Atomic Masses and Fundamental Constants 5ed J H Sanders and A H Wapstra (New York: Plenum), p. 545-51 (1976).

4 A SUGGESTION FOR A DIFFERENT WAY OF REALISING THE AMPERE

A major aim of these measurements is to determine the ratio of the maintained ampere to the SI ampere, denoted by K, by combining the result with that of the weak field method (Cohen and Taylor 1973). We take this opportunity to draw attention to a possible way of determining K directly which needs only minor modifications to the strong field apparatus described above.

NPL-1

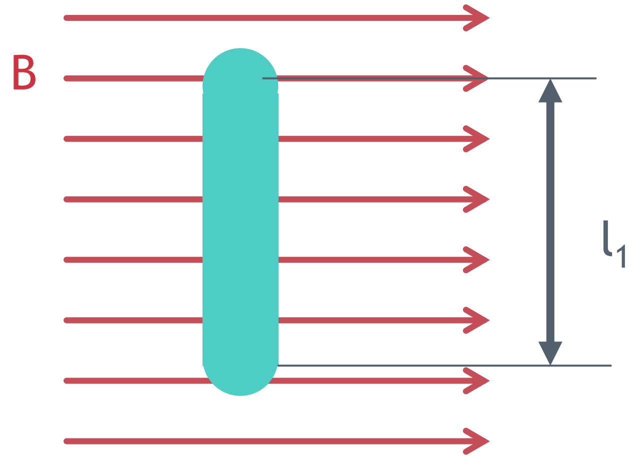




B. P. Kibble, I.A. Robinson, J.H. Beliss, metrologia **27**, 173 (1990).

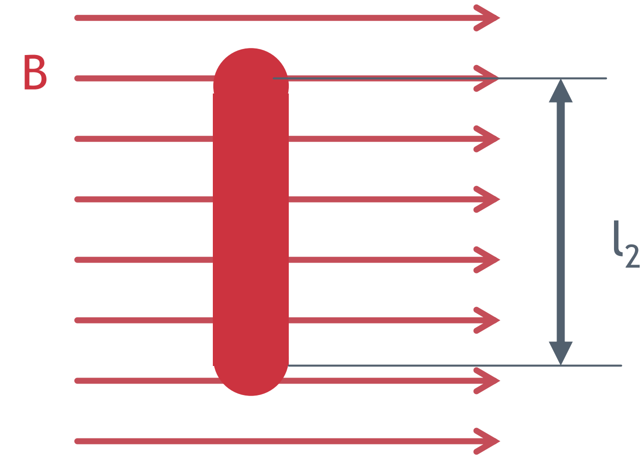
One shortcoming

velocity mode (wire cold)



$$\frac{V}{v} = N l_1 B$$

weighing mode



$$\frac{mg}{I} = l_2 B$$

ohmic heating

↓

change in temperature

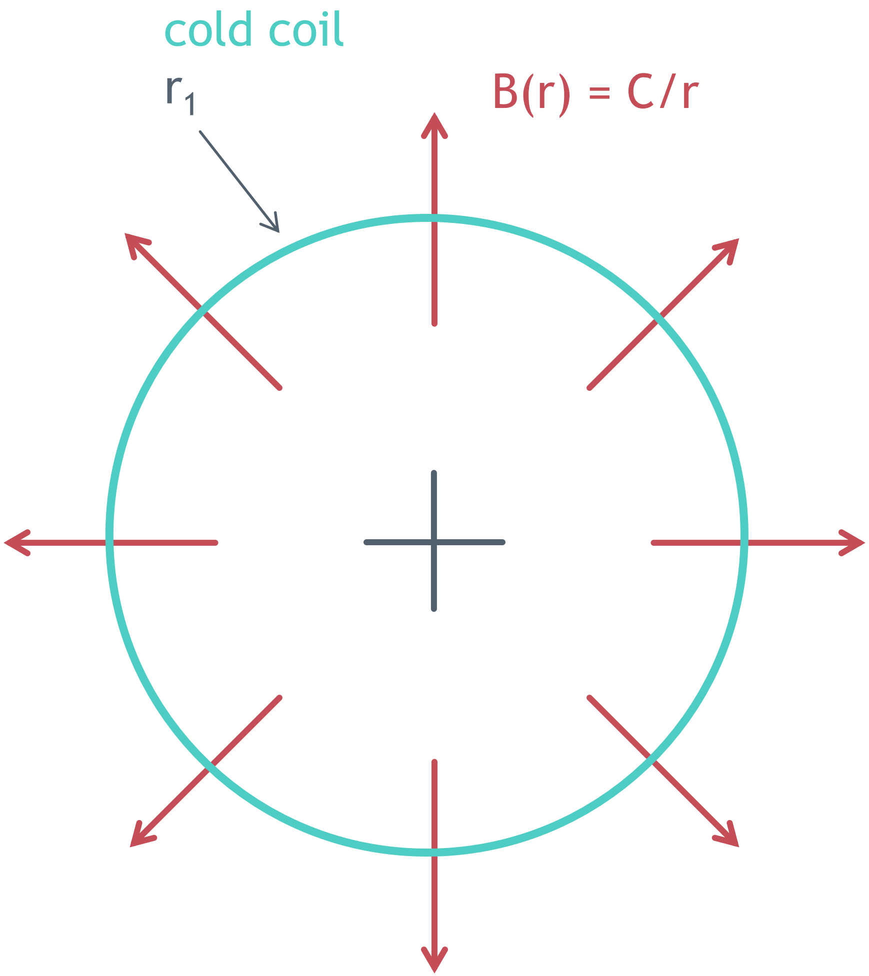
↓

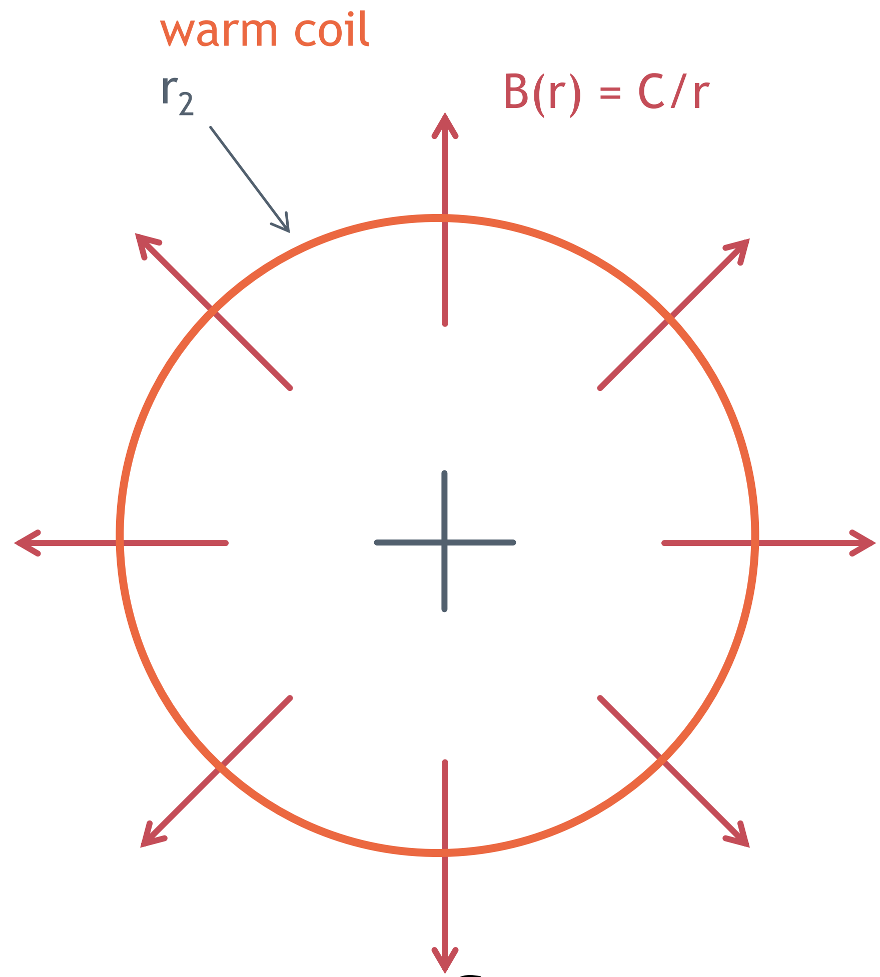
$l_2 = l_1(1+\alpha\,\Delta T)$ change in l

Copper: α=16.6 10^{-6} K^{-1}

I=10 mA, R=100 Ω

The solution (idea by P.T. Olsen)

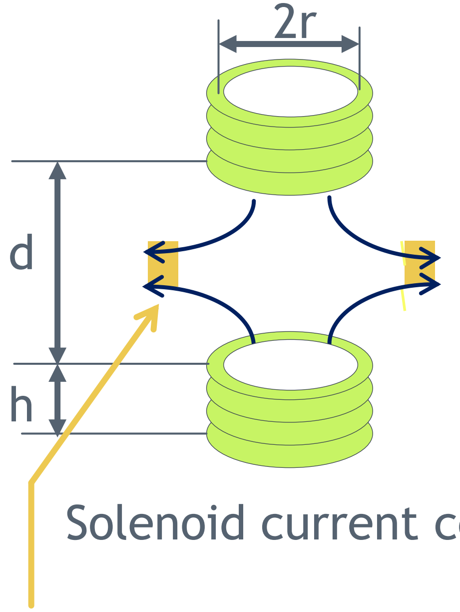




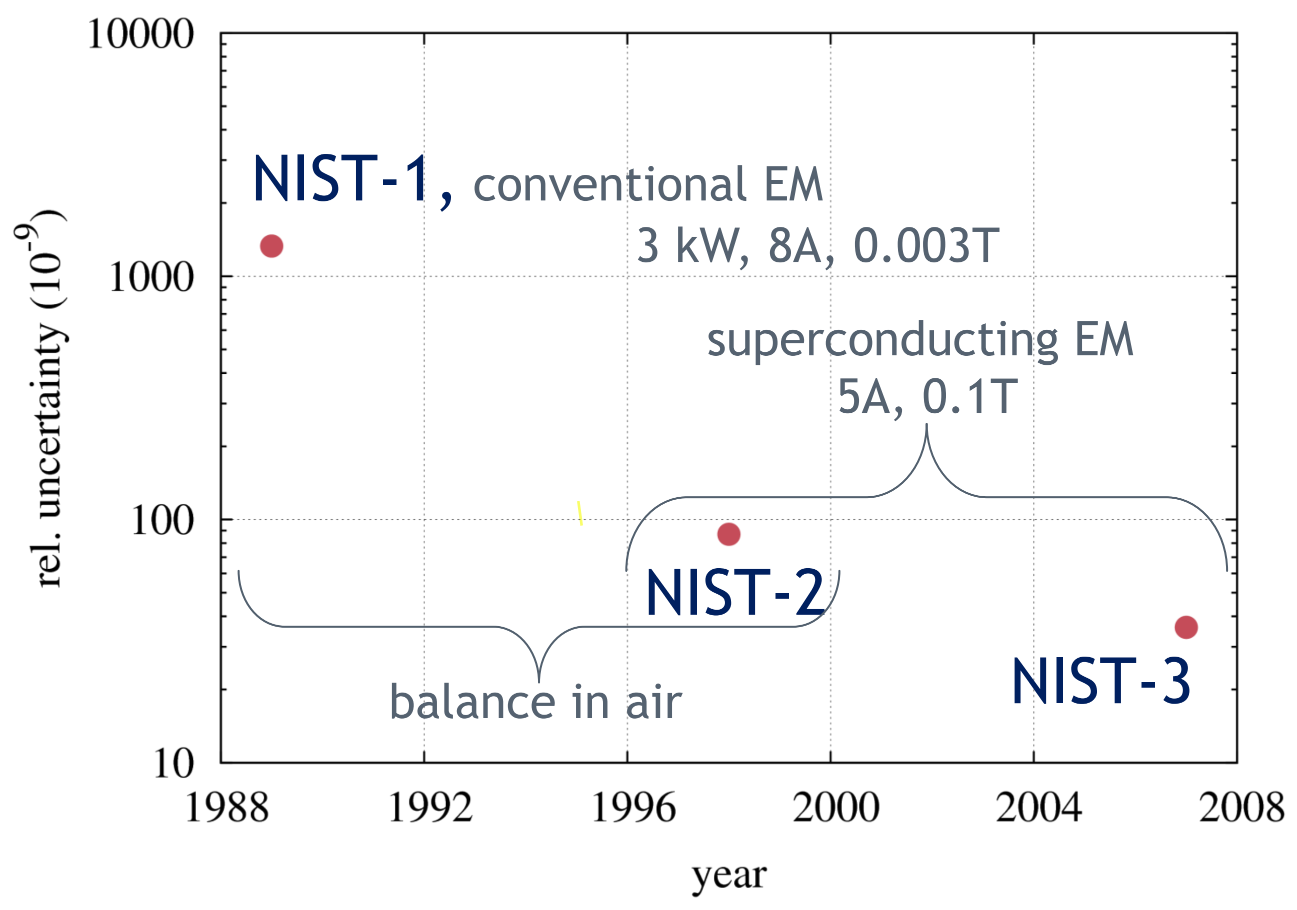
$$\frac{V}{v} = N 2\pi r_1 \frac{C}{r_1} = N 2\pi C$$

$$\frac{mg}{I} = N 2\pi r_2 \frac{C}{r_2} = N 2\pi C$$

The NIST way

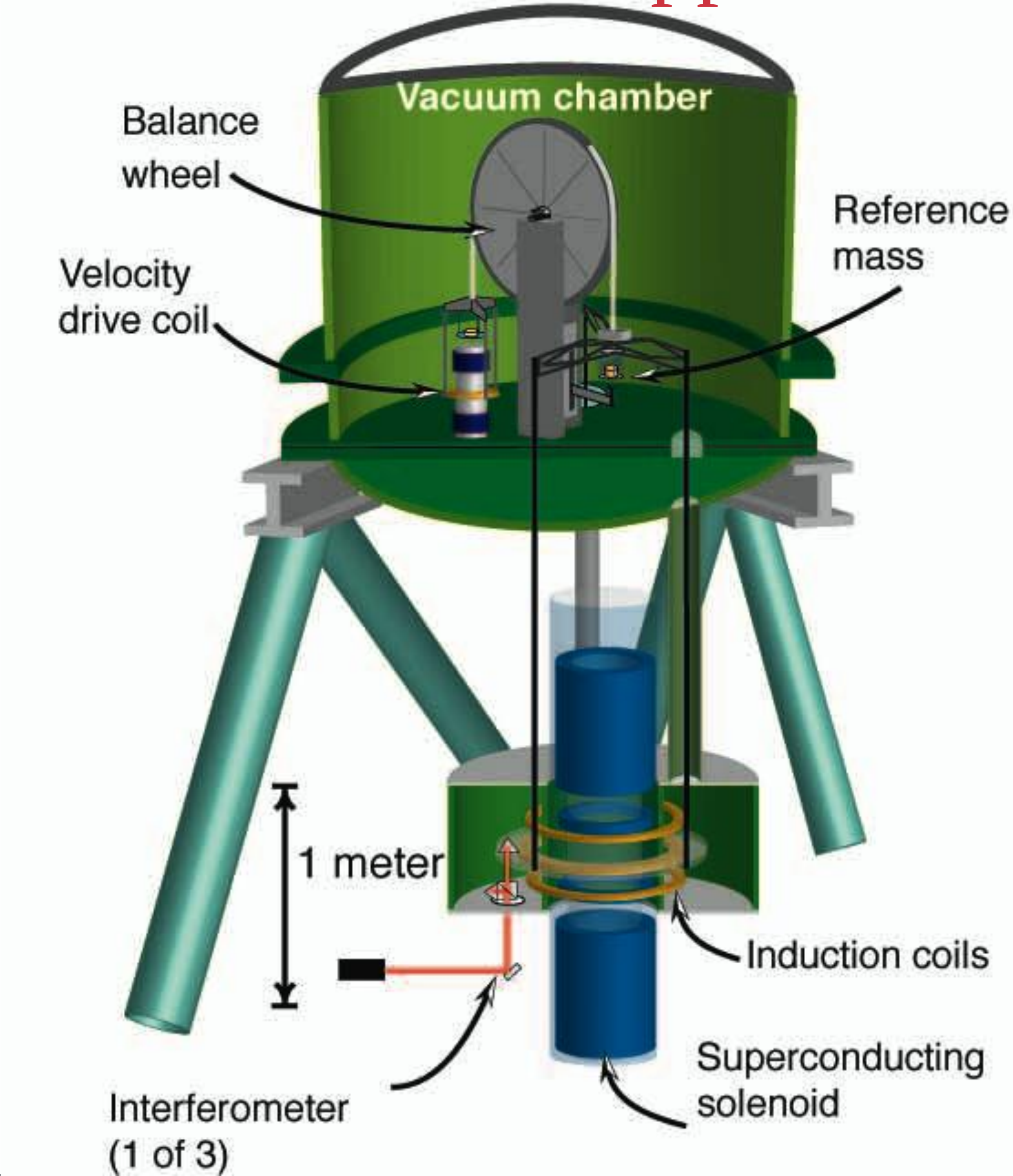


Solenoid current cw
2r
d
h
Solenoid current ccw
region, where B(r)=C/r

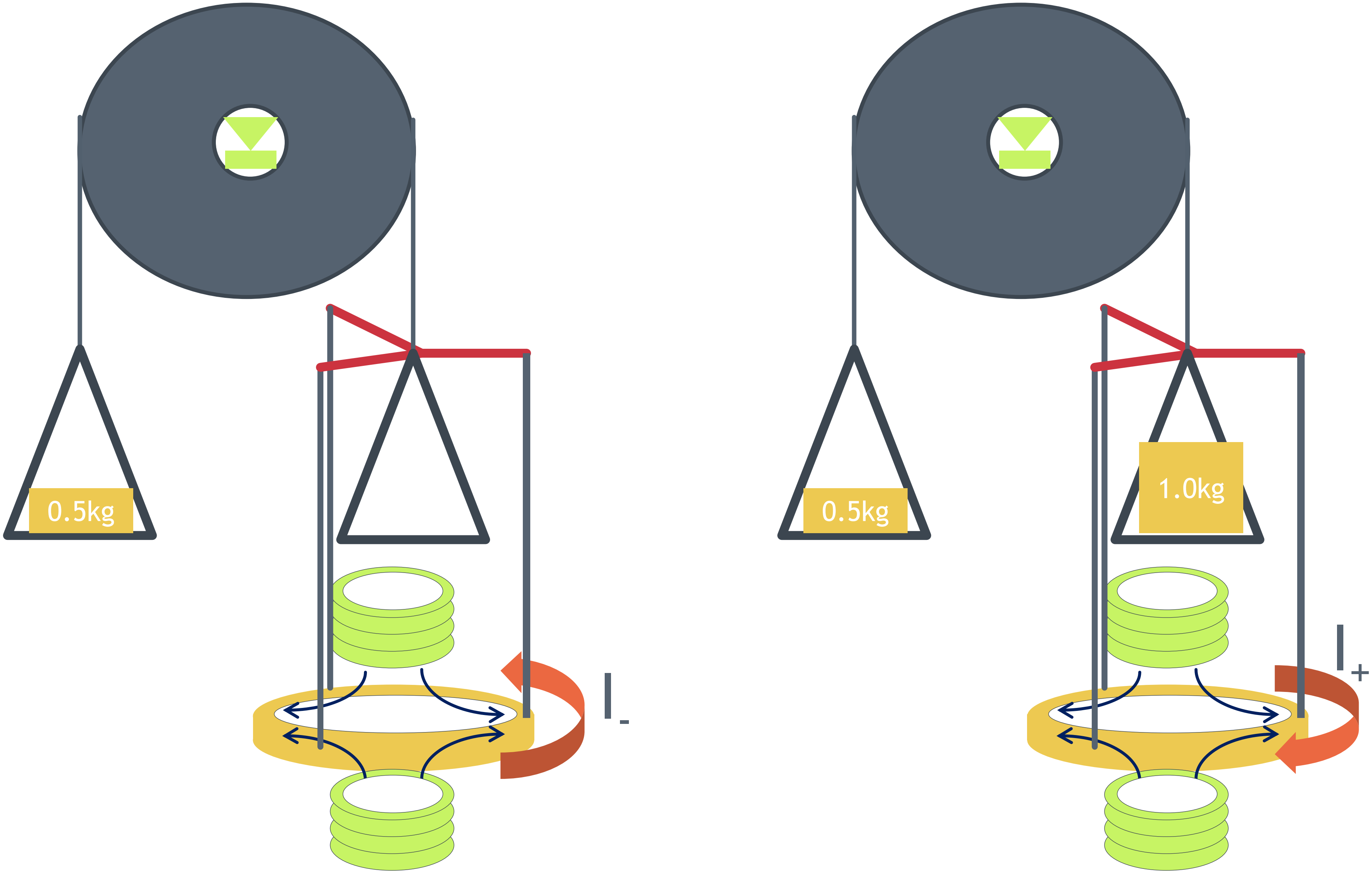


10000
1000
100
10
rel. uncertainty (10^{-9})
1988
1992
1996
2000
2004
2008
year
NIST-1, conventional EM
3 kW, 8A, 0.003T
superconducting EM
5A, 0.1T
NIST-2
NIST-3
balance in air

