The revised international system of units (SI)

April 17th 2019

APS Mid-Atlantic Senior Physicists Group

College Park, MD

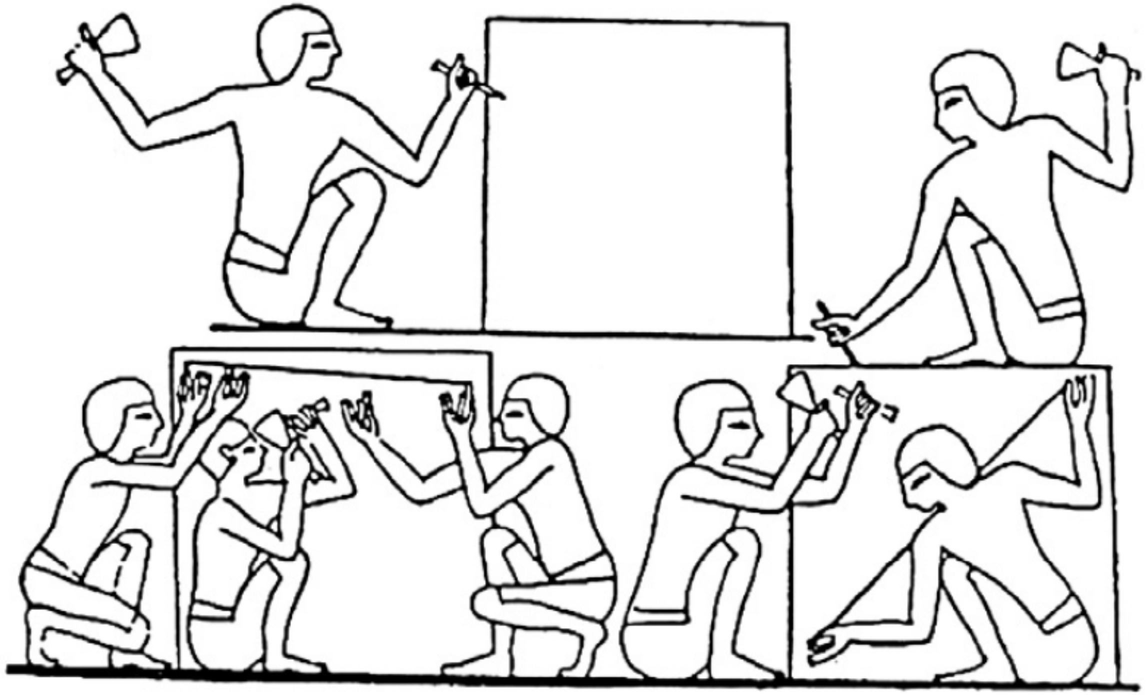
Stephan Schlamminger[+],

Darine Haddad, Frank Seifert, Leon Chao, David Newell, Jon Pratt

[+]Stephan.Schlamminger@nist.gov

The art of measurement has a long history

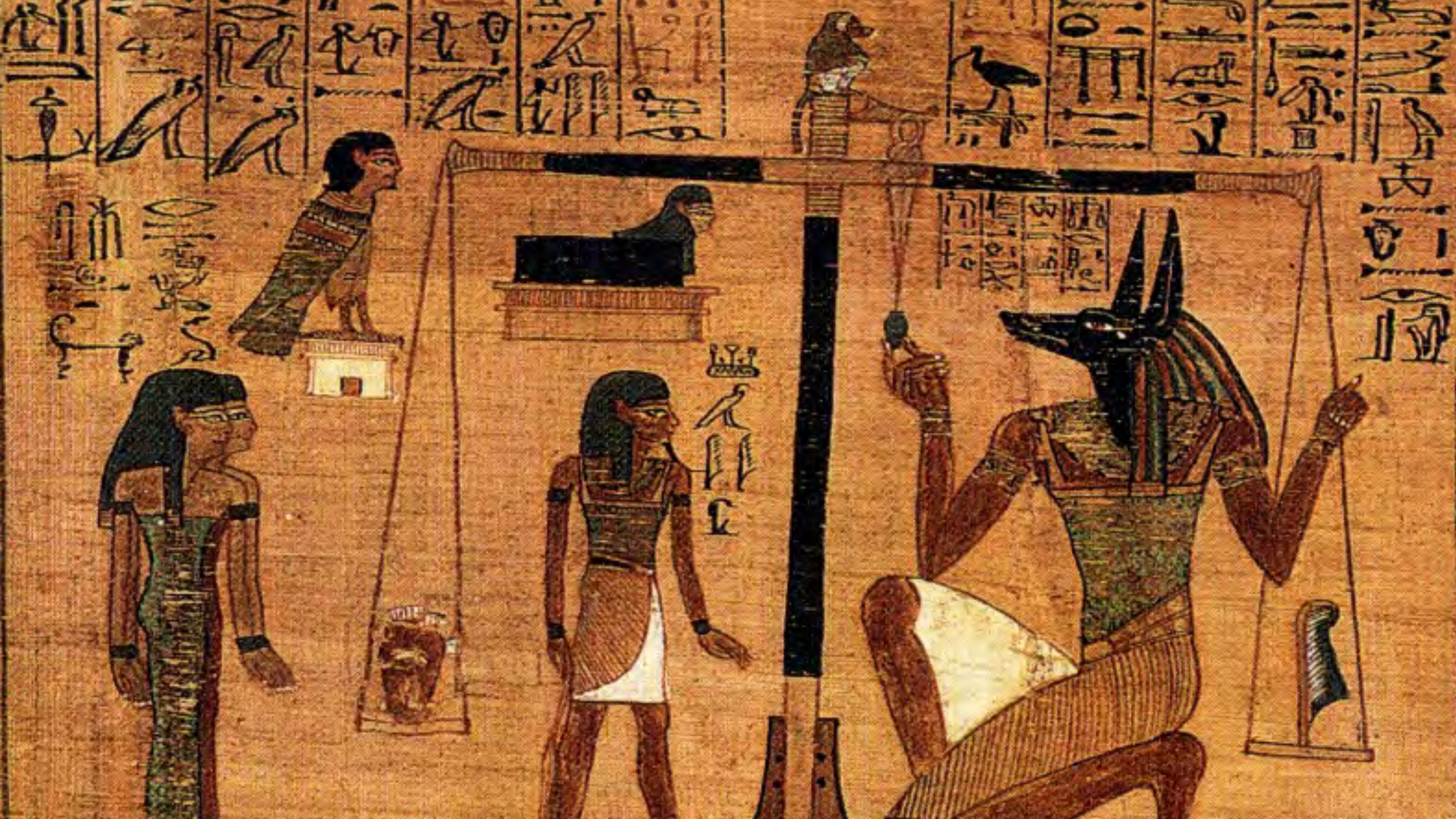
Egyptian wall painting in the tomb of Rekh-mi-re’ (1440 BC)