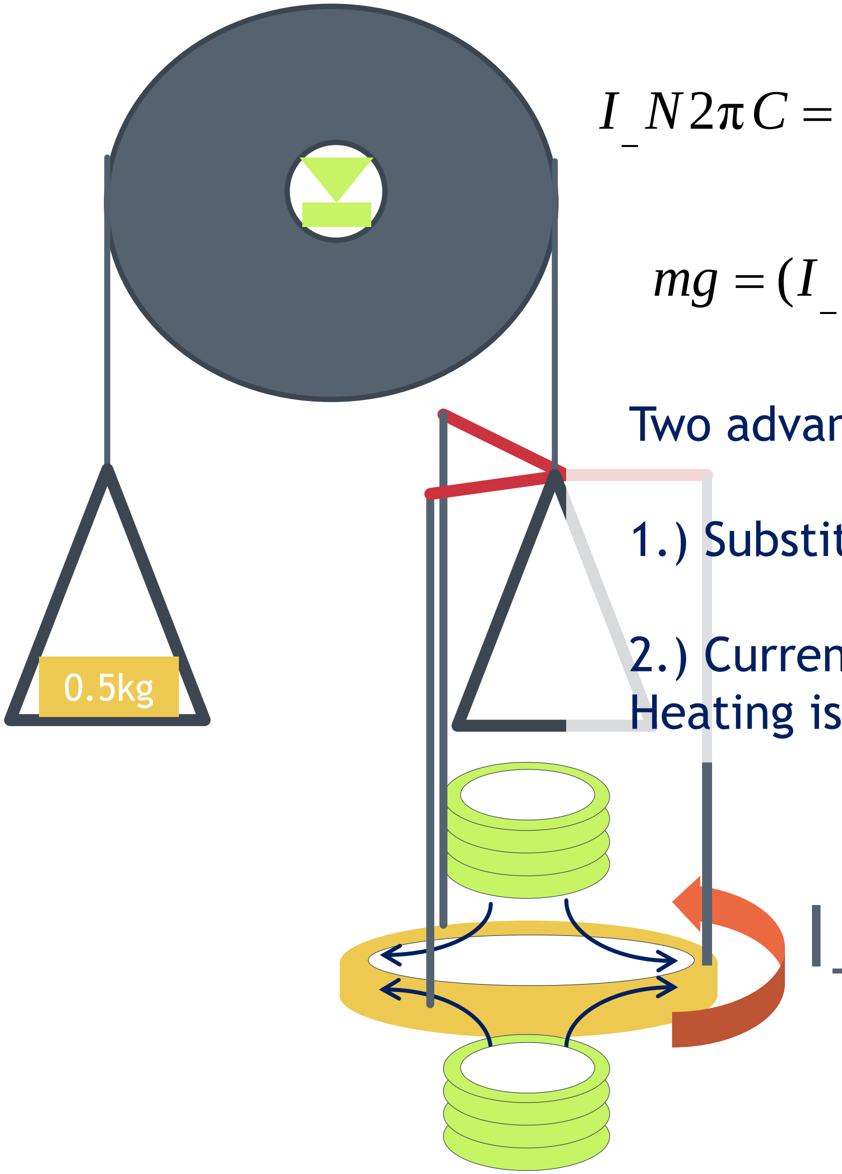
NIST-3 Apparatus



The weighing mode (in more detail)



The weighing mode (in more detail)



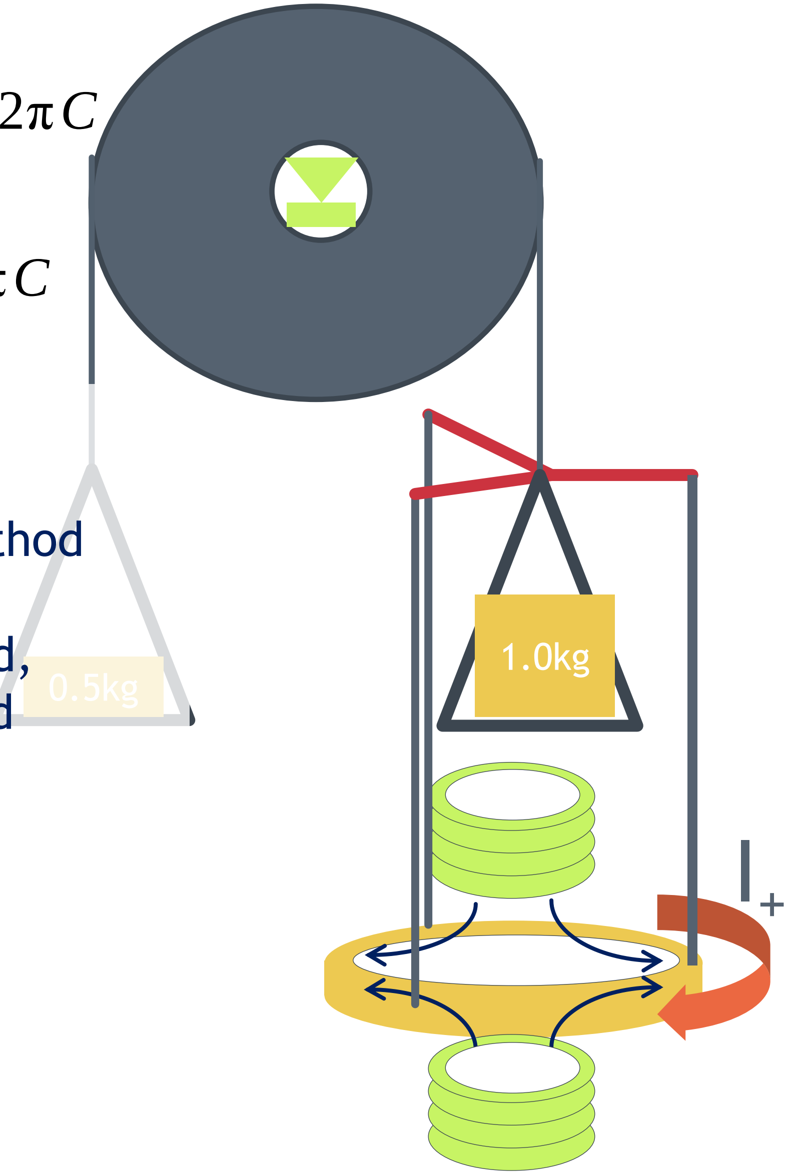
$$I_- N 2\pi C = mg - I_+ N 2\pi C$$

$$mg = (I_- + I_+) N 2\pi C$$

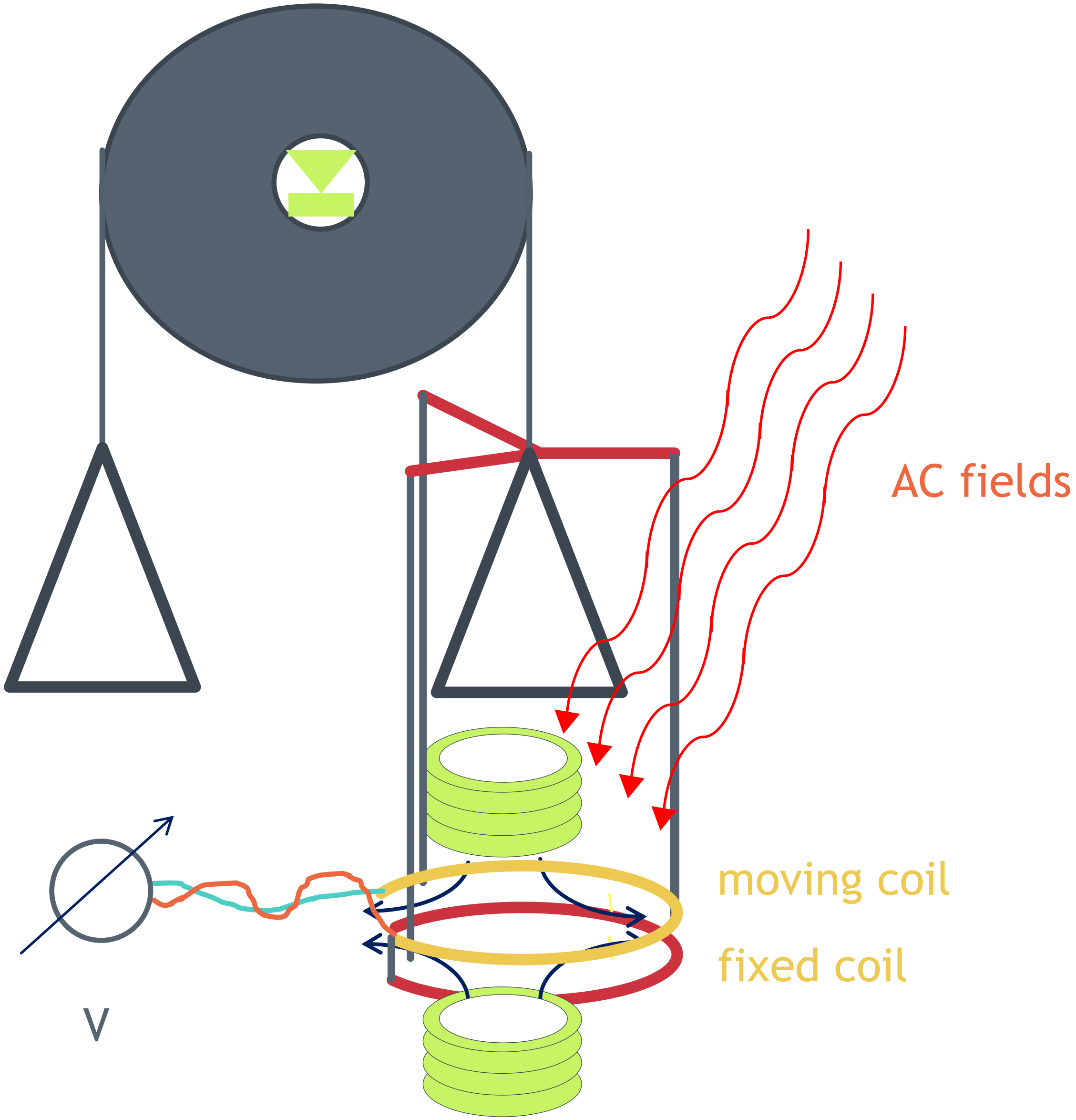
Two advantages:

1.) Substitution method

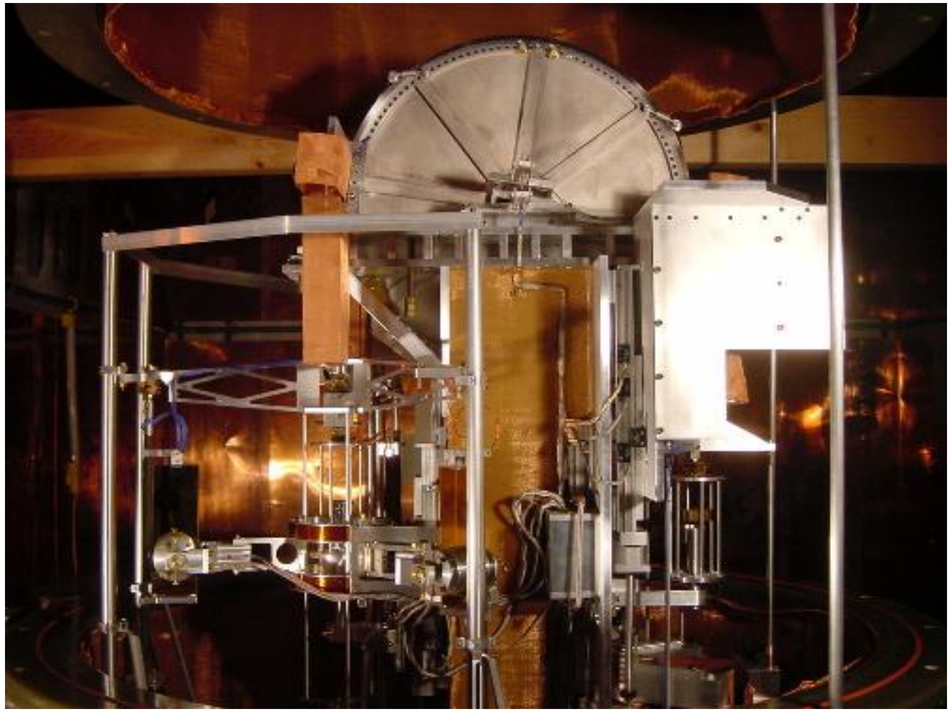
2.) Current is halved,
Heating is quartered

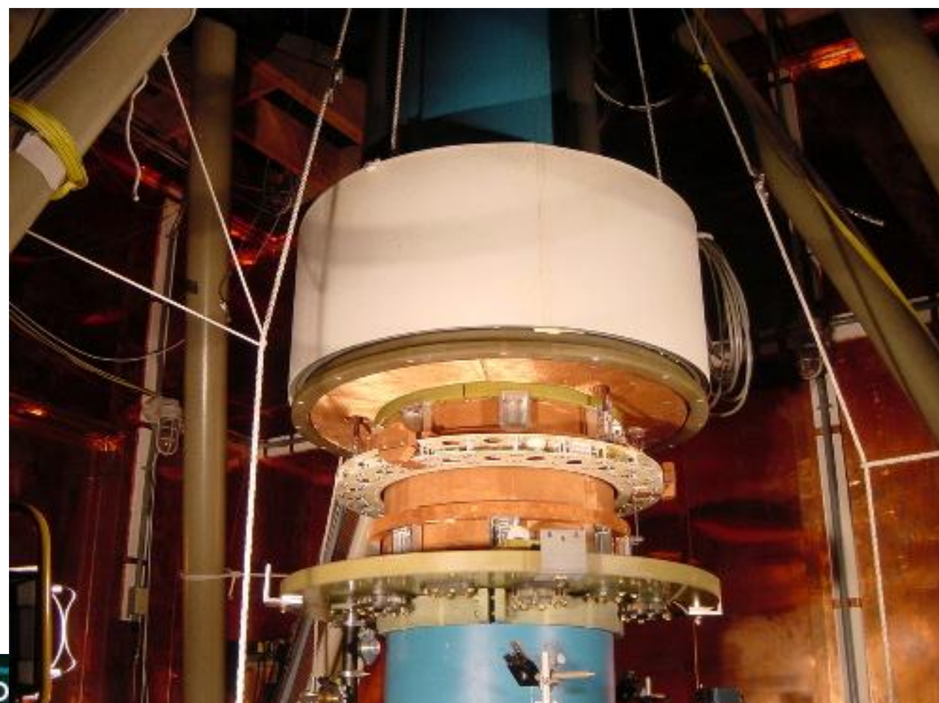


The velocity mode (in more detail)

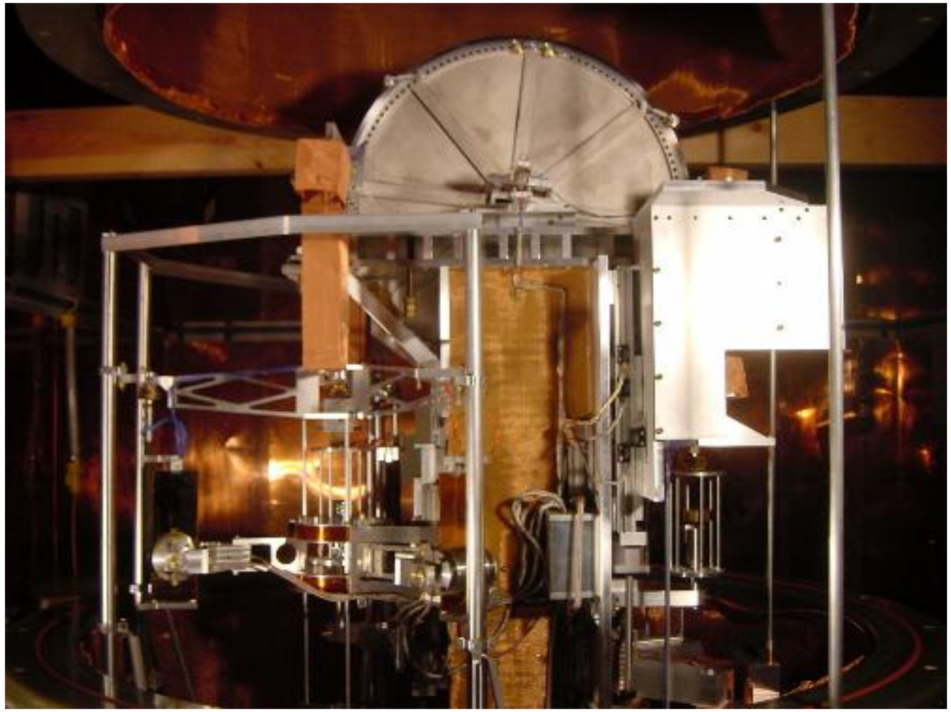


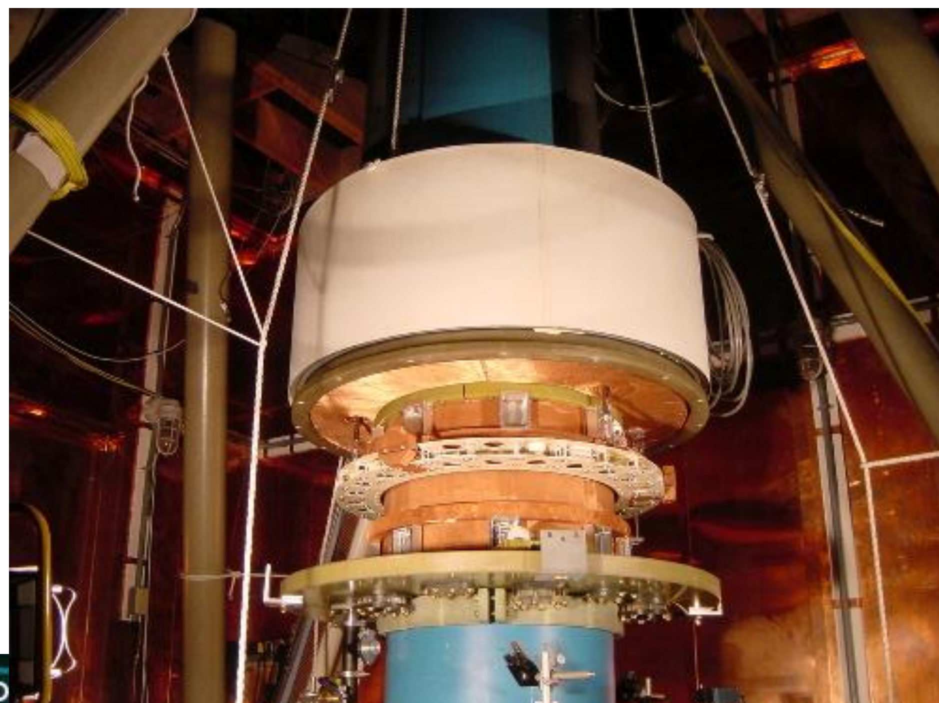
10 cm



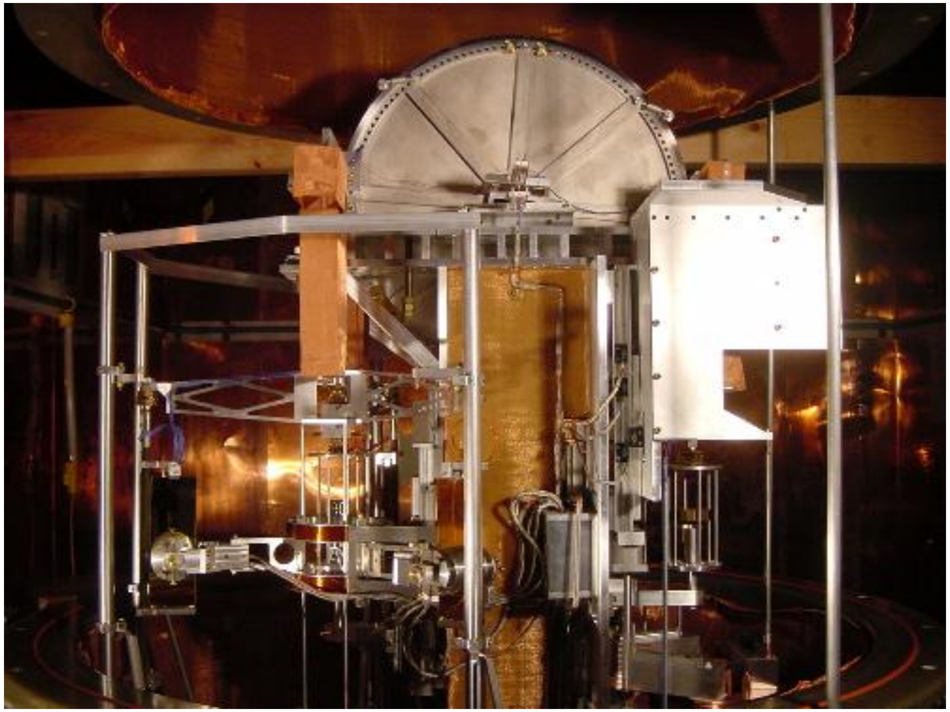


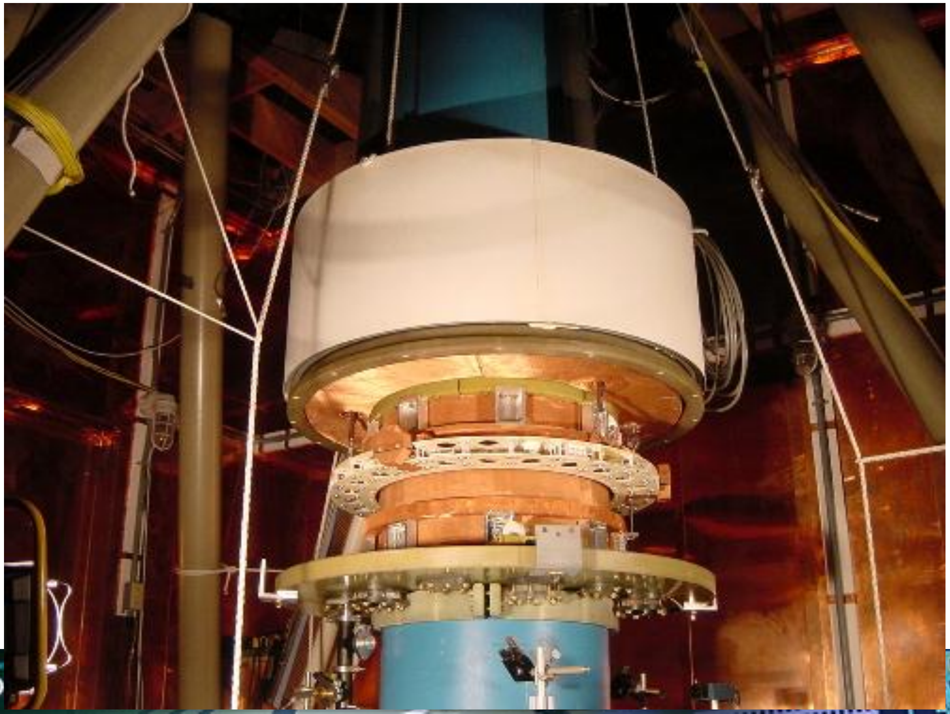
9 cm



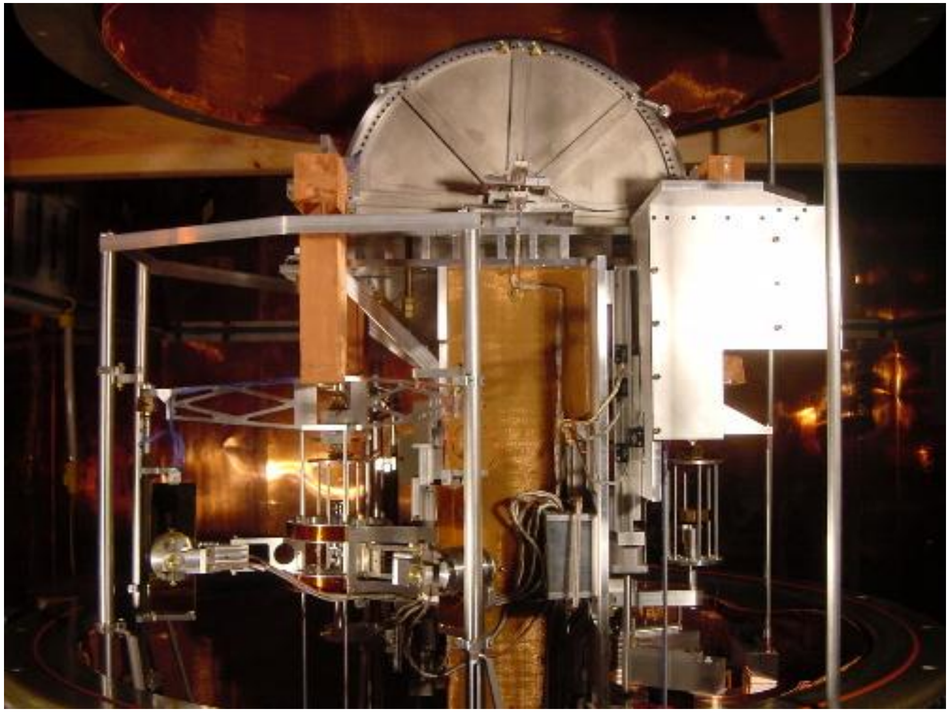


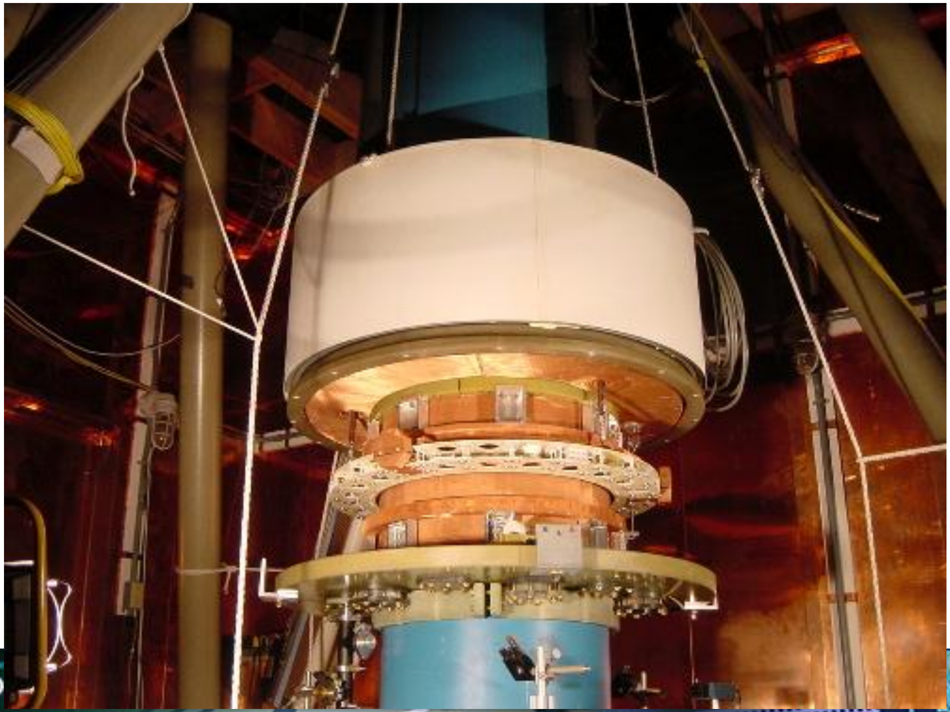
8 cm



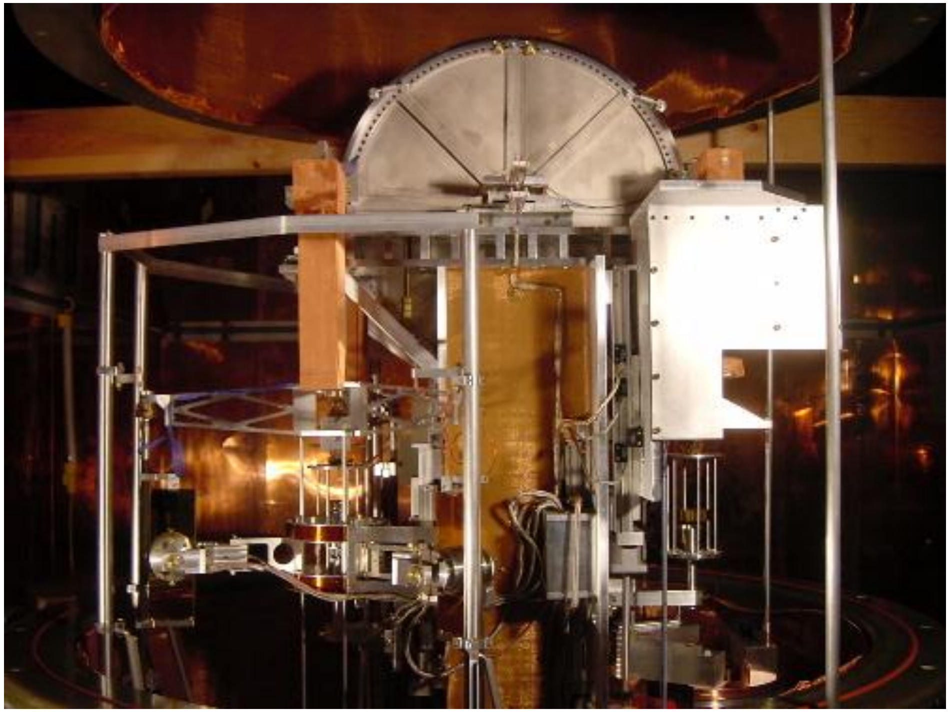


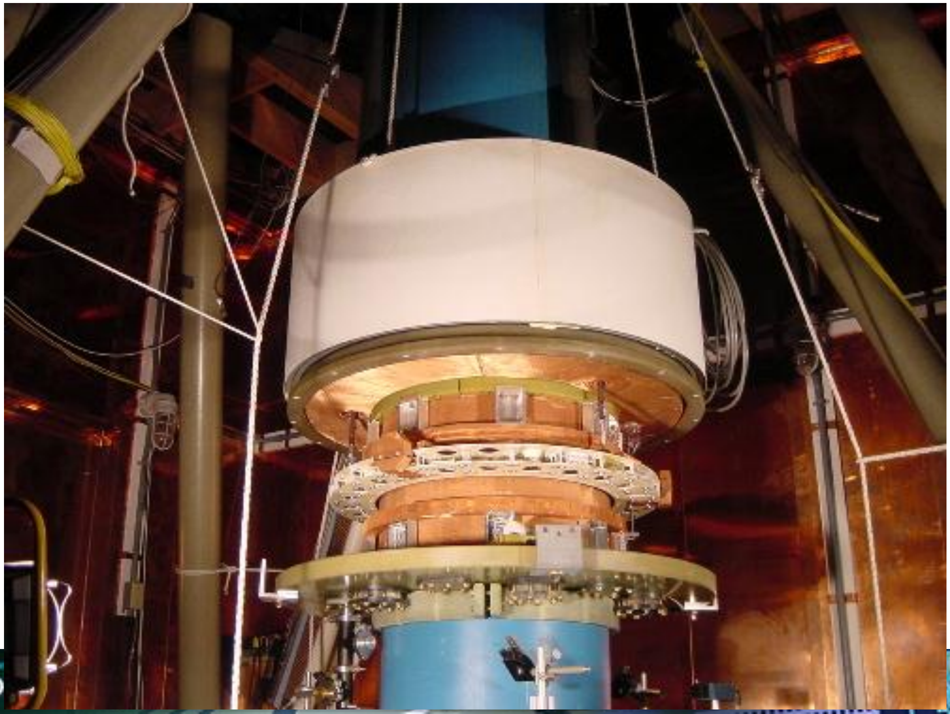
7 cm



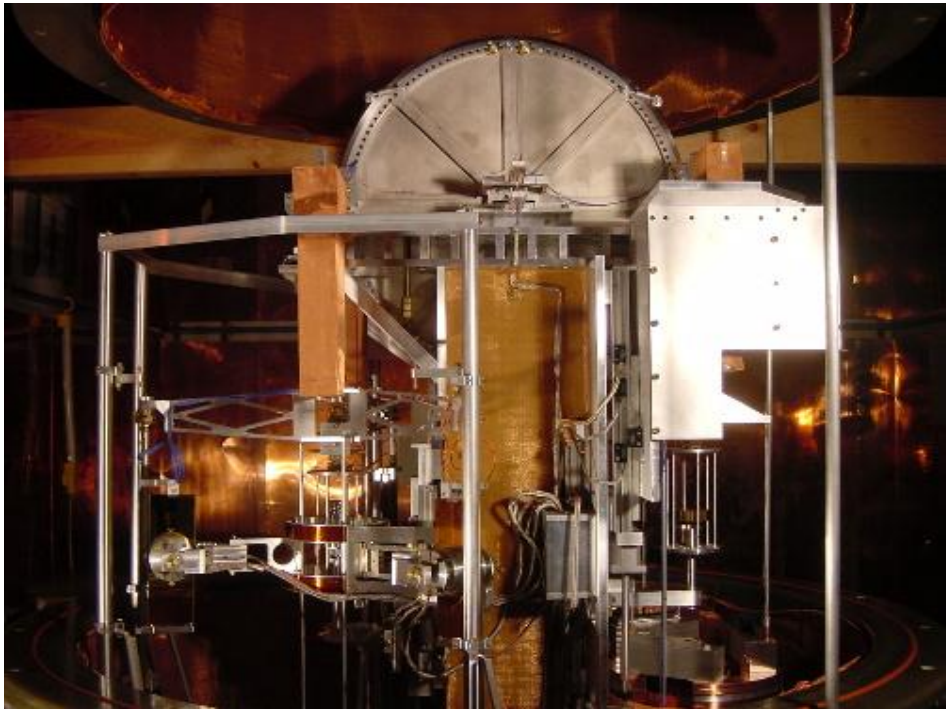


6 cm

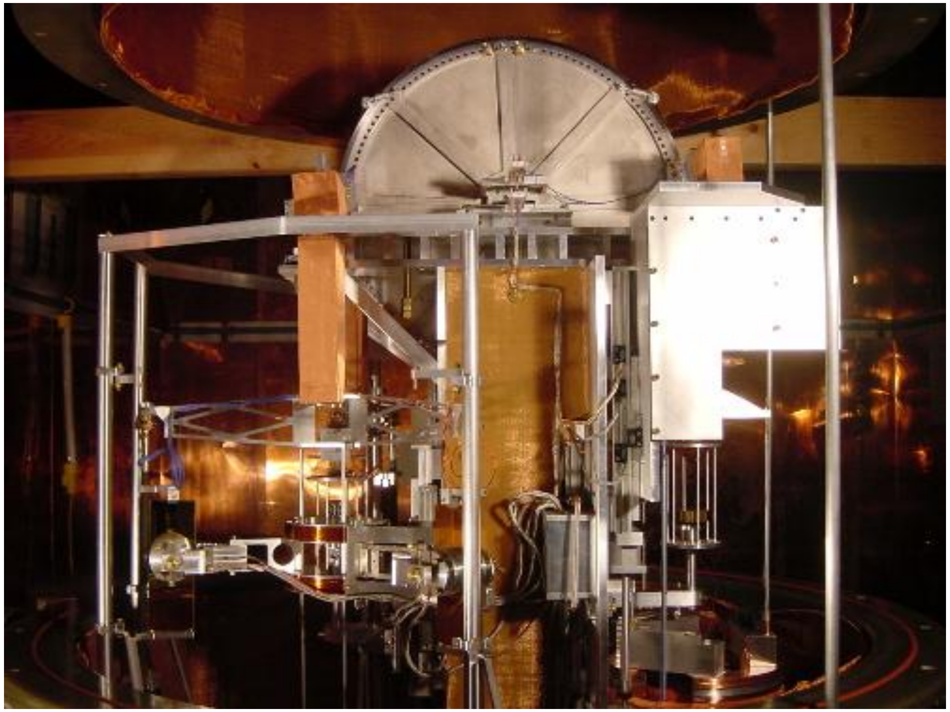




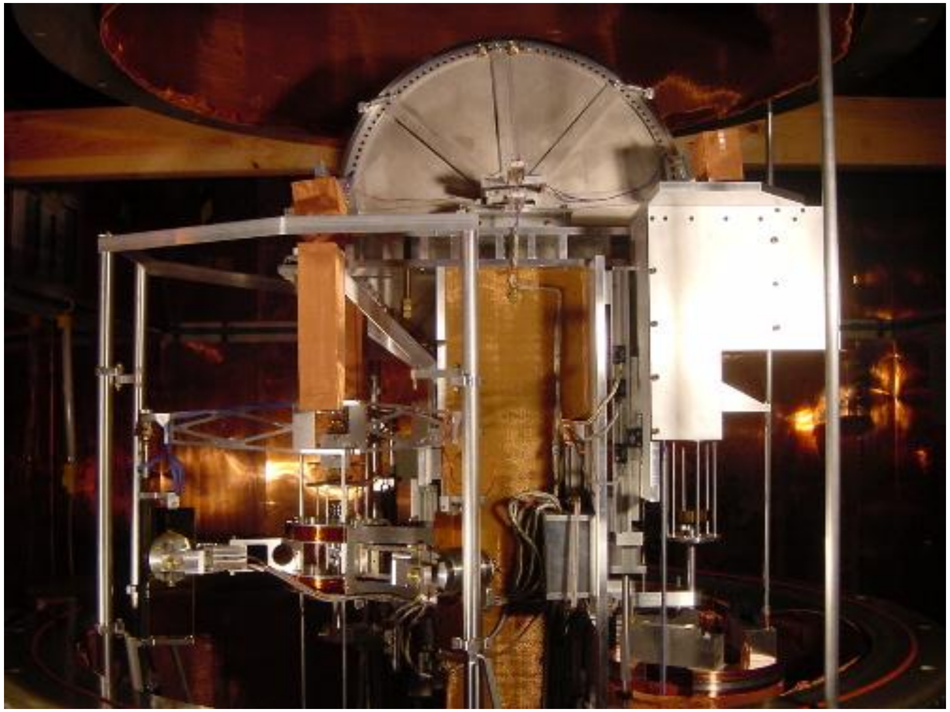
5 cm

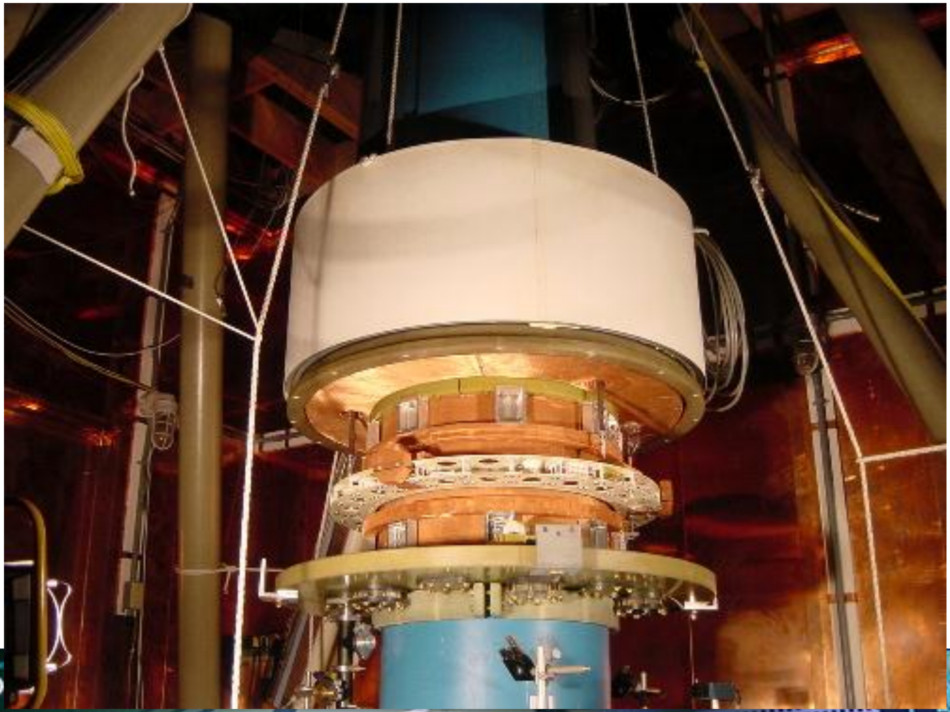