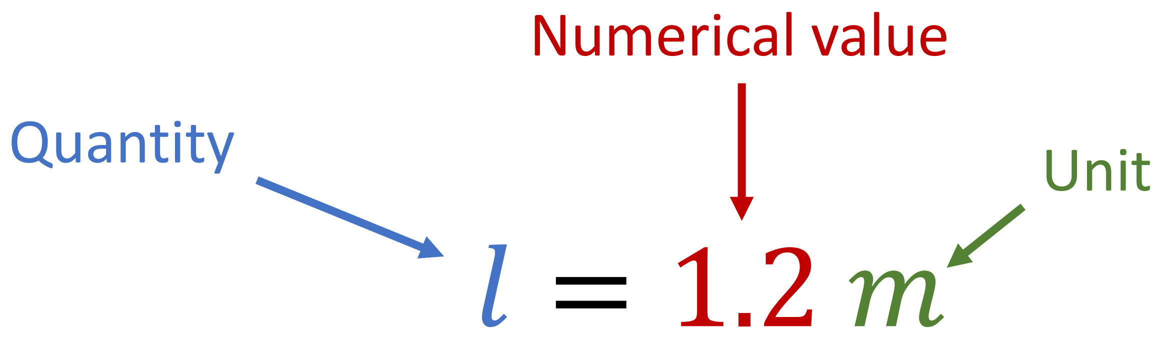
One of the earliest record of geometrical measurements.

Chinese Han dynasty (9 BC)

Vernier caliper



Quantities, units ..



Numerical value

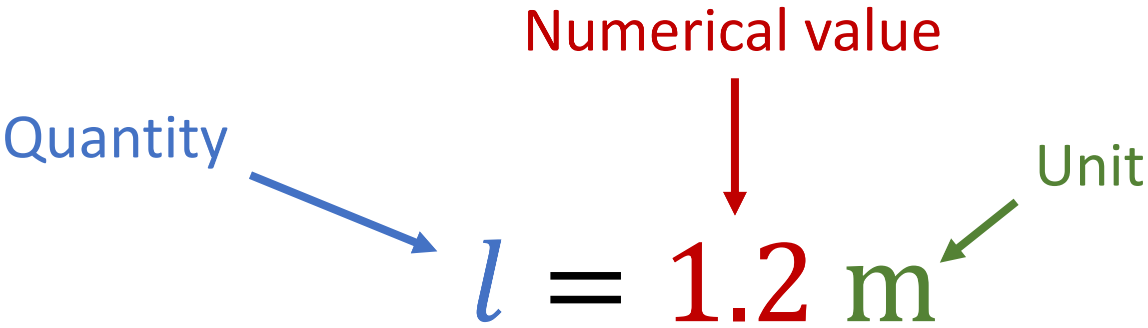
Quantity

Unit

$$l = 1.2\,m$$

$$l = \{l\}[l]$$

Quantities, units ..


Numerical value

Quantity

Unit

$$l = 1.2\ \mathrm{m}$$

$$l = \{l\}[l]$$

$$l = 4\ \mathrm{ft}$$

Units for many quantities

new Unit

$$A = w \times l = 0.8\ \mathrm{m} \times 1.2\ \mathrm{m} = 0.96\ \mathrm{m}^2$$

$$v = \frac{\Delta s}{\Delta t} = \frac{0.8\ \mathrm{m}}{0.1\ \mathrm{s}} = 8\,\frac{m}{s}$$

Do we need an infinite amount of units to measure the quantities in our lives?

Units for many quantities

new Unit

$$A = w\times l = 0.8\ \mathrm{m}\times 1.2\ \mathrm{m} = 0.96\ \mathrm{m}^2$$

$$v = \frac{\Delta s}{\Delta t} = \frac{0.8\ \mathrm{m}}{0.1\ \mathrm{s}} = 8\,\frac{m}{s}$$

Do we need an infinite amount of units to measure the quantities in our lives?

No, we can use a set of base units and combine these to new units.

A system of units

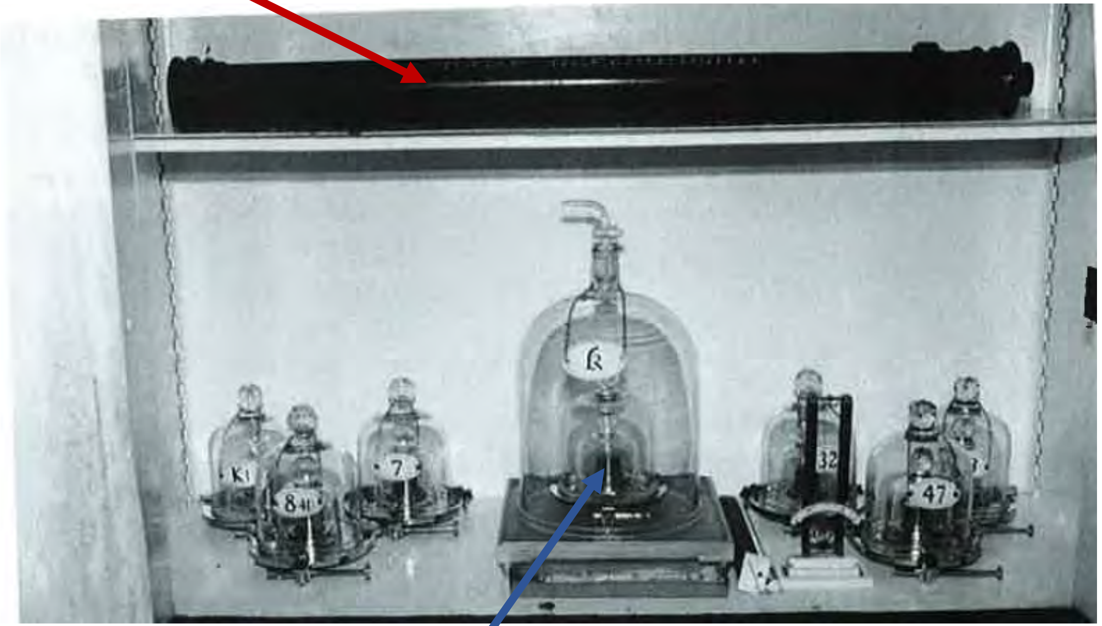
The international system of units (SI)

traces back to the meter convention

held from March 1st – May 20th 1875
in Paris.

17 signatory states.

International Prototype of the meter



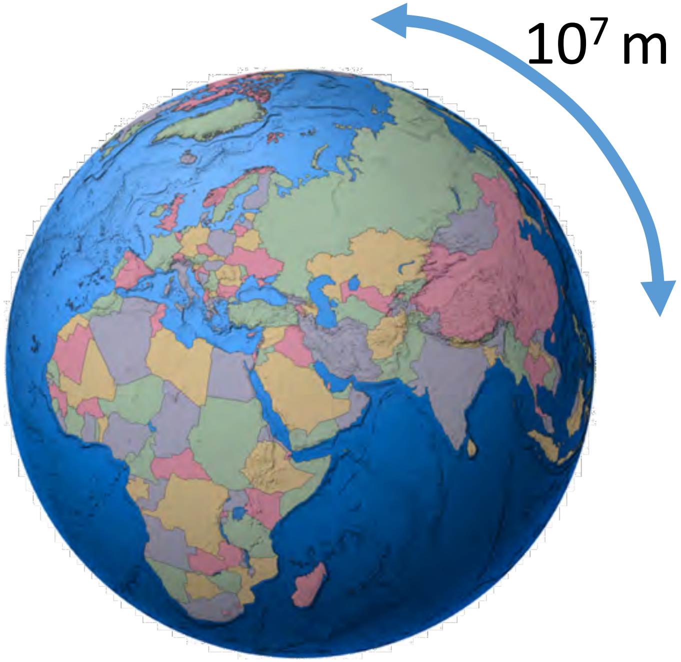
International Prototype of the kilogram

Historical evolution

before 1789

regional weights and measures
decreed by regional rulers

since 1875


10^7 m

international w/m
Earth sets standard

1980

c

international w/m
fundamental constants

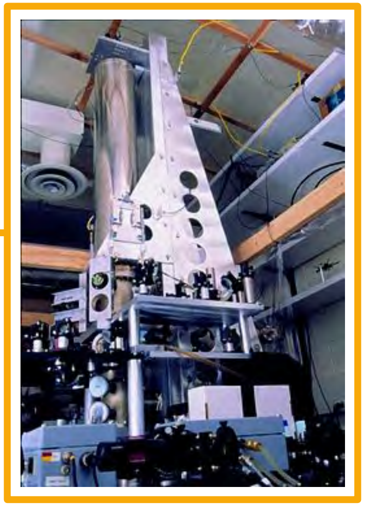
"For all times, for all people"