4 cm

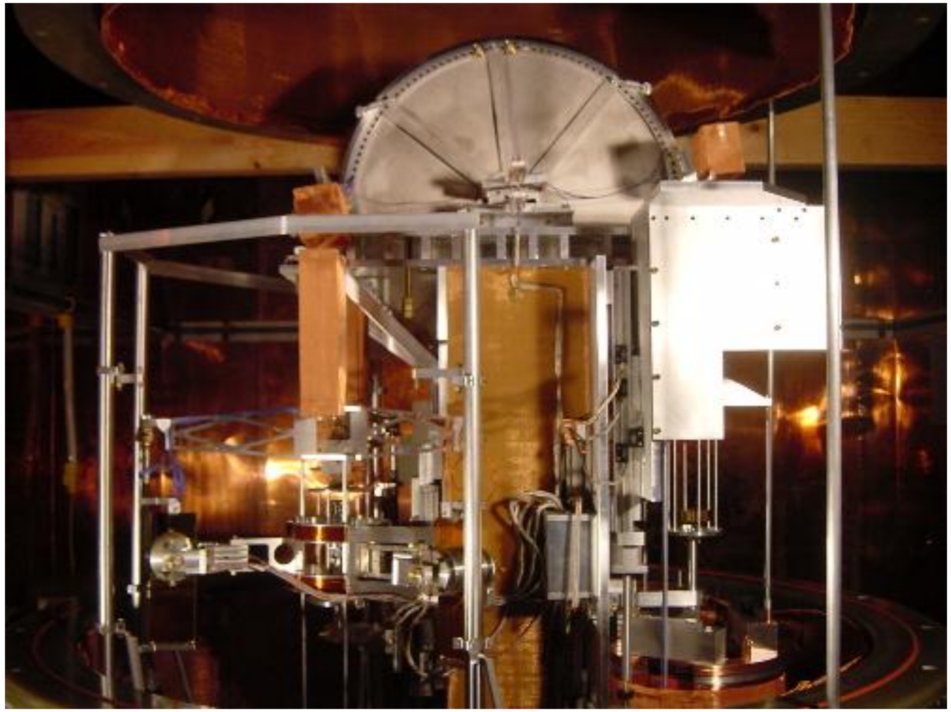


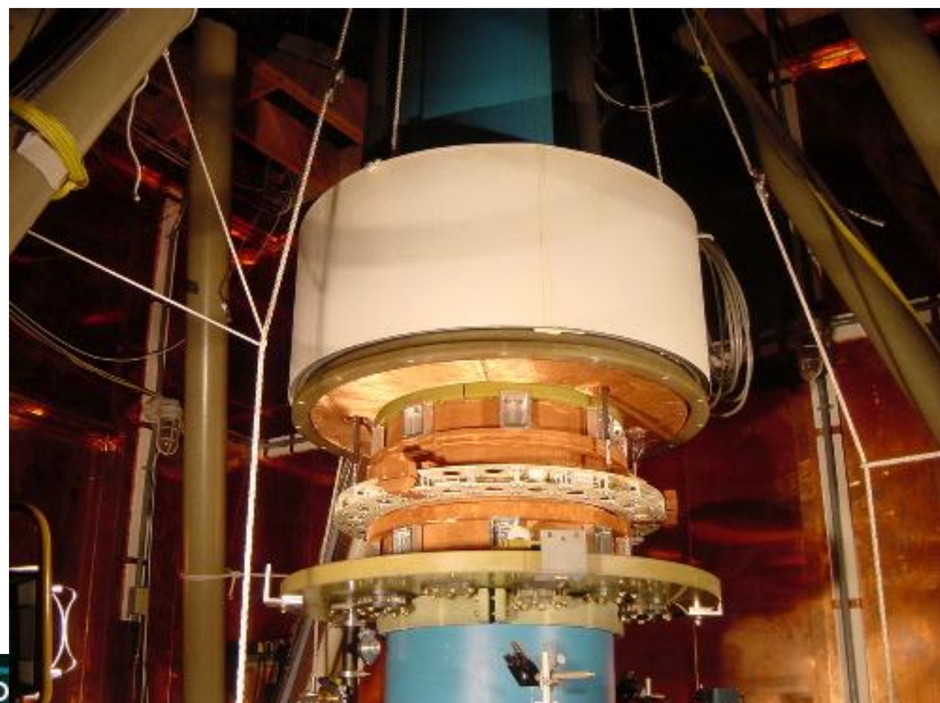
3 cm





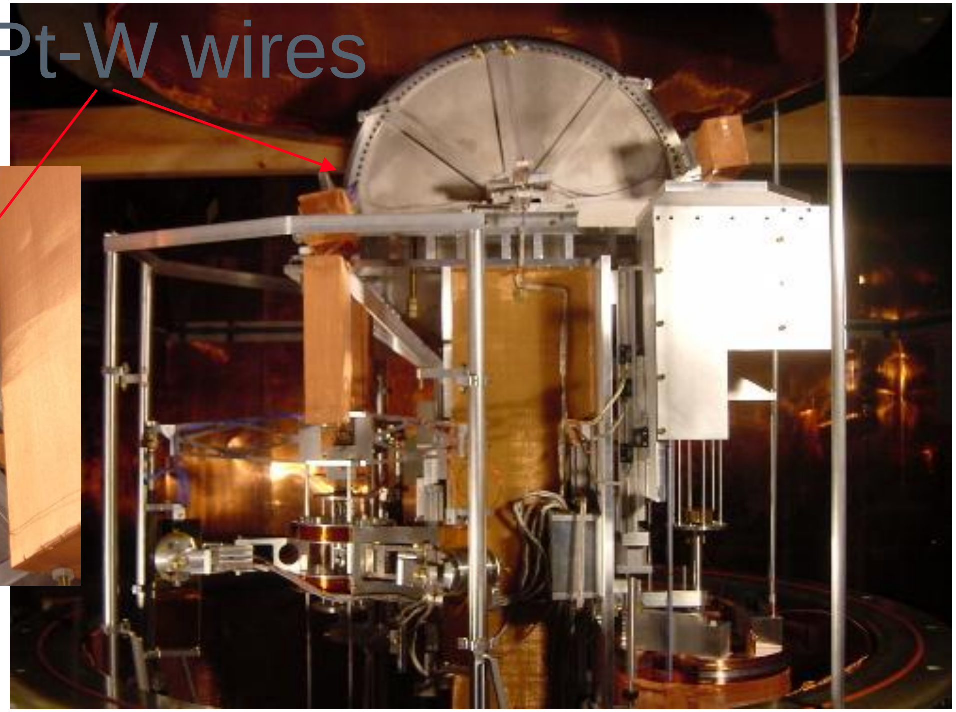
2 cm



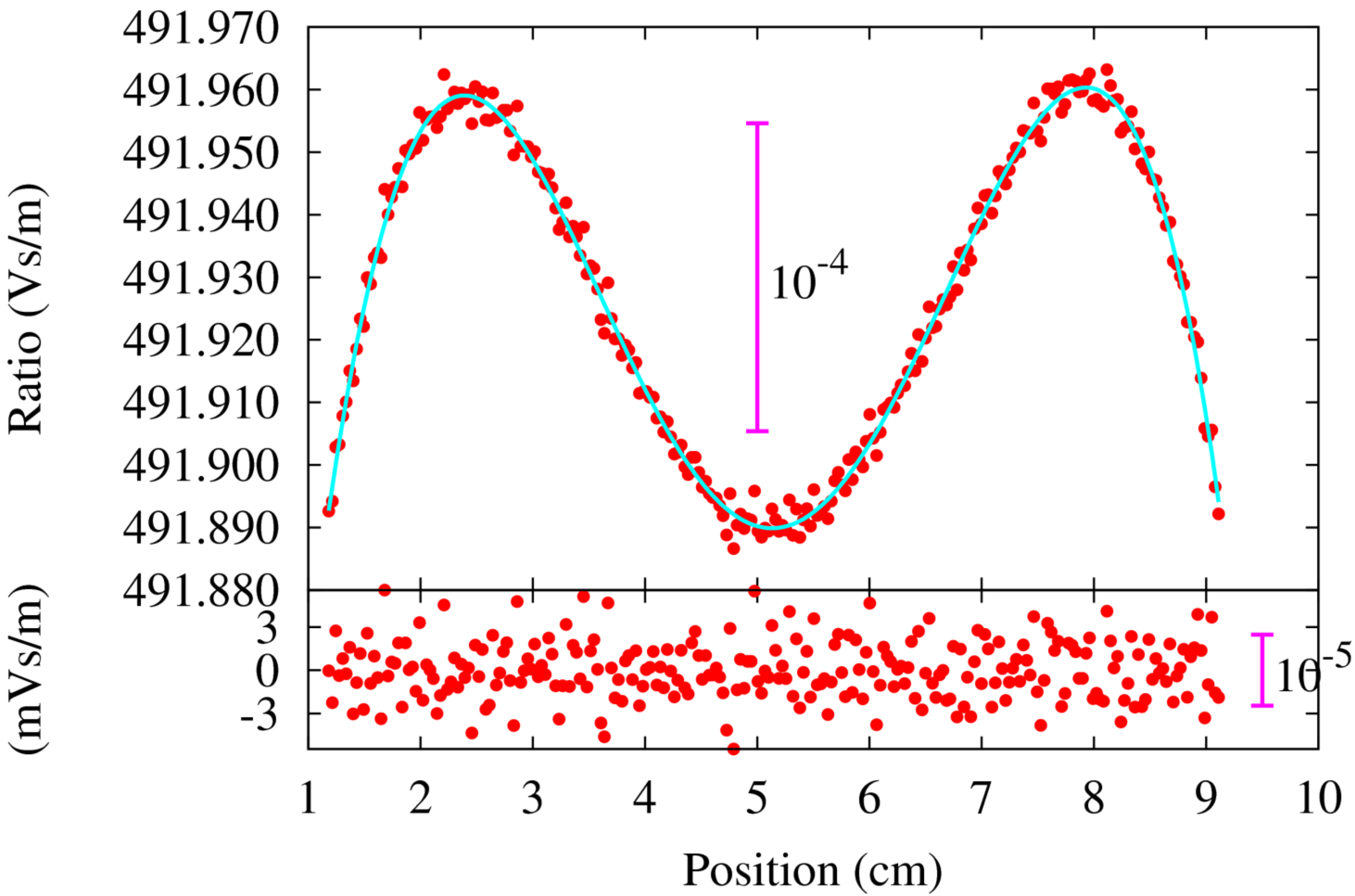


Band of 70 Pt-W wires

1 cm



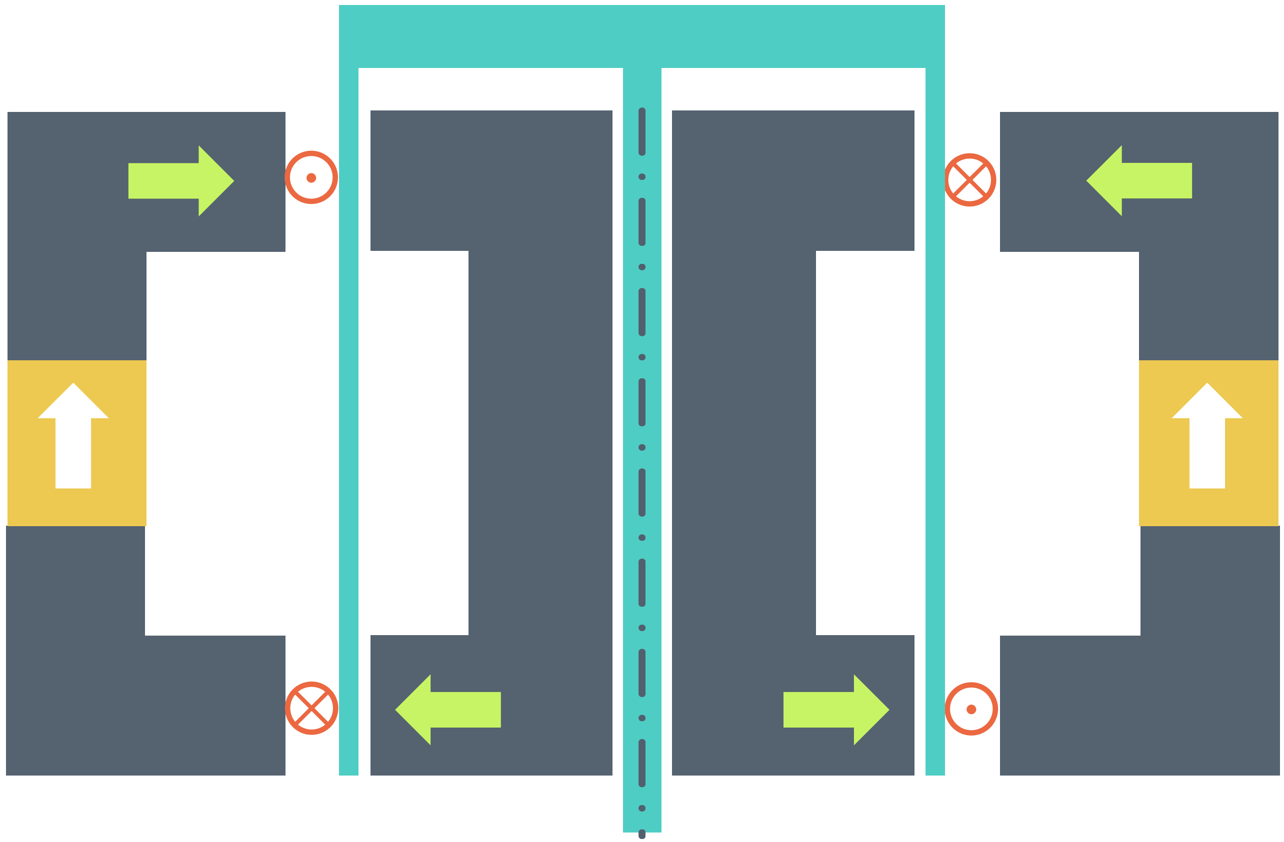
Voltage velocity measurement



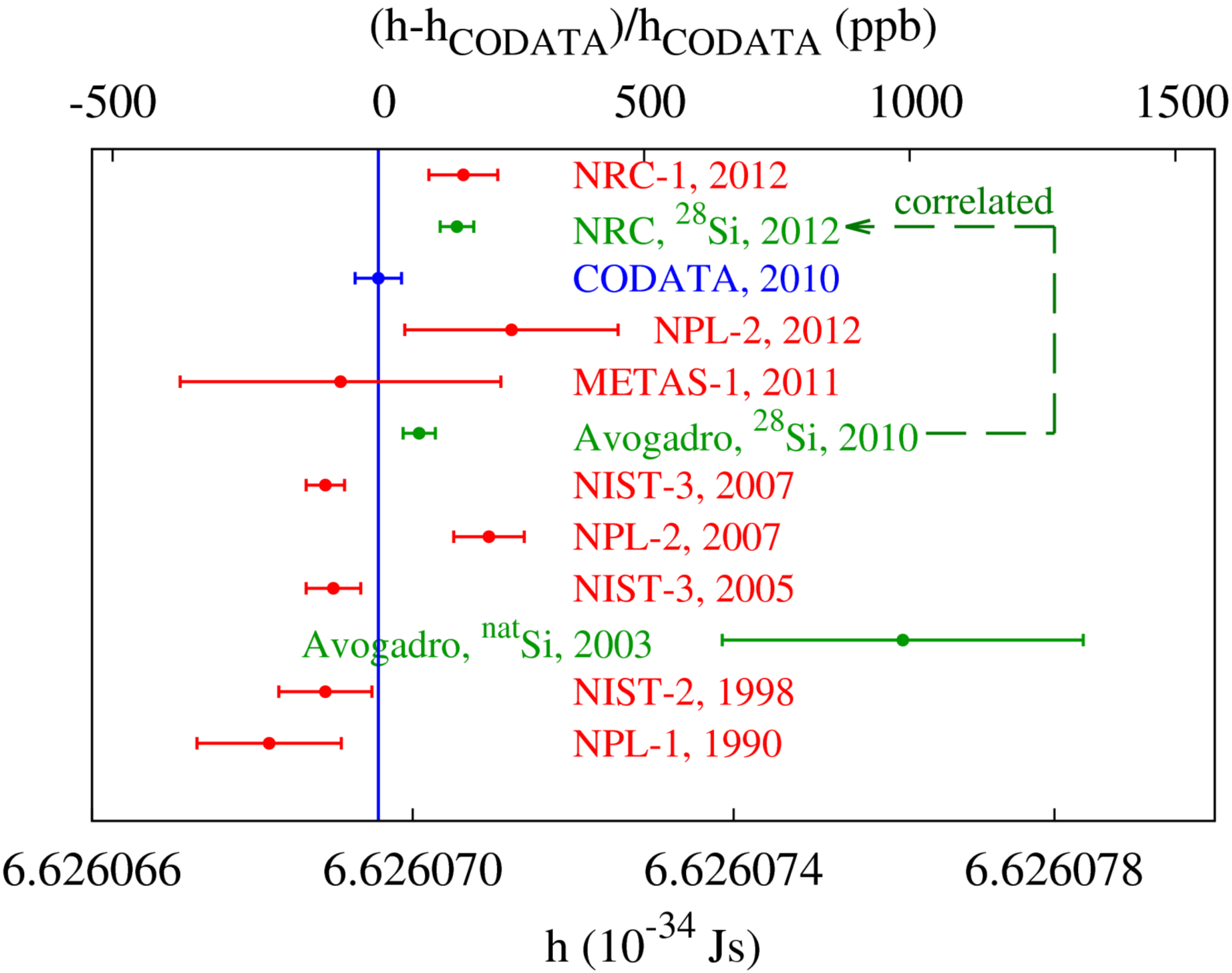
NPL-2 = NRC-1

- equal arm beam balance
- design with 3 knives
- SmCo permanent magnet
- radial field

NRC-1 magnet design

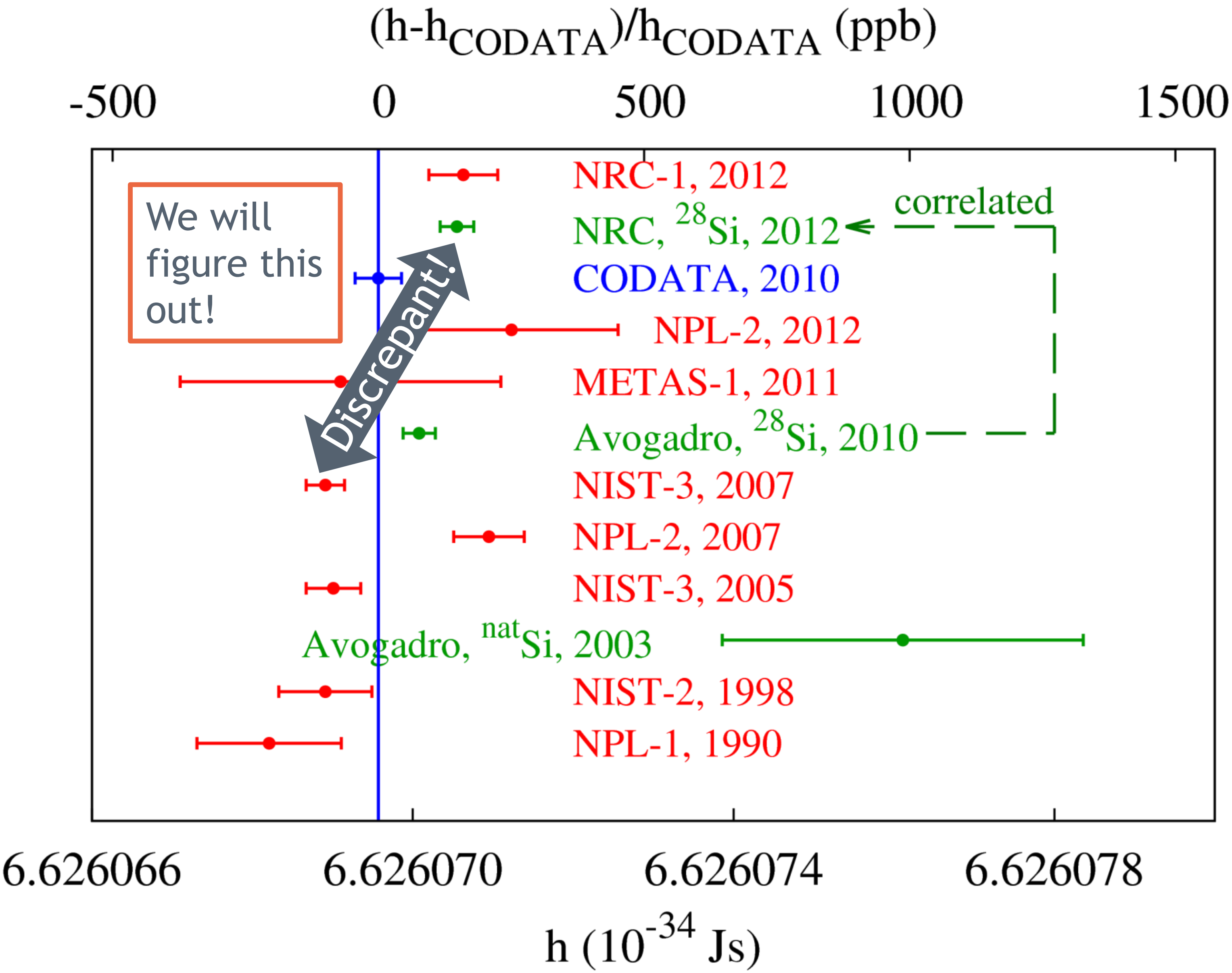


The present!