The present SI has seven base units

Do not confuse the logos

LaLiga

The present SI has seven base units

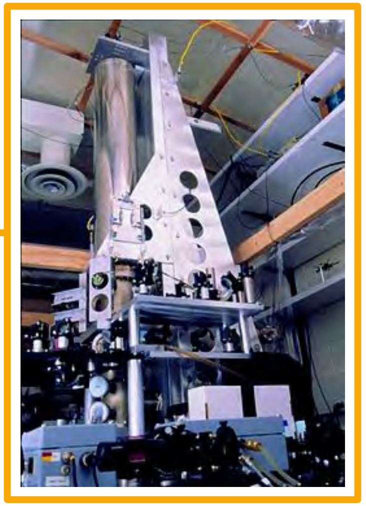


The present SI has seven base units

$c = 299\,792\,458\ \mathrm{m/s}$

The present SI has seven base units

$c = 299\ 792\ 458\ \mathrm{m/s}$



The present SI has seven base units

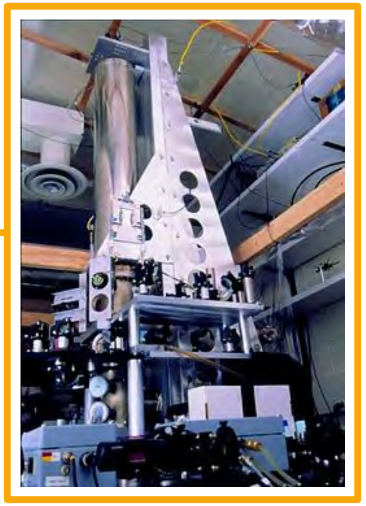
$K_{cd} = 683\ \mathrm{lm\ W^{-1}}$

$c = 299\ 792\ 458\ \mathrm{m/s}$

The present SI has seven base units

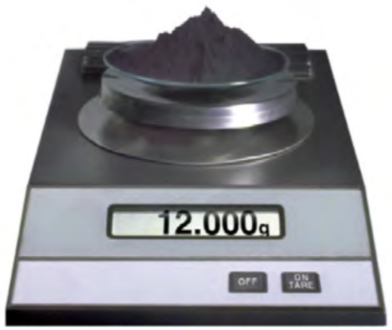
$K_{cd} = 683 \text{ lm W}^{-1}$

$c = 299\ 792\ 458 \text{ m/s}$

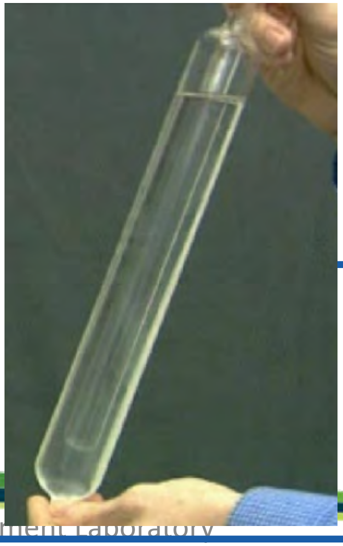


The present SI has seven base units

$K_{cd} = 683 \text{ lm W}^{-1}$



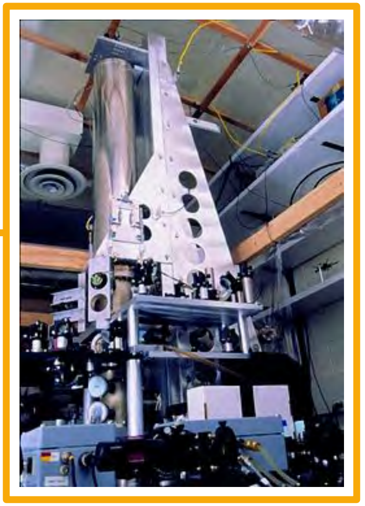
$c = 299\ 792\ 458 \text{ m/s}$

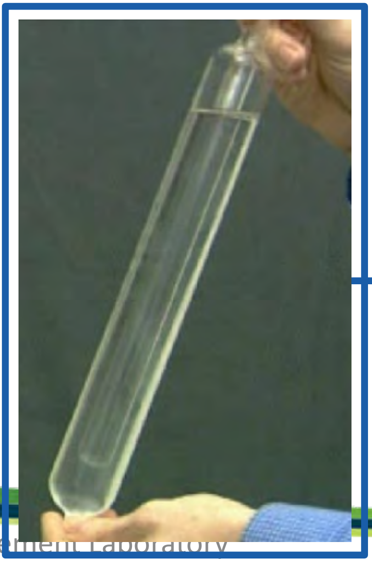


The present SI has seven base units

$K_{cd} = 683 \text{ lm W}^{-1}$

$c = 299\ 792\ 458 \text{ m/s}$



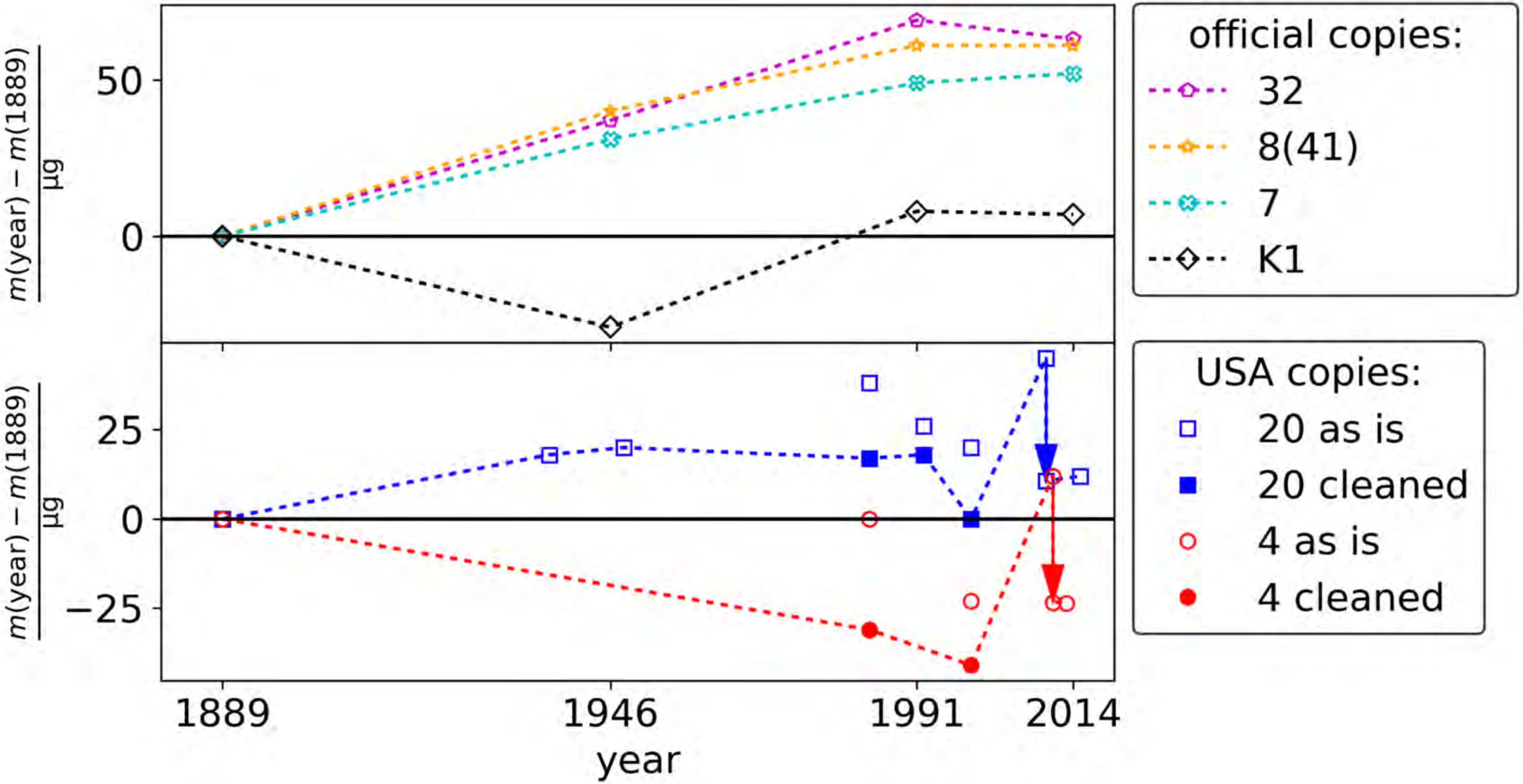


The definition of the kilogram

The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram. (1889)


3sat

Long term stability and accessibility



Why change the present SI?