(h-h$_{CODATA}$)/h$_{CODATA}$ (ppb)
-500
0
500
1000
1500
NRC-1, 2012
correlated
NRC, ^{28}Si, 2012
CODATA, 2010
NPL-2, 2012
METAS-1, 2011
Avogadro, ^{28}Si, 2010
NIST-3, 2007
NPL-2, 2007
NIST-3, 2005
Avogadro, ^{nat}Si, 2003
NIST-2, 1998
NPL-1, 1990
6.626066
6.626070
6.626074
6.626078
h (10^{-34} Js)

The present!



(h-h_CODATA)/h_CODATA (ppb)
-500
0
500
1000
1500
We will figure this out!
NRC-1, 2012
NRC, 28Si, 2012
correlated
CODATA, 2010
Discrepant!
NPL-2, 2012
METAS-1, 2011
Avogadro, 28Si, 2010
NIST-3, 2007
NPL-2, 2007
NIST-3, 2005
Avogadro, natSi, 2003
NIST-2, 1998
NPL-1, 1990
6.626066
6.626070
6.626074
6.626078
h (10-34 Js)

NIST-3 v2.0

- A new, (mostly independent) team will measure a last data point with NIST-3.

- Blind measurement: The mass group will measure our masses. But they will add a to us unknown offset (-500ppb…500ppb) to the values.

Relevance

**RECOMMENDATIONS OF THE
CONSULTATIVE COMMITTEE FOR MASS AND RELATED QUANTITIES
SUBMITTED TO THE INTERNATIONAL COMMITTEE FOR WEIGHTS AND MEASURES**

**RECOMMENDATION G 1 (2010)
Considerations on a new definition of the kilogram**

The Consultative Committee for Mass and Related Quantities (CCM)

recalling its previous Recommendation to the CIPM on the "Conditions for a new definition of the kilogram", CCM G 1 (2005), and

considering

recommends

- that the following conditions be met before the kilogram is redefined in terms of fundamental constants:

    1. at least three independent experiments, including work both from watt balance and from International Avogadro Coordination projects, yield values of the relevant constants with relative standard uncertainties not larger than 5 parts in 10^8. At least one of these results should have a relative standard uncertainty not larger than 2 parts in 10^8,

    2. for each of the relevant constants, values provided by the different experiments be consistent at the 95 % level of confidence,

    3. traceability of BIPM prototypes to the international prototype of the kilogram be confirmed,

- that the CODATA recommended values be adopted for the relevant fundamental constants,

- that the associated CODATA relative standard uncertainties be suitably considered when the initial uncertainty is assigned to the mass of the international prototype of the kilogram,

Timeline

01Oct 2011

now, 09/01/12

01Oct 2013

new team starts working on NIST-3

Solenoid cooled down

Assessment
Planning
Order of
hardware

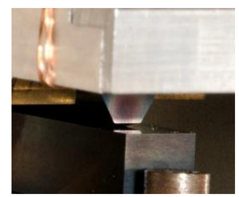
Alignment

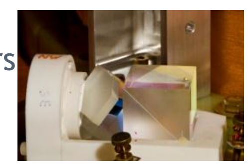
first quarter of data
uncertainties ≈100 x10^{-9}

Change of electrical
grounding
New PJVS

New
- current source
- vacuum pumps
- knife edge

Upgrades to
- Interferometers





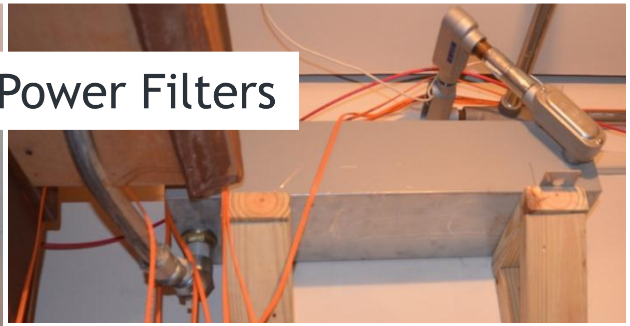
Comparison of
absolute gravimeter

Comparison of
JJ Voltage Std.

Data analysis, systematic tests

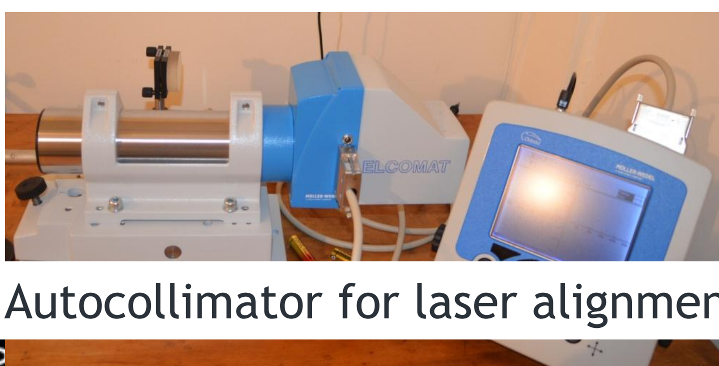
Goal: An independent data point with NIST-3
Independent measurement & uncertainty analysis

Changes

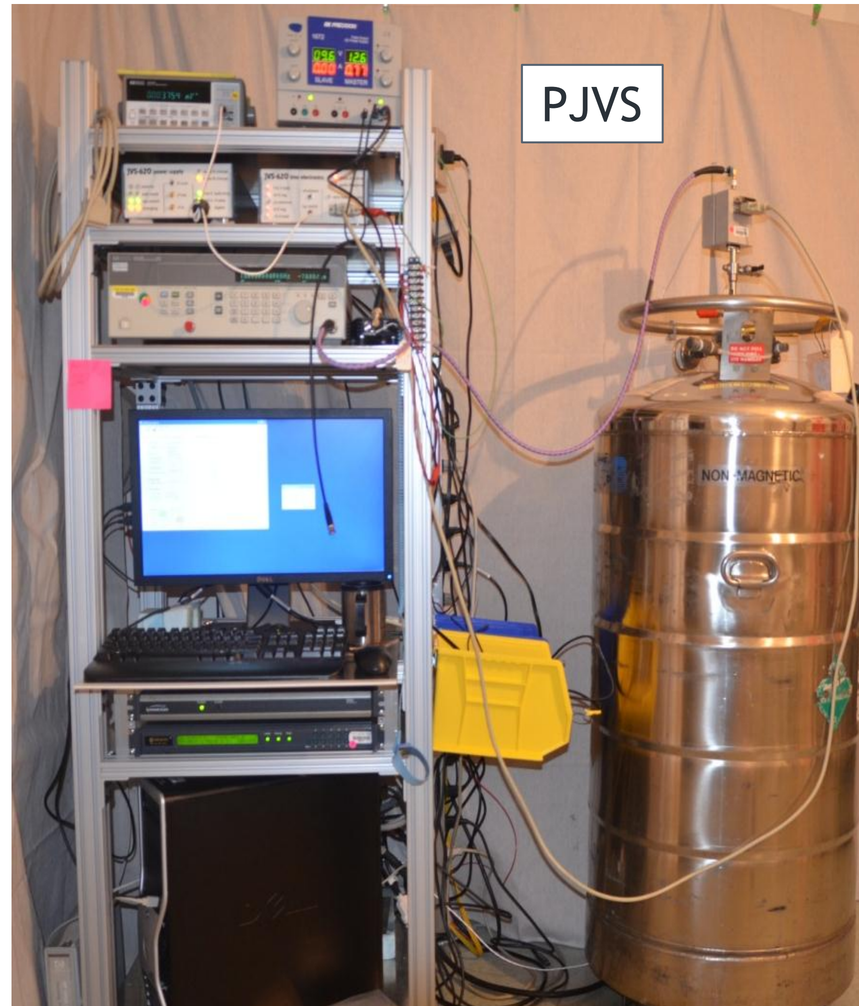


Power Filters

GND STAR

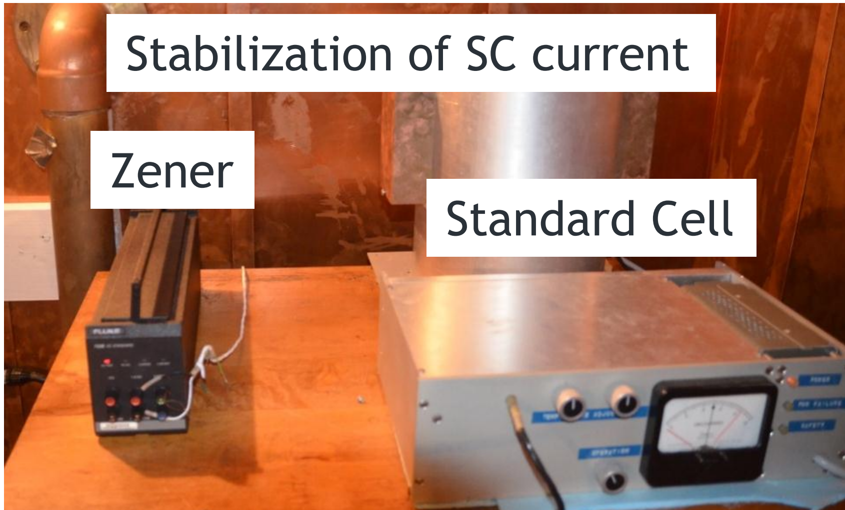


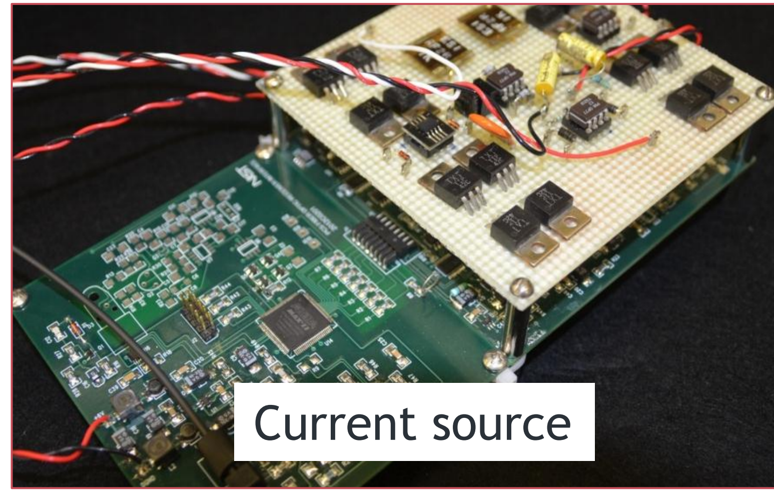
Autocollimator for laser alignment

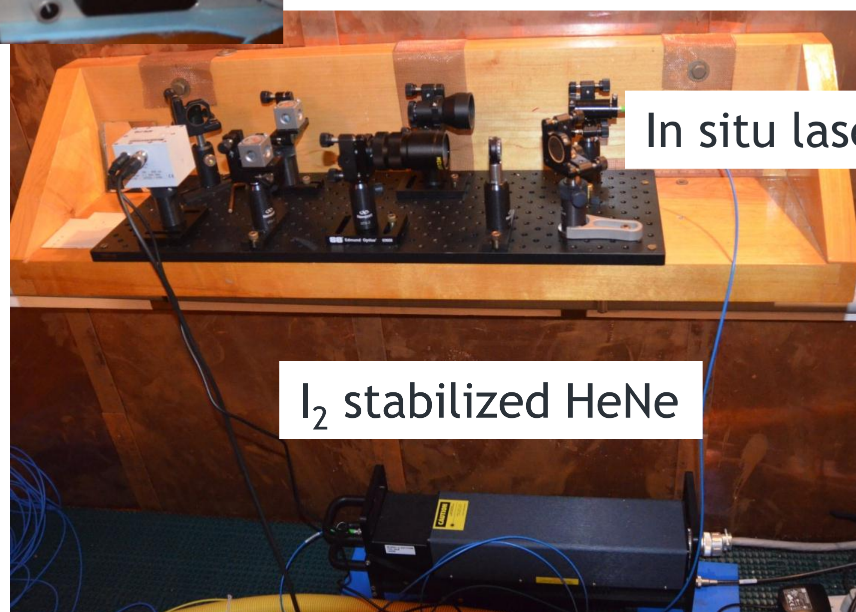


PJVS

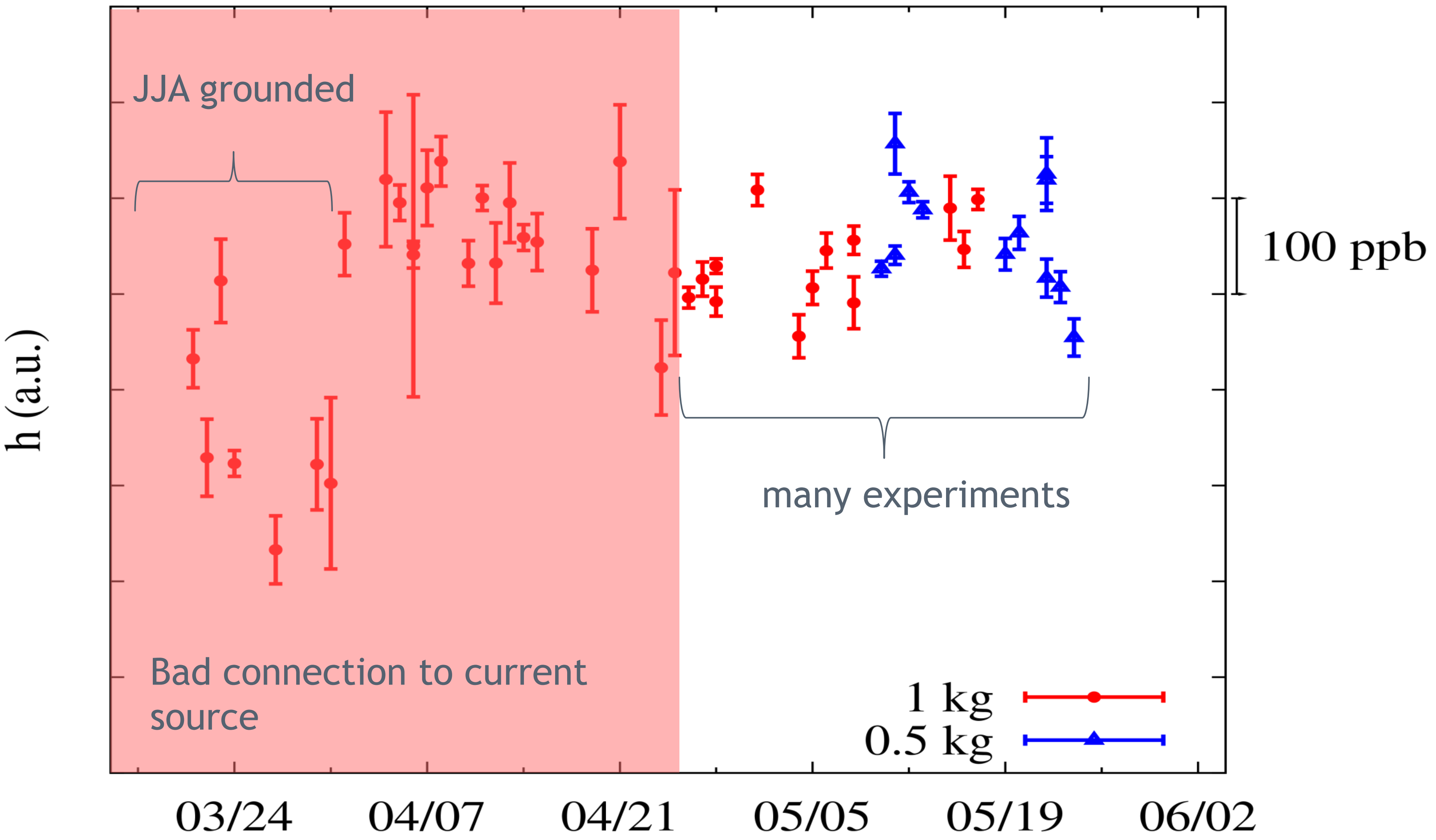
Changes continued







Results



JJA grounded
Bad connection to current source
many experiments
100 ppb
h (a.u.)
1 kg
0.5 kg
03/24
04/07
04/21
05/05
05/19
06/02

In the meantime we are building NIST-4

NIST-3

“The Thoroughbred is a horse breed best known for its use in horse racing. Thoroughbreds are considered "hot-blooded" horses, known for their agility, speed and spirit.”

- Watt balance to measure *h*
- Optimized for the best measurement of *h*
- Highly sophisticated
- Costly to operate and maintain
- Lifetime: $\approx$10 years

NIST-4

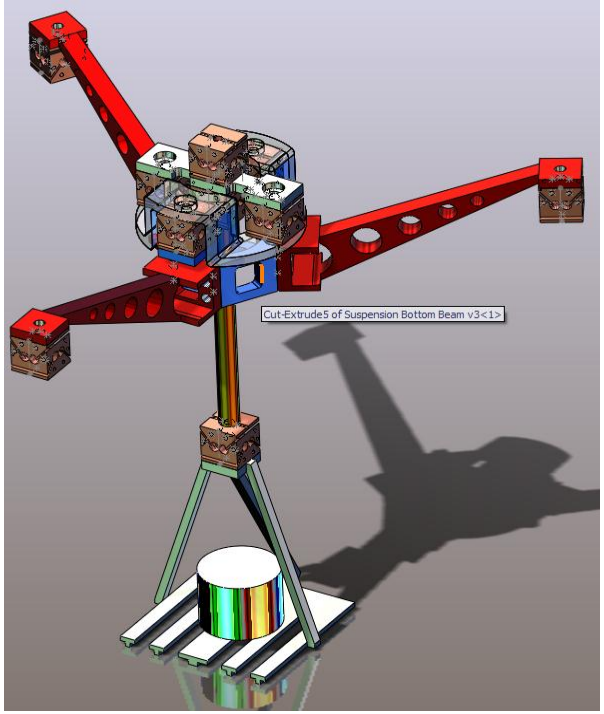
“The Clydesdale is a cold blooded horse appreciated for its strength, style, and versatility. Like all cold-blooded horses the Clydesdale has a stolid demeanor and is not suitable for sports other than hauling or pulling.”

- Watt balance to realize the kg
- Optimized for reliable operation
- Ease of Use
- Low maintenance
- Lifetime: $>$30 years

picture source: wikipedia.org

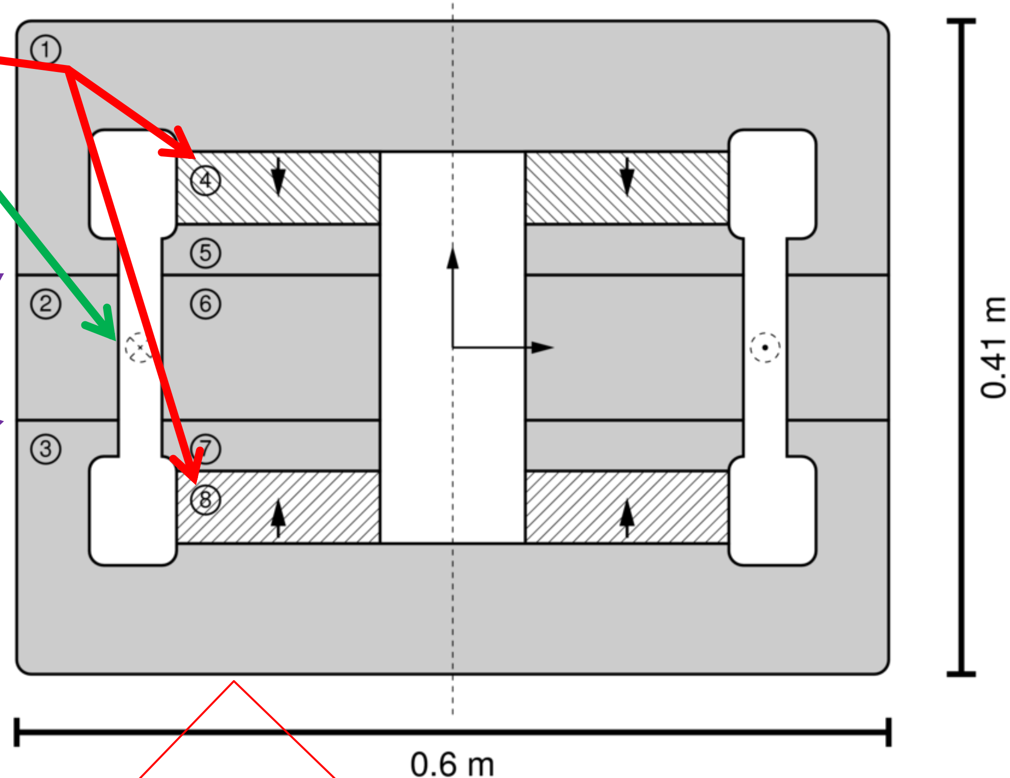
NIST-4

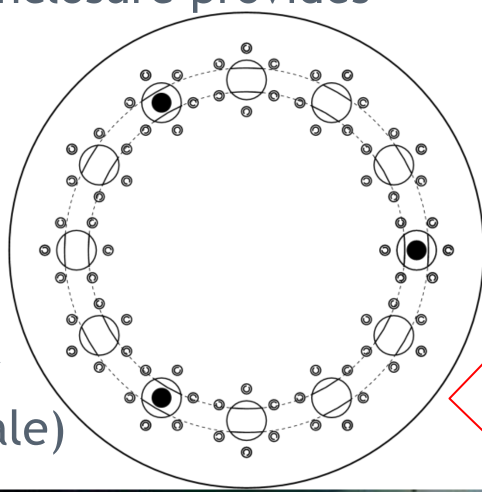
- Design has started.
- Baseline design: Wheel balance with permanent magnet system.
- Magnet is designed and plans are at a manufacturer
- Prepare the infrastructure.
- Think about alternative design ideas.



NIST-4 magnet

- SmCo magnet rings provide field.
- wide gap (3cm) 15 cm long.
- symmetric design, field minimum.
- Field ≈0.54 T.
- Magnet separates.
- full iron enclosure provides shielding





Top view
(not to scale)

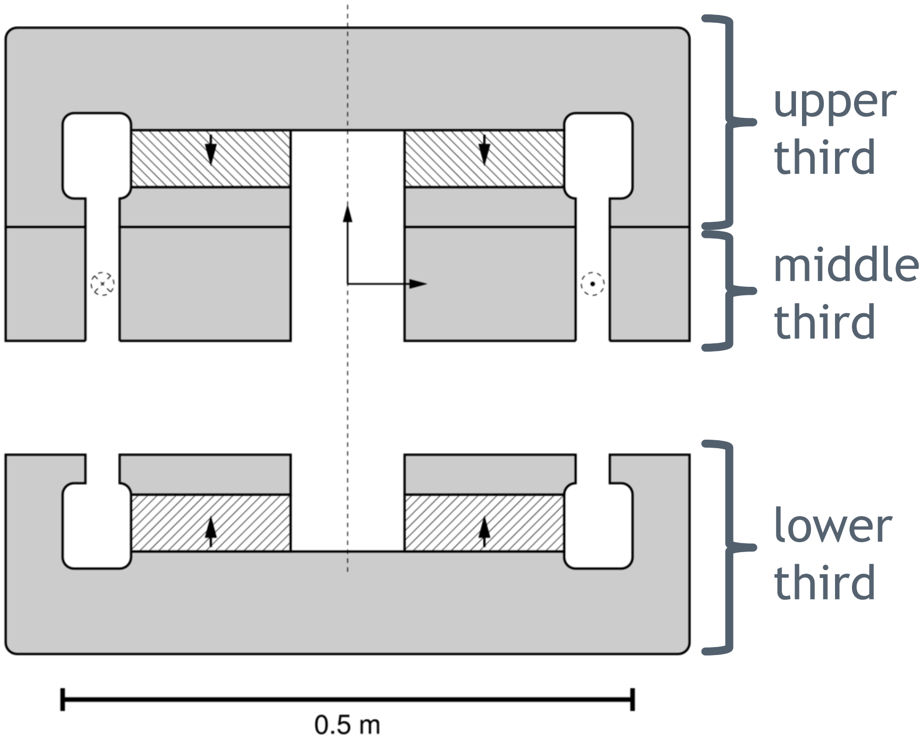
Construction is about to start!

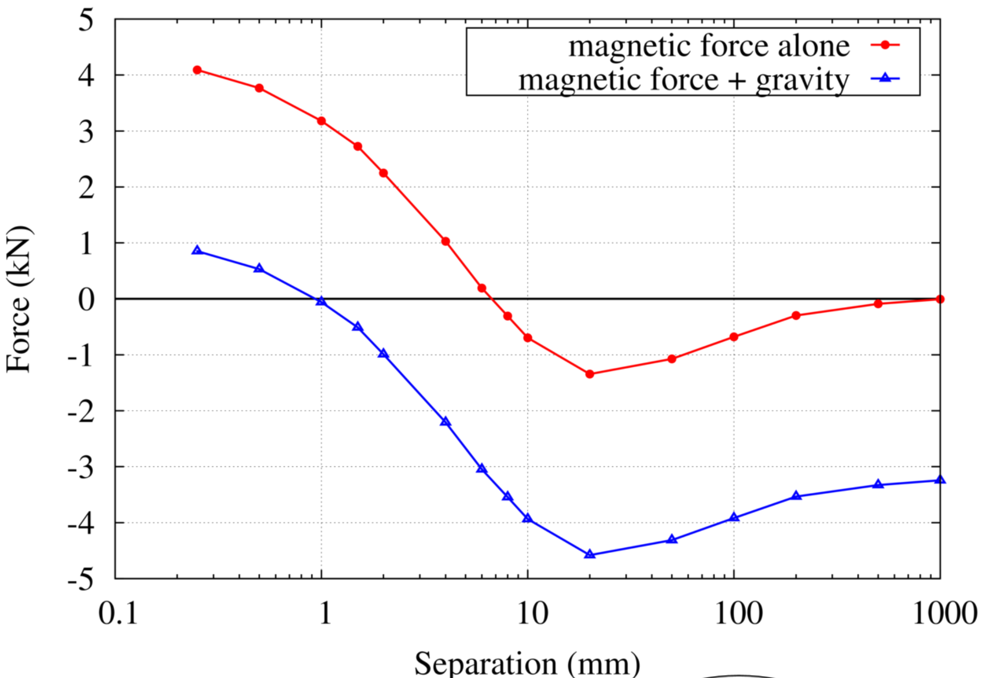
8 parts:

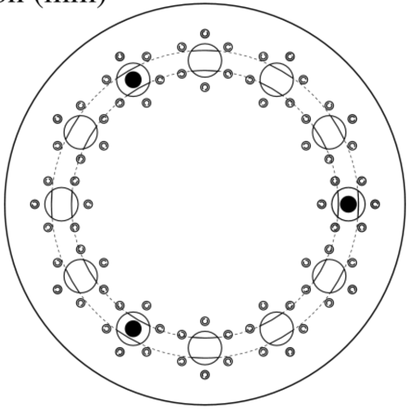
1-3	outer yoke
5-7	inner yoke
4 & 8	magnets

Access to the coil

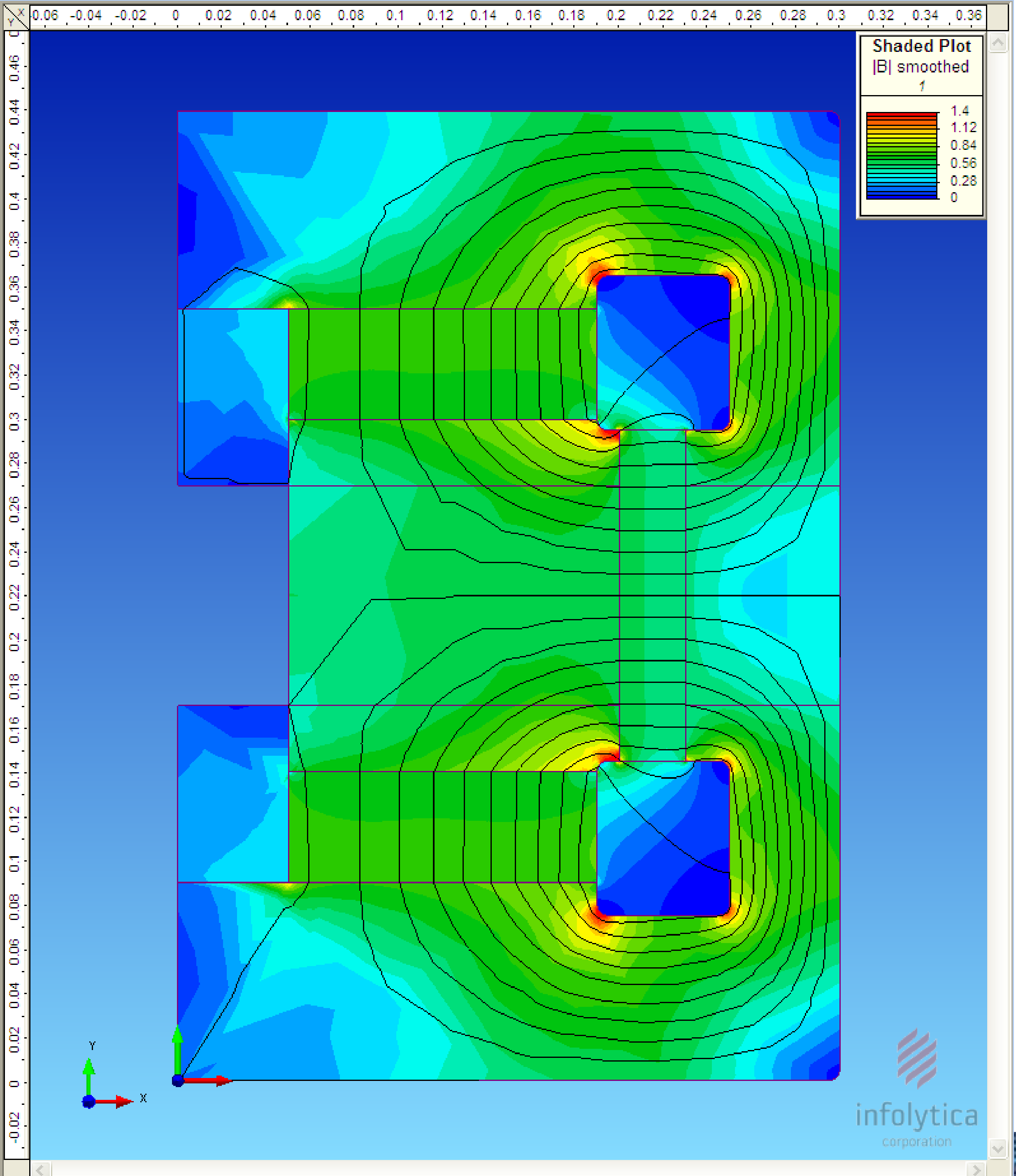
To access the coil, the magnet separates:







Total mass of the magnetic circuit: 850 kg
Mass of one magnet ring: 44 kg
Mass of the lower part of the magnet: 330 kg



NIST National Institute of Standards and Technology U.S. Department of Commerce

FEA results

- calculated by MagNet
- Flux in Iron < 0.9 T

- Field in gap is 0.54 T
- coil radius is 21.5 cm

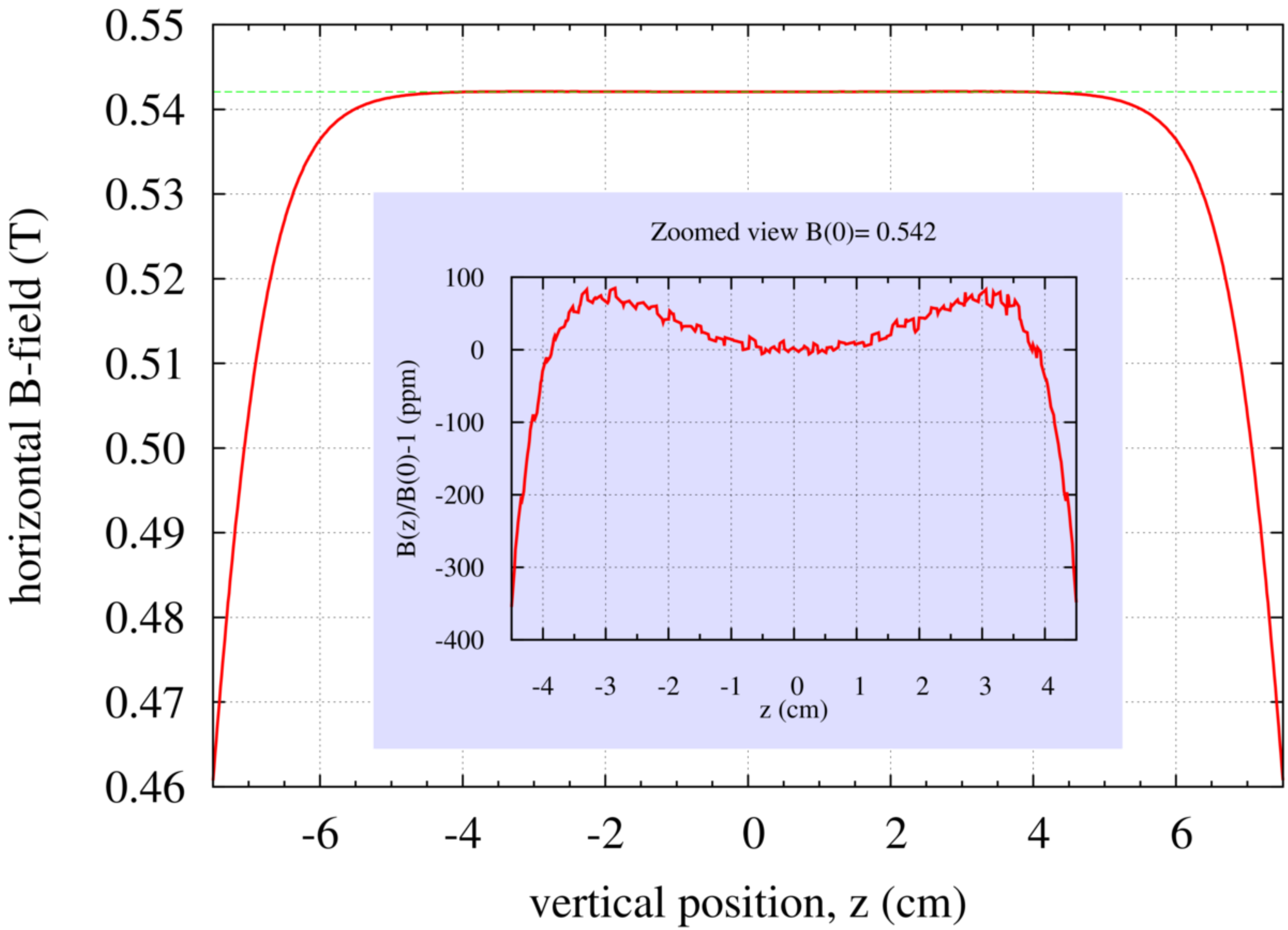
$$F = N 2\pi r B I$$

$$\frac{F}{I} = N 0.732 \frac{\mathrm{N}}{\mathrm{A}}$$

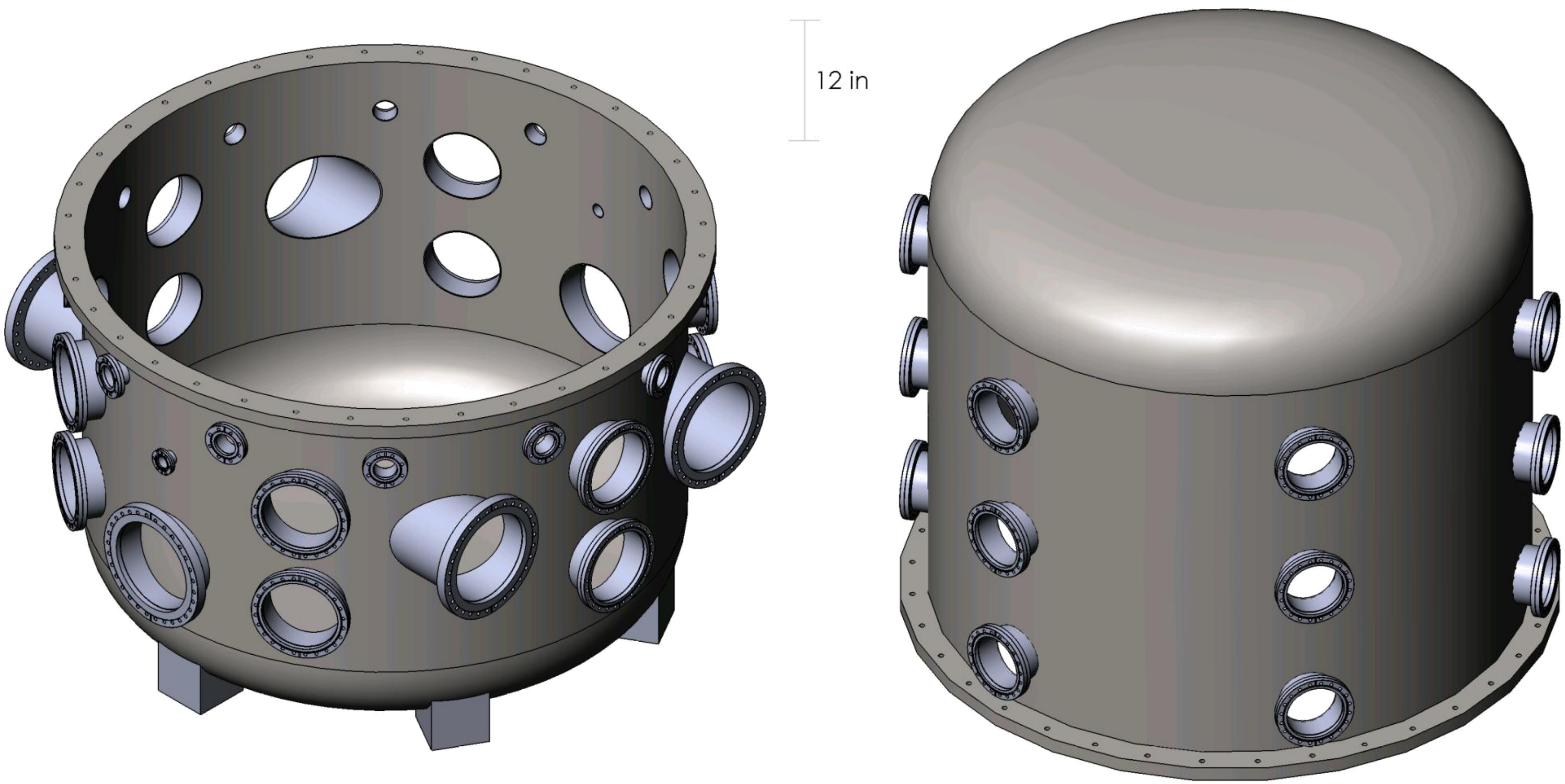
$$\frac{F}{I} = 500..1000 \frac{\mathrm{N}}{\mathrm{A}}$$