Brian Josephson

awarded the Nobel Prize in 1973

Josephson effect

Quantum voltage

$$U = n\frac{h}{2e}f$$

Josephson constant

$$K_j = \frac{2e}{h}$$

Klaus von Klitzing

awarded the Nobel Prize in 1985

integral quantum Hall effect

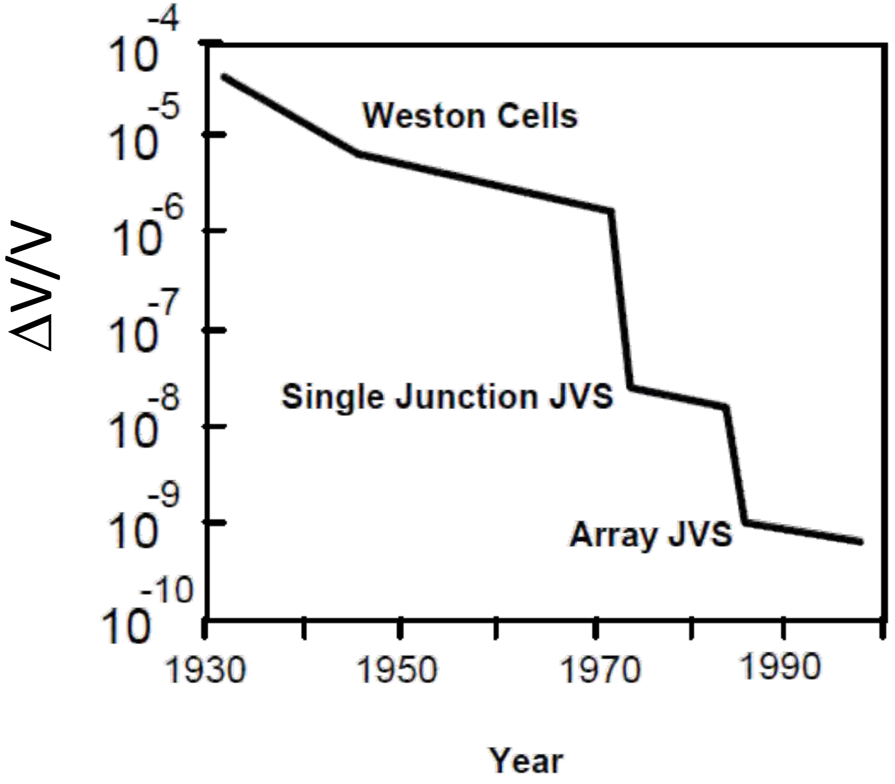
Quantum resistance

$$R = \frac{1}{n}\frac{h}{e^2}$$

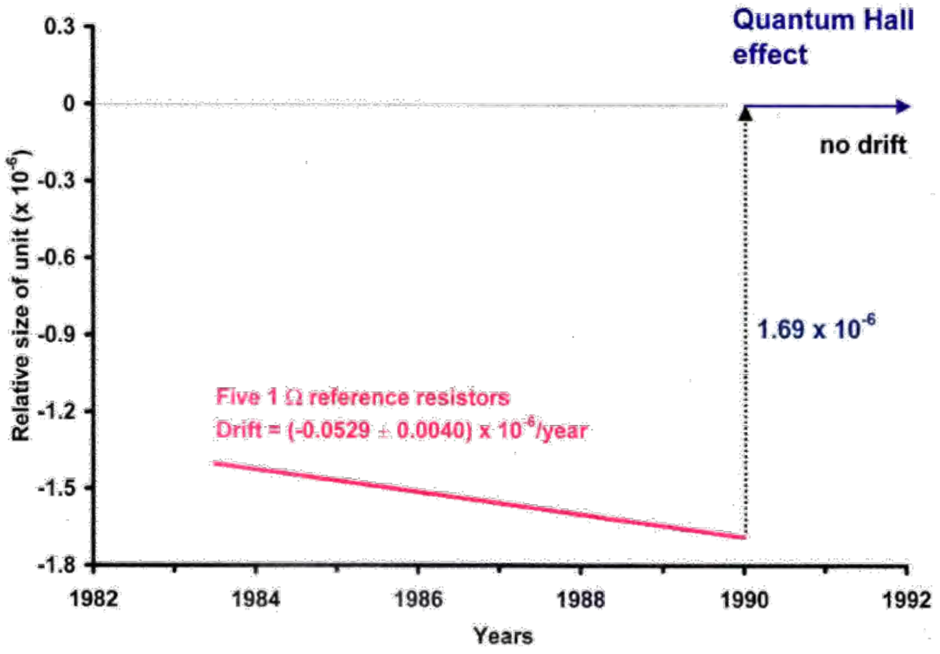
von Klitzing constant

$$R_K = \frac{h}{e^2}$$

Quantum leap in electrical metrology



C.A. Hamilton, *Rev. Sci. Instr.* **71** ,3611 (2000).



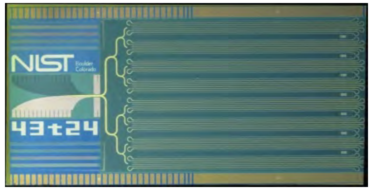
R.E. Elmquist et al., *J. Res. NIST* **106** ,65 (2001).

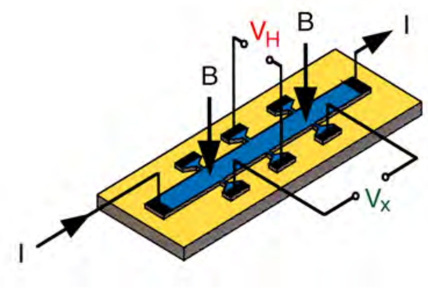
From “artifacts” to quantum standards

Weston cell

Thomas-type resistor

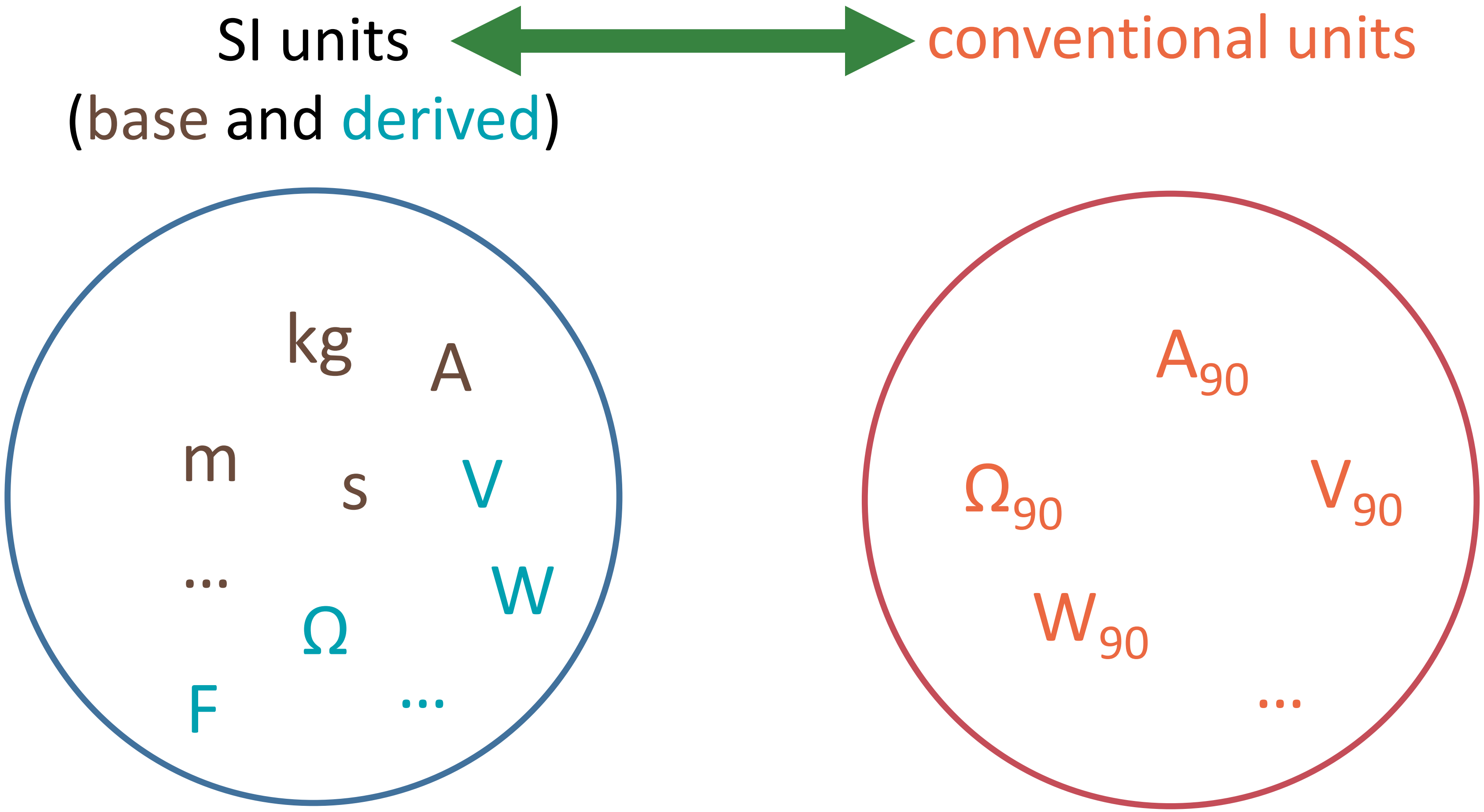
Josephson
Junction
Array





Quantum Hall Device

A schism in metrology



SI units ⟷ conventional units

(base and derived)