$$N = 680..1370$$

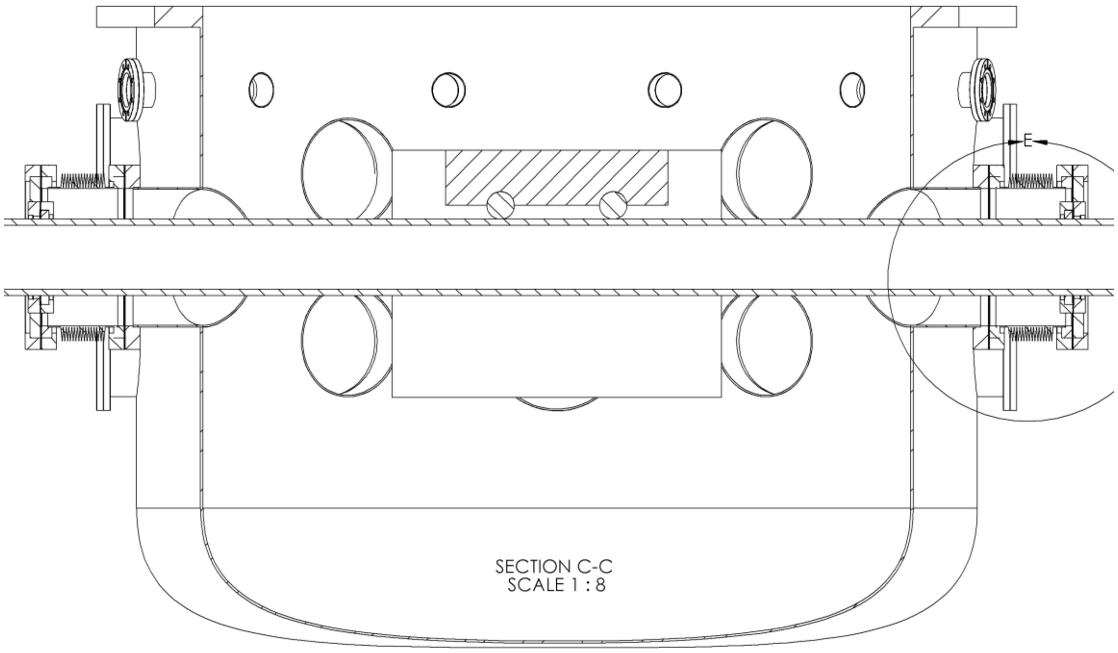
Field quality

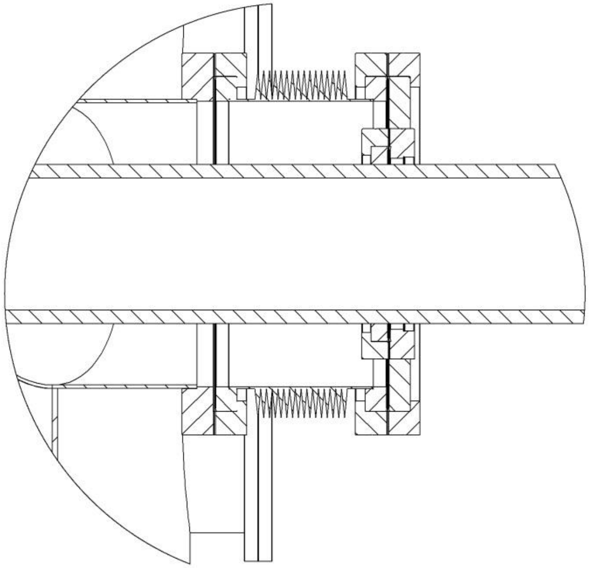


Vacuum Vessel is designed



Magnet in vacuum vessel





NIST-4 next steps

- Prepare infrastructure (crane, clean power, vacuum pumps).
- Building a device to verify the magnet.
- Work on an alternative mechanical design.
- Design load lock for the masses.

Outline

1. ~~The SI and the definition of the kg~~

2. ~~The principle of the Watt balance~~

3. ~~The past, the present, & the future of the Ekg~~

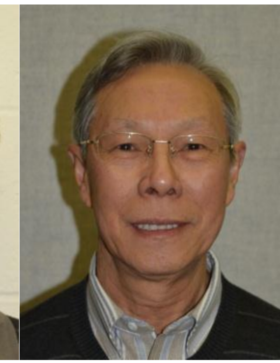
Thank you for your attention!

Jon Pratt,
group leader

Stephan
Schlamminger

Darine
Haddad

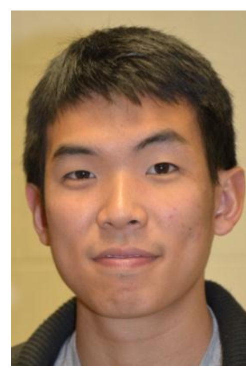
Frank
Seifert



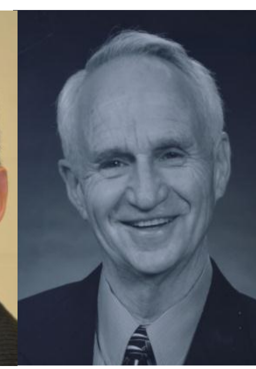
Ruimin
Liu

David
Newell

part-time:



Leon
Chao



Ed
Williams

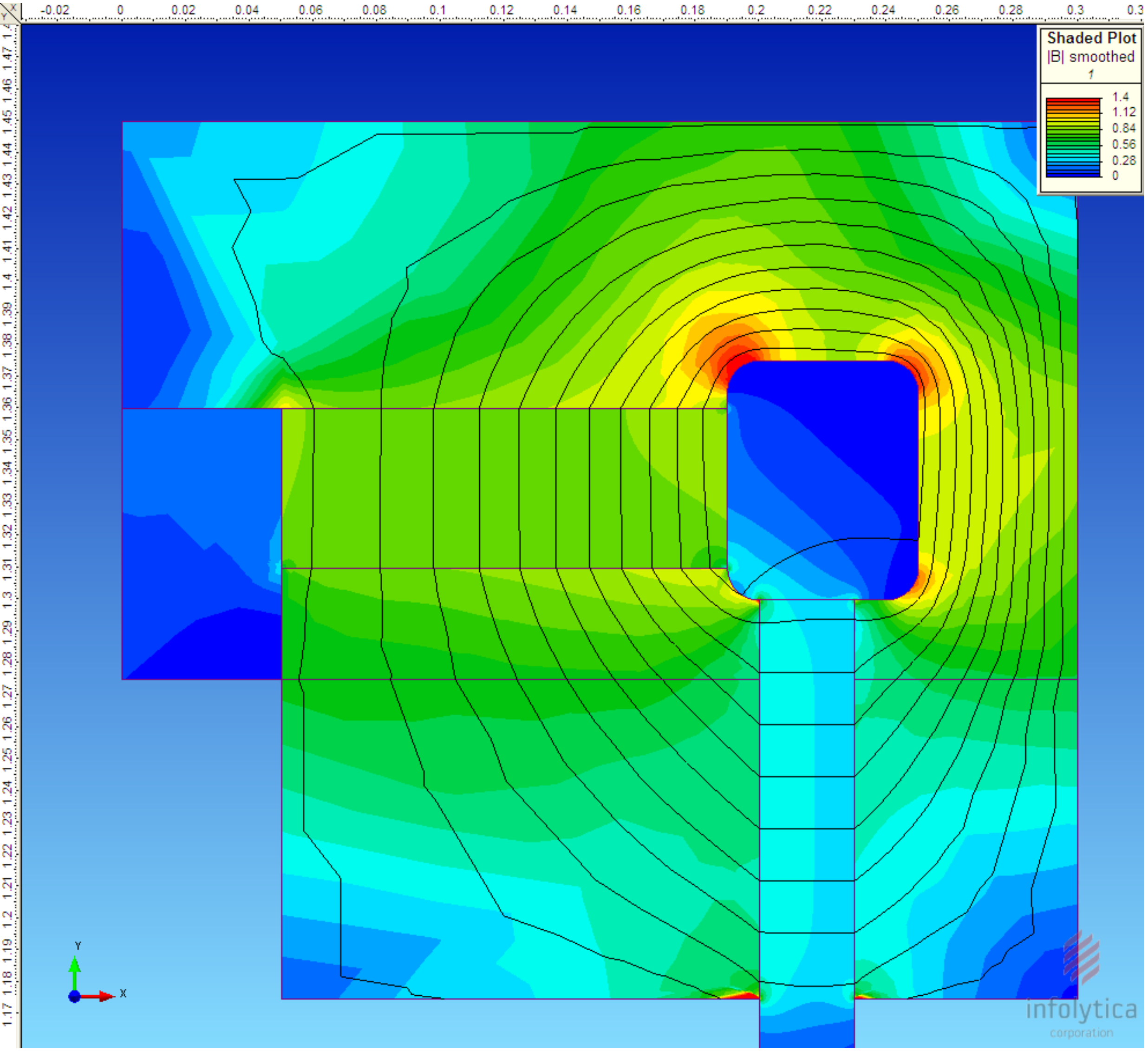
Shawn
Zhang

The End

Summary

- Current definition of the kilogram.
- Shortcomings of the definition.
- Quantum electrical metrology.
- Principle of the Watt balance (velocity mode, weighing mode).
- Account of the past, present and future of the electronic kilogram
- Introduction to the NIST-4 Watt balance

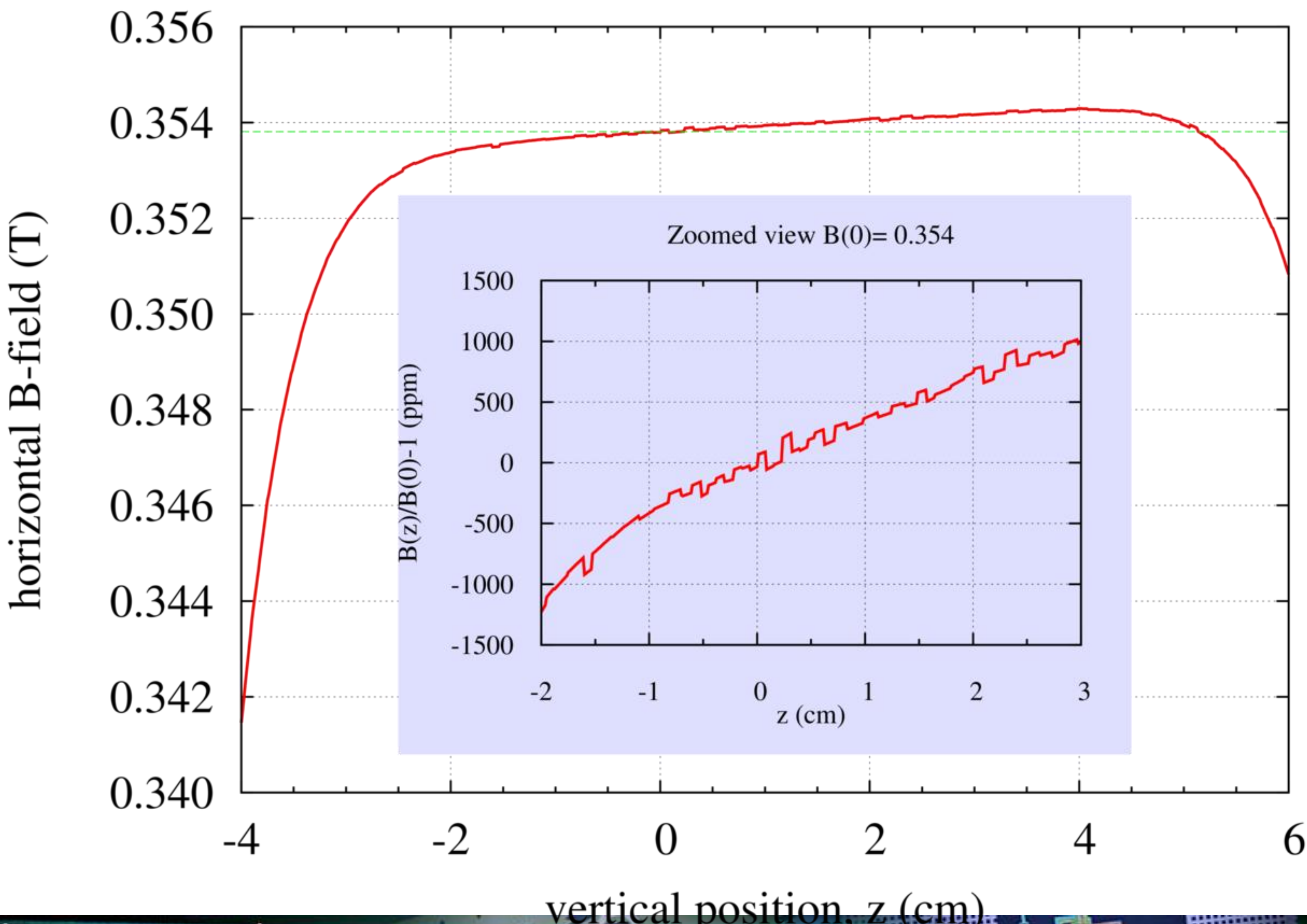
FEA results



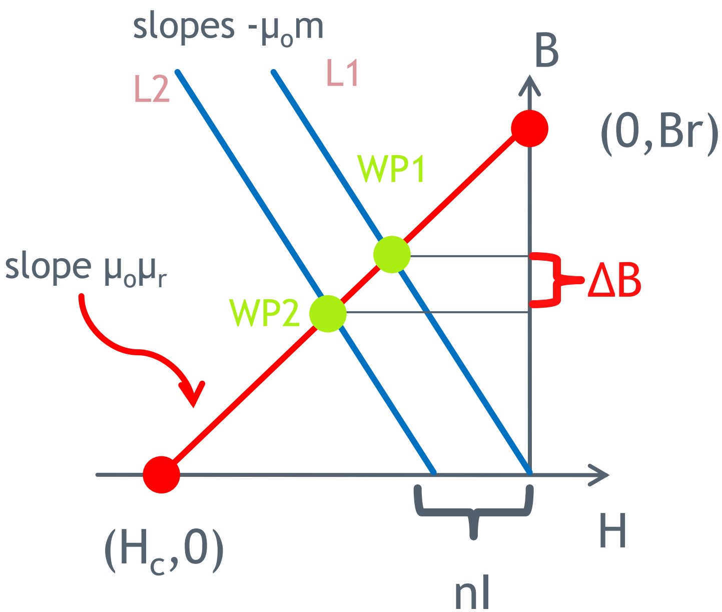
- Top 2/3 of the magnet.
- The lower 1/3 is a meter below.

- At the beginning we can use the top 2/3 of the magnet to tweak our system. We should be able to make a complete Watt balance experiment with that, until we have settled for an induction coil.
- The field is not as uniform.
- Shielding is not perfect.

Field quality of the upper 2/3



External Demagnetizing field on PM



recoil curve: $B(H) = B_r + \mu_0\mu_r H$

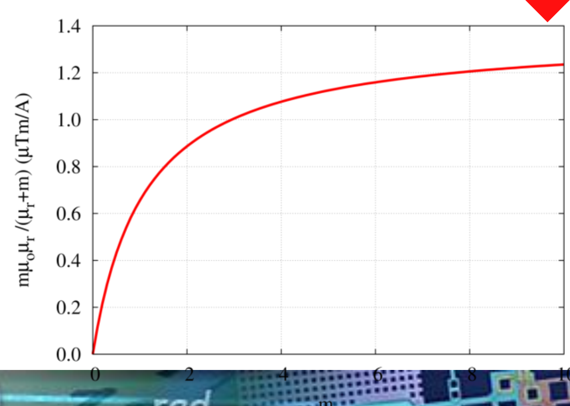
load curve 1: $B(H) = -\mu_0 mH$

load curve 2: $B(H) = -\mu_0 mH - \mu_0 mnI$

working point 1: $H_{wp1} = \dfrac{-B_r}{\mu_o(\mu_r + m)}$ $\quad B_{wp1} = \dfrac{m}{\mu_r + m} B_r$

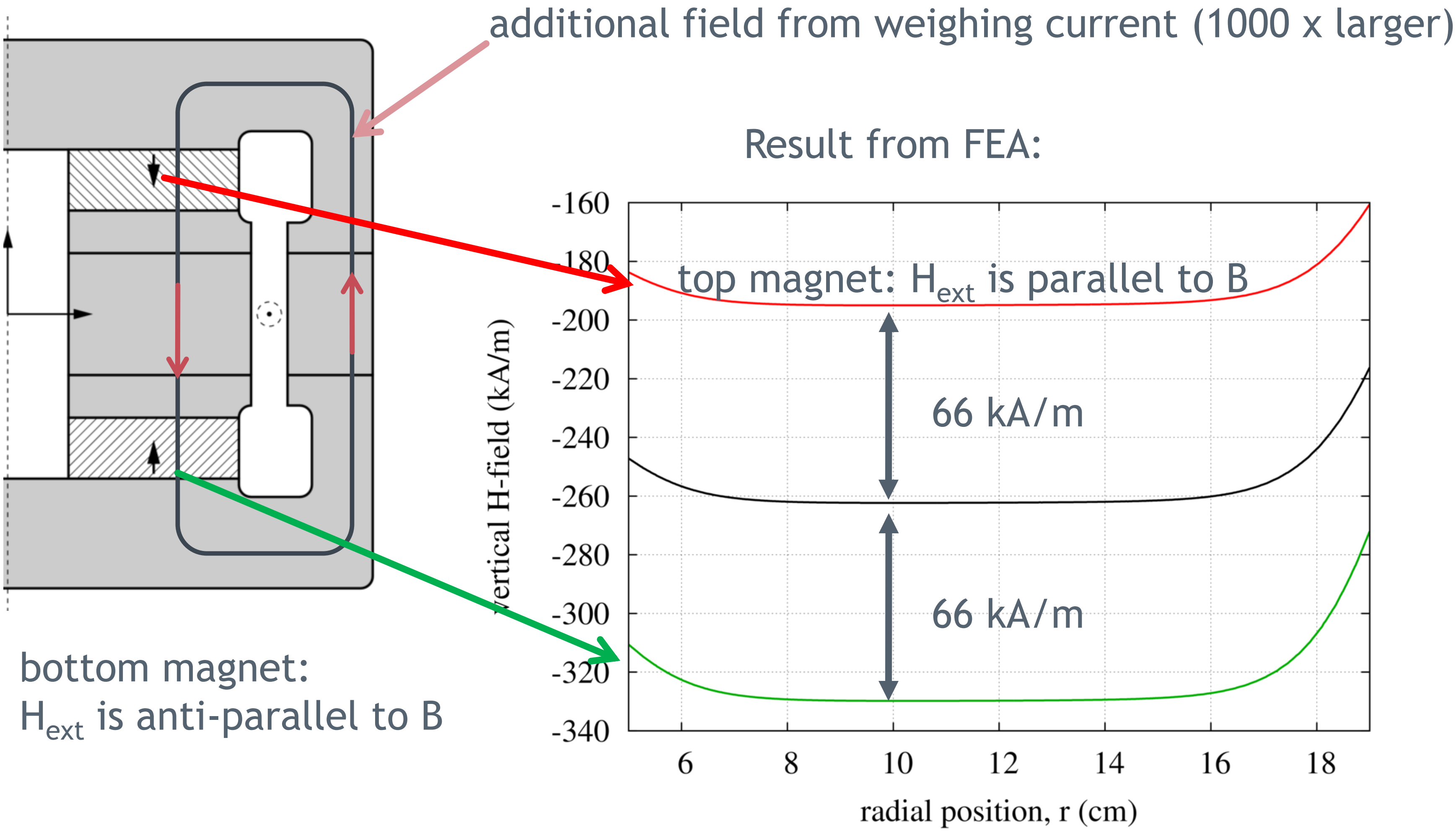
working point 2: $H_{wp2} = \dfrac{-B_r}{\mu_o(\mu_r + m)} - \dfrac{mnI}{(\mu_r + m)}$ $\quad B_{wp2} = \dfrac{mB_r - m\mu_o\mu_r nI}{\mu_r + m}$