kg A

m s V

… W

Ω

F …

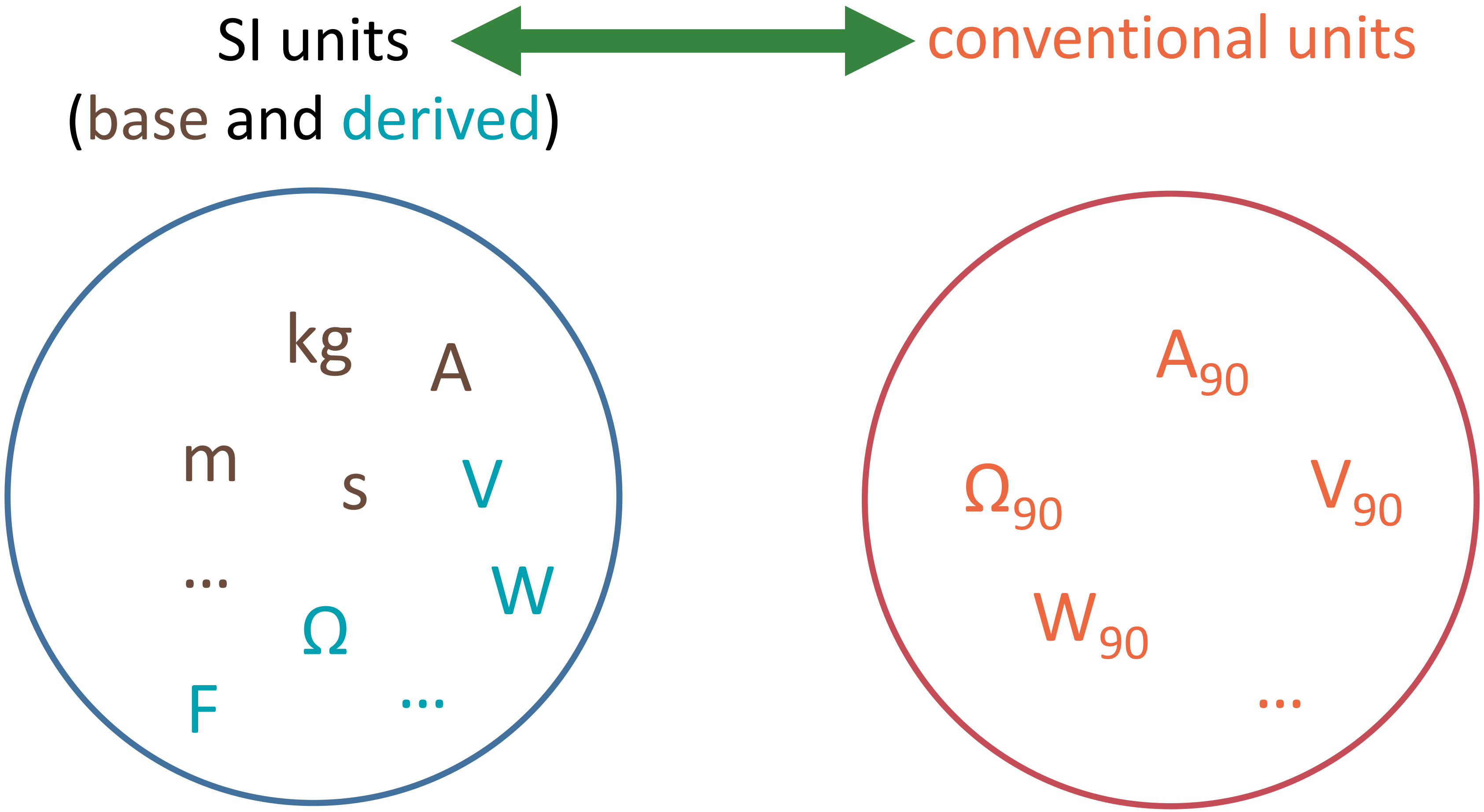
A_{90}

Ω_{90} V_{90}

W_{90}

…

A schism in metrology



SI units ↔ conventional units

(base and derived)