change in B: $\Delta B = \dfrac{m\mu_o\mu_r nI}{\mu_r + m}$



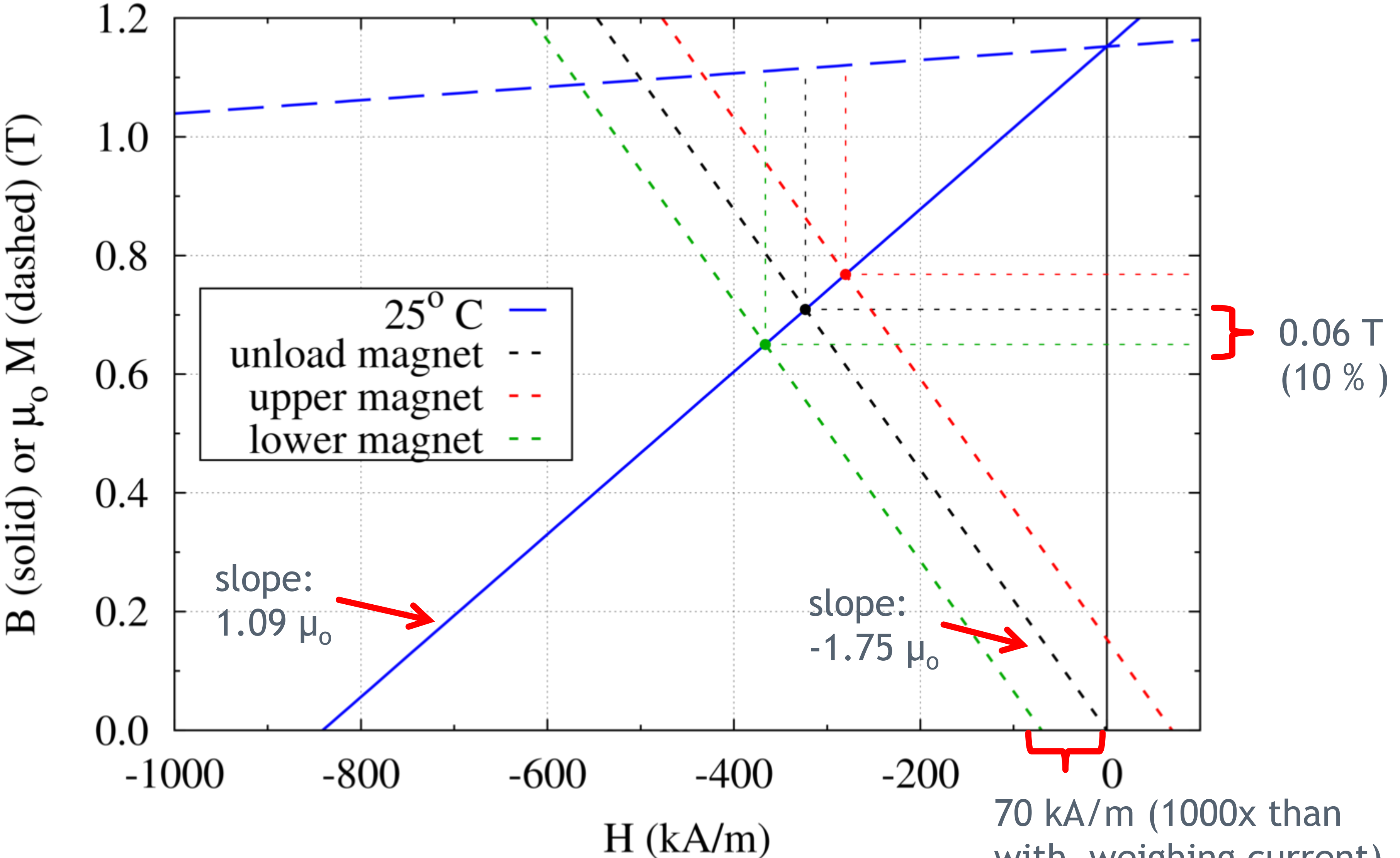
smaller m is better

Additional H-field at the Magnet

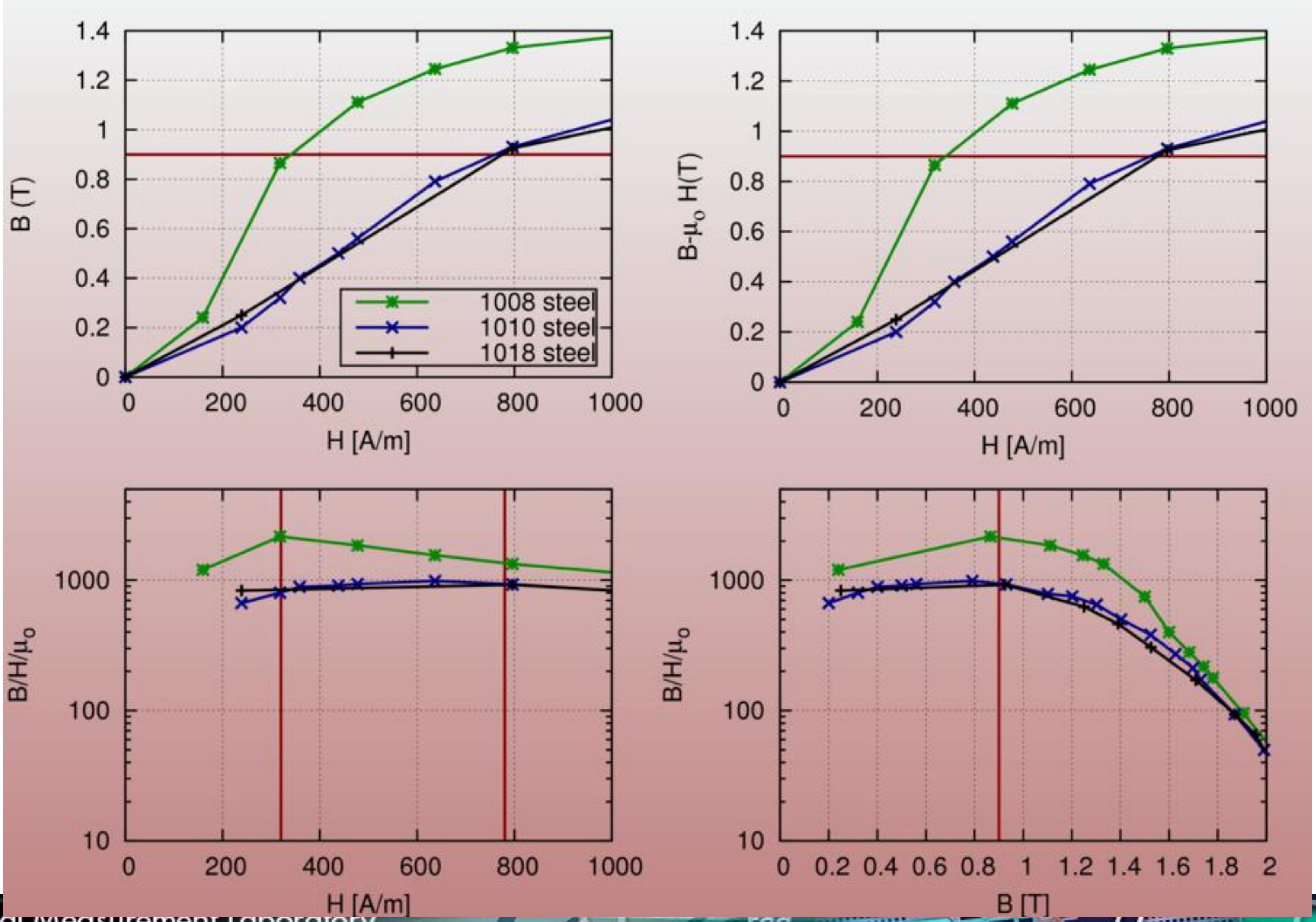


Since there are two magnets, the effect cancels.
One magnet gets stronger, the other weaker by the same amount!

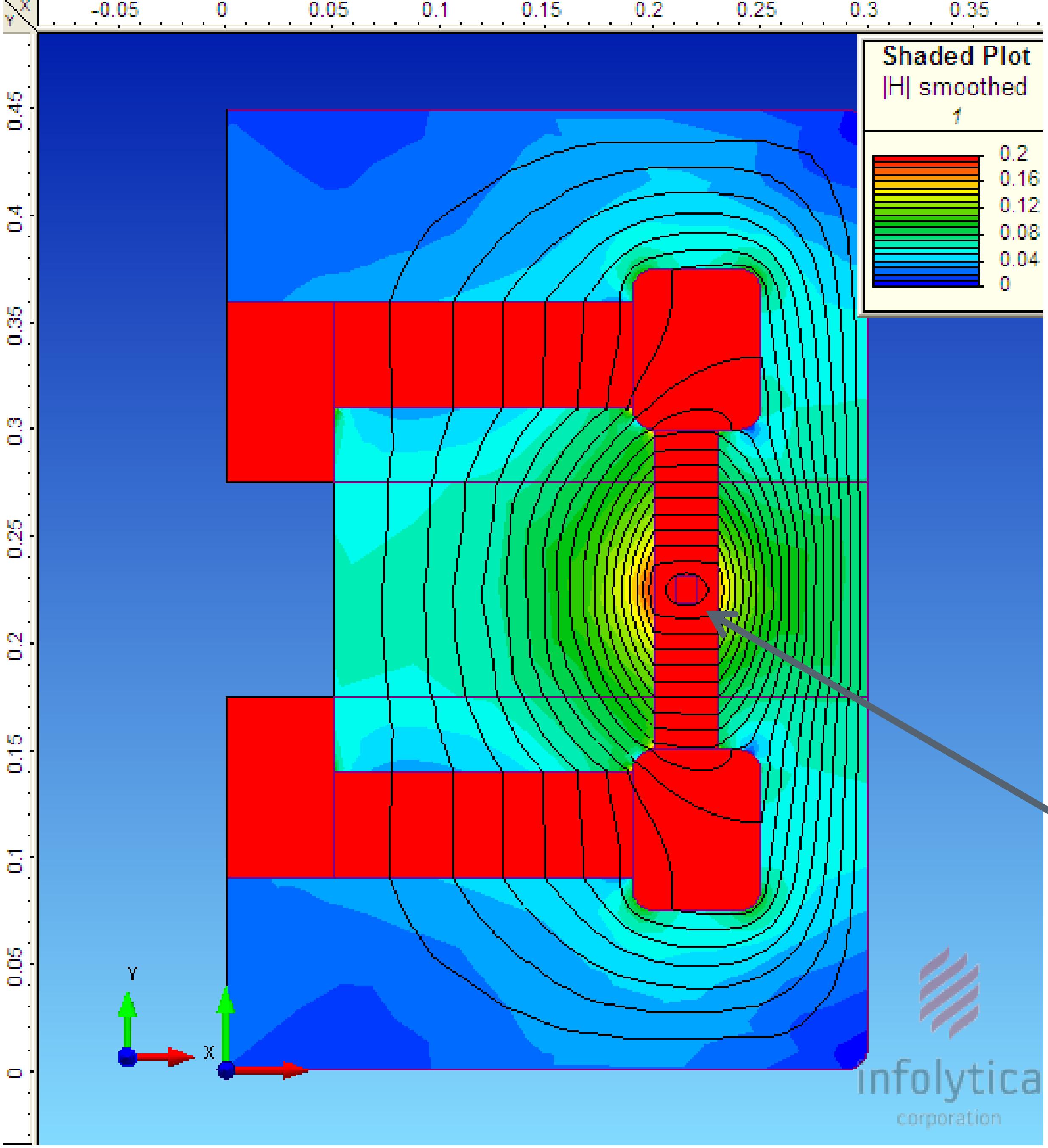
Semi Analytic result



Yoke Material at working point



H-field in Yoke due to weighing current

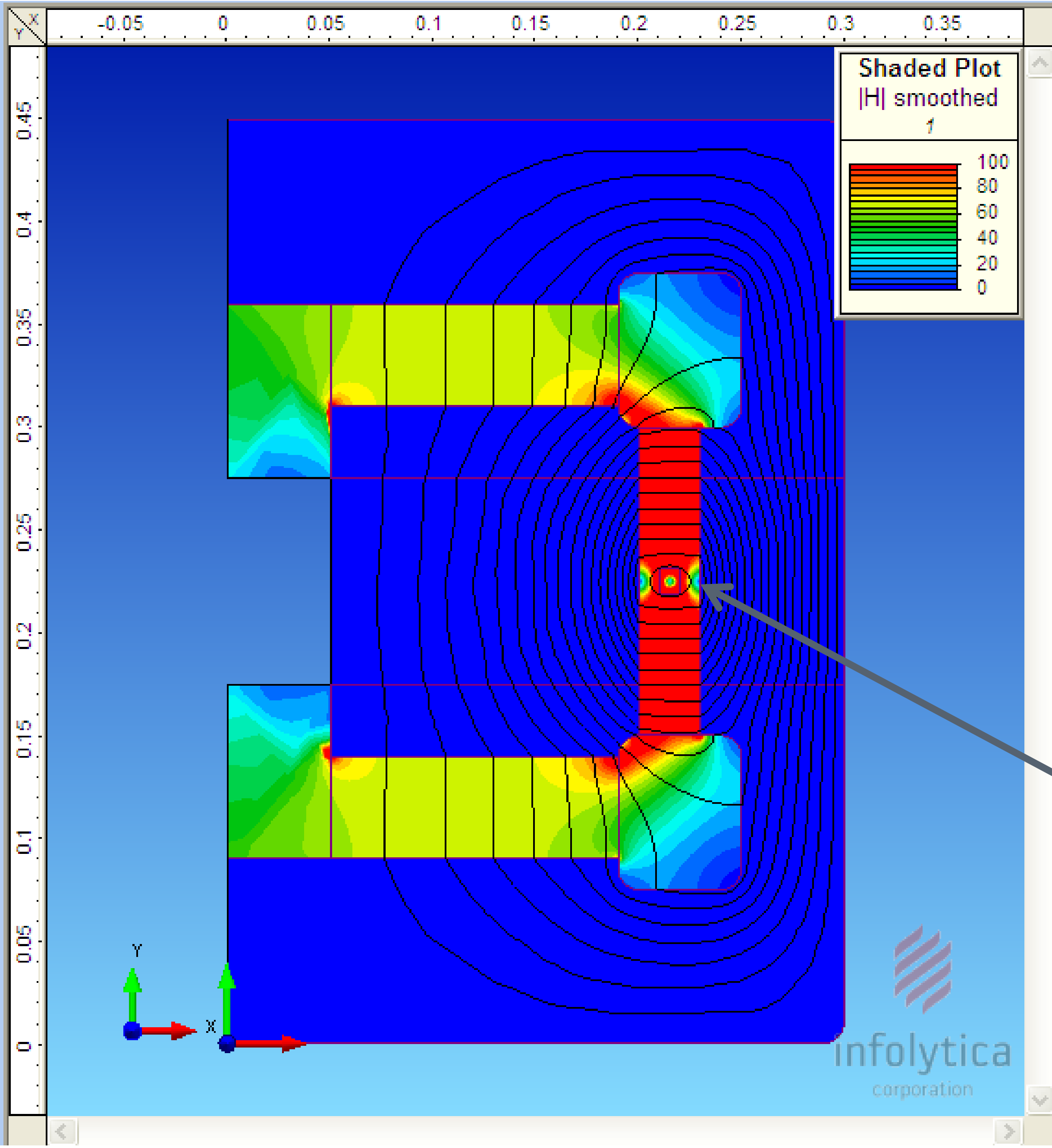


H in Yoke < 0.2 A/m

0.2/800 = 250 ppm

6.82 A-t
(10 mA 682
turns)

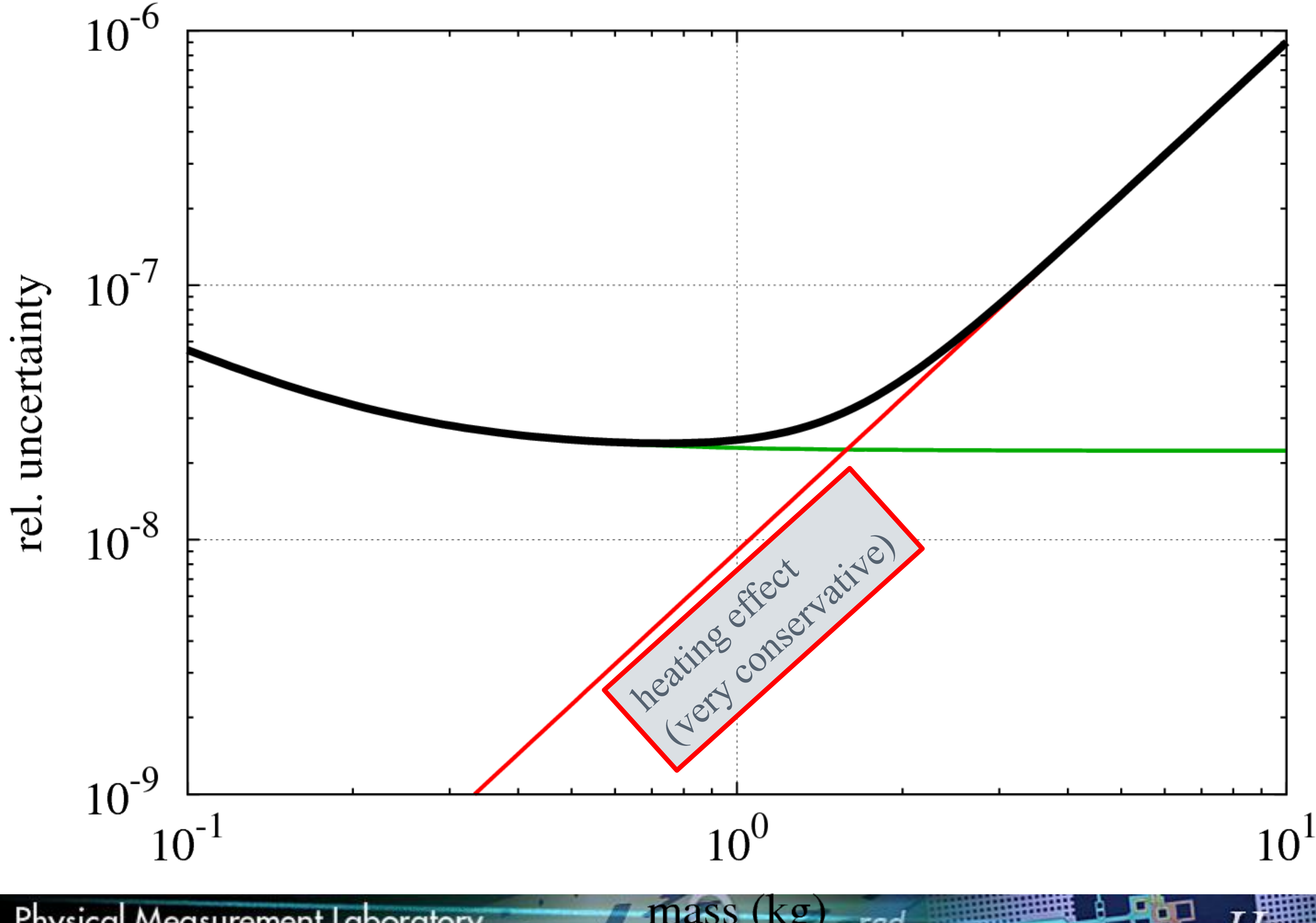
H-field in PM caused by weighing current



H in Magnet ≈70 A/m, see next slide.

6.82 A-t
(10 mA 682 turns)

Uncertainty as a function of mass



The Watt balance master equation

$$\frac{h}{h_{90}} = \frac{W_{90}}{W} = \frac{\{mgv\}_{SI}}{\{UI\}_{90}}$$

with

$$h_{90} = \frac{4}{K_{J-90}^{2} R_{K-90}} = 6.626\,068\,854\,36 \times 10^{-34}\ \text{Js}$$