kg A

m s V

… W

Ω

F …

A_{90}

Ω_{90} V_{90}

W_{90}

…

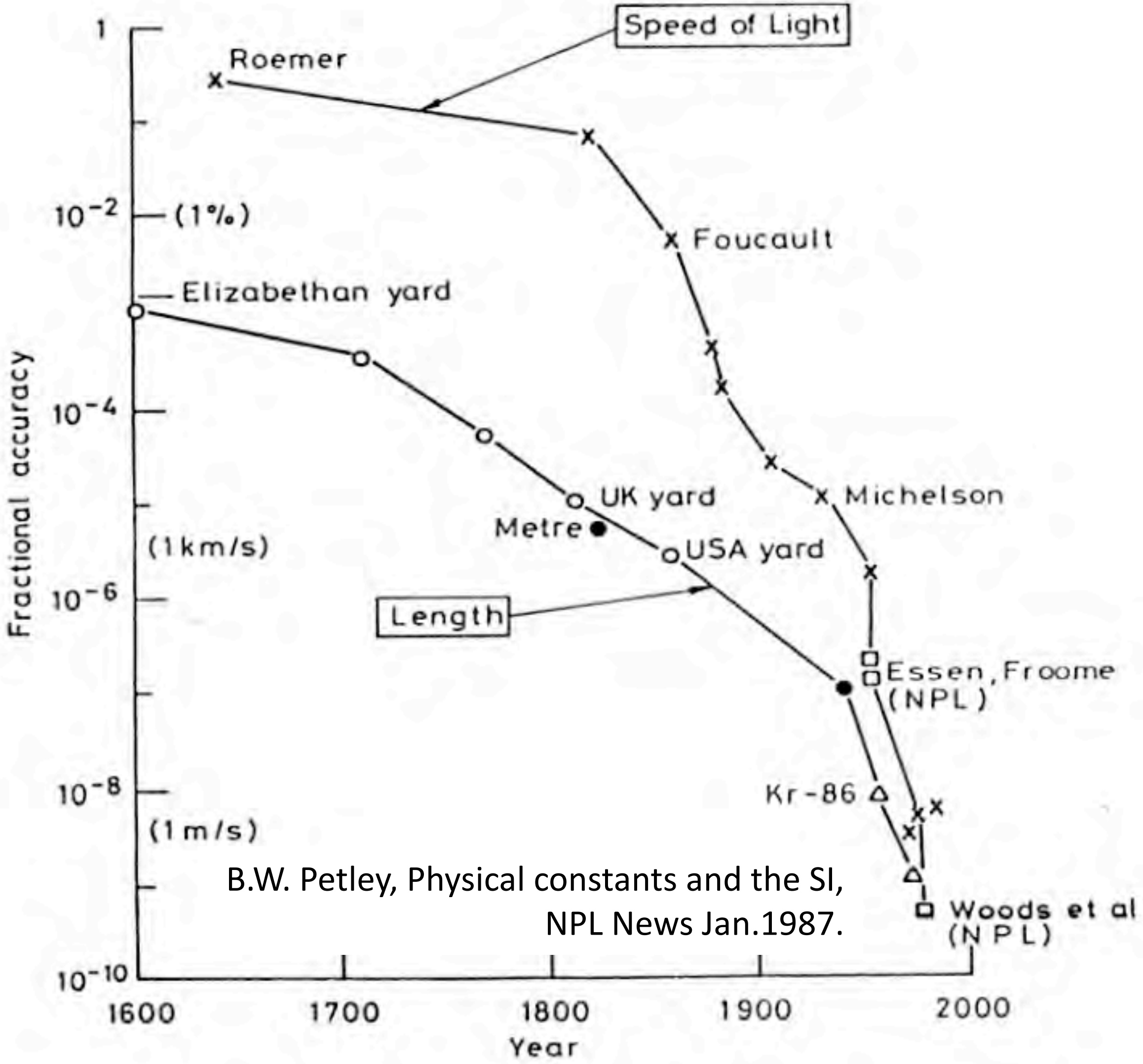
presently

$$\frac{V_{90}}{V} = 1 + 9.83\times10^{-8}$$

$$\frac{\Omega_{90}}{\Omega} = 1 + 1.77\times10^{-8}$$

$$\frac{W_{90}}{W} = 1 + 17.9\times10^{-8}$$

Fundamental constants – better than artifacts



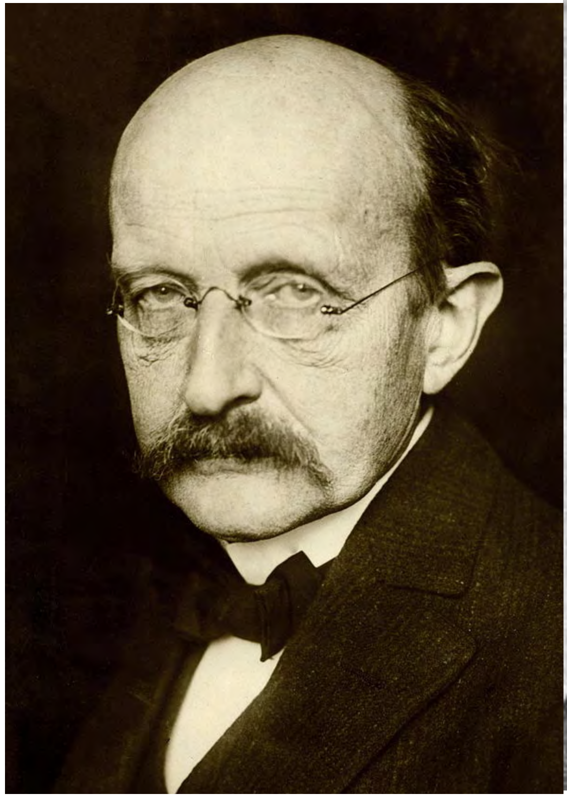
B.W. Petley, Physical constants and the SI, NPL News Jan.1987.

Fundamental constants are:

- stable
- accessible
- scale invariant
- measurements can be improved

$$\underset{\underset{\text{given}}{\uparrow}}{c} = \underset{\underset{\text{given}}{\uparrow}}{299\ 792\ 458}\ \underset{\underset{\text{unknown}}{\uparrow}}{\mathrm{m}}/\underset{\underset{\text{given}}{\nwarrow}}{\mathrm{s}}$$

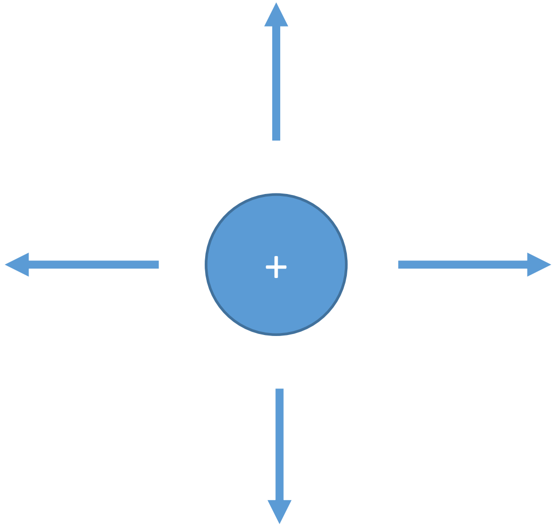
Four additional constants will be fixed on 5/20/19



h

N_A

k



e

The new defining constants

Planck constant $h = 6.626\,070\,15{\times}10^{-34}\ \mathrm{kgm^2s^{-1}}$ ⇨ kg

The new defining constants

Planck constant $h = 6.626\,070\,15{\times}10^{-34}\ \mathrm{kgm^2s^{-1}}$ ⇨ kg

Avogadro constant $N_A = 6.022\,140\,76{\times}10^{23}\ \mathrm{mol^{-1}}$ ⇨ mol

The new defining constants

Planck constant $h = 6.626\,070\,15{\times}10^{-34}\ \mathrm{kgm^2s^{-1}}$ ⇒ kg

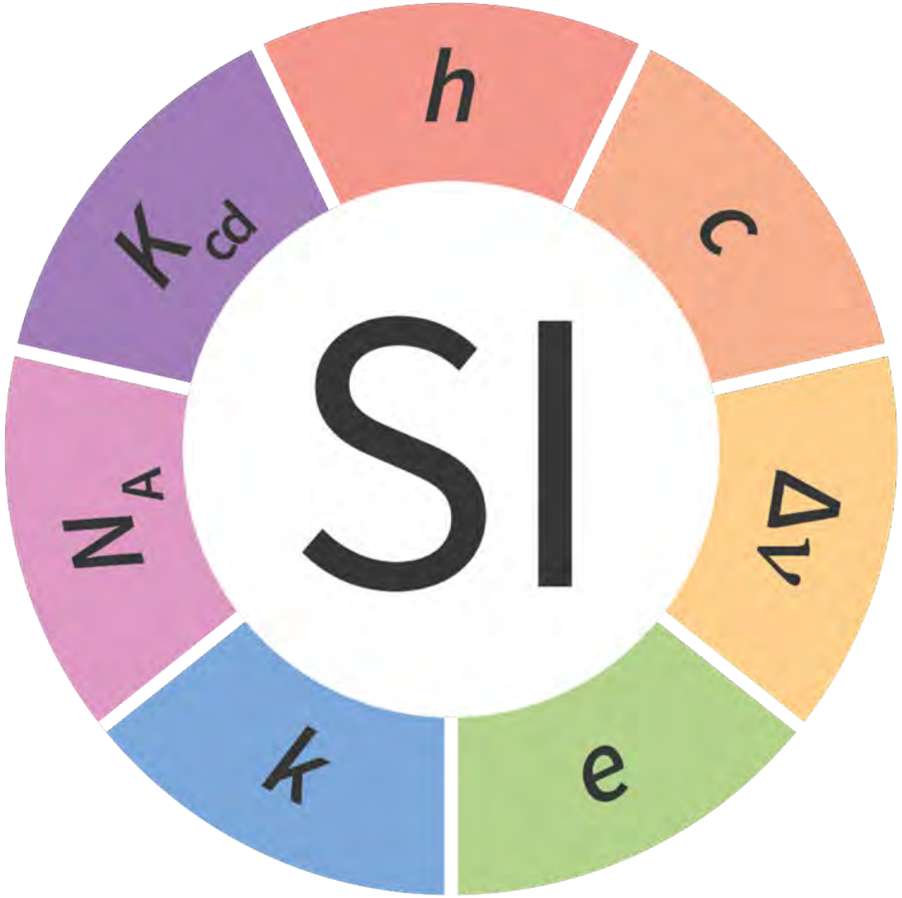
Avogadro constant $N_A = 6.022\,140\,76{\times}10^{23}\ \mathrm{mol^{-1}}$ ⇒ mol

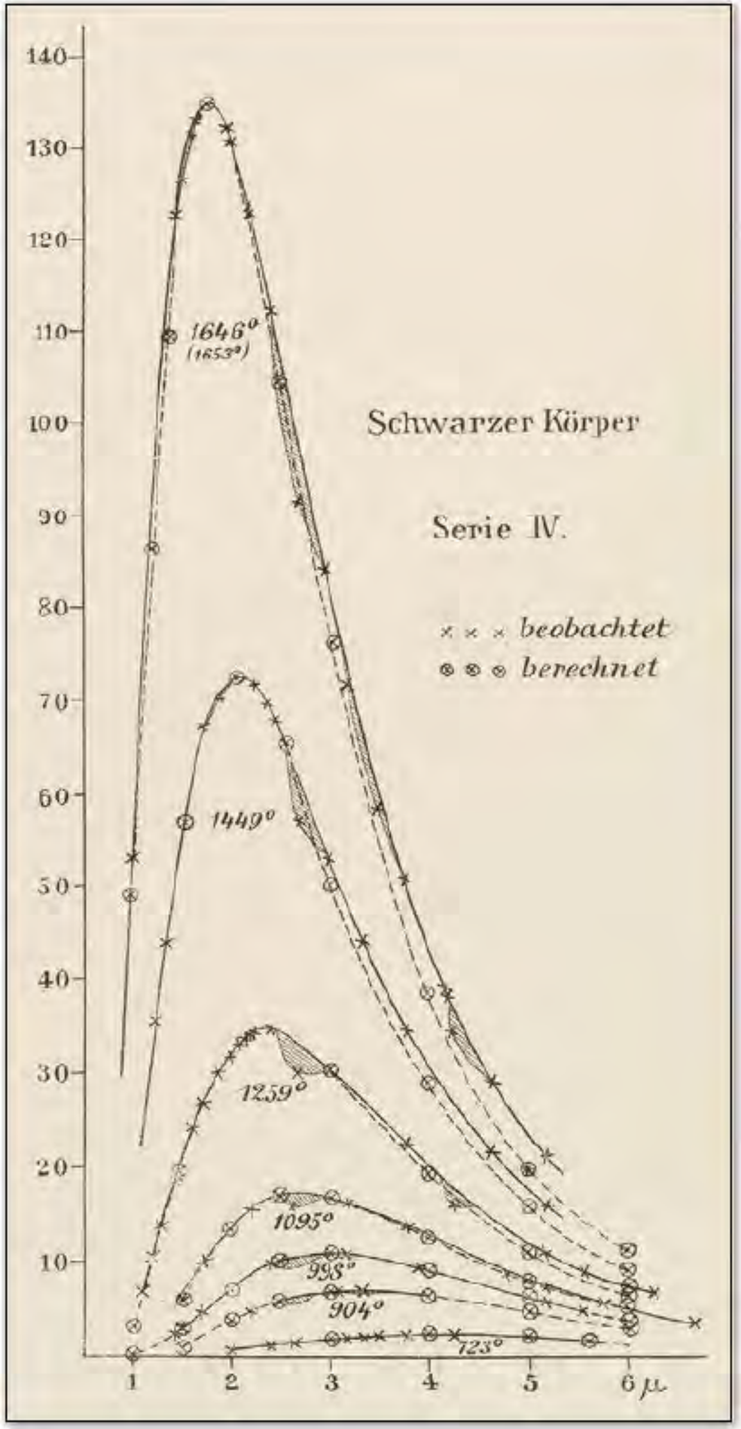
Boltzmann constant $k = 1.380\,649{\times}10^{-23}\ \mathrm{kgm^2s^2K^{-1}}$ ⇒ K

The new defining constants

Planck constant	$h = 6.626\ 070\ 15{\times}10^{-34}\ \mathrm{kgm^2s^{-1}}$	⇨	kg
Avogadro constant	$N_A = 6.022\ 140\ 76{\times}10^{23}\ \mathrm{mol^{-1}}$	⇨	mol
Boltzmann constant	$k = 1.380\ 649{\times}10^{-23}\ \mathrm{kgm^2s^2K^{-1}}$	⇨	K
Elementary charge	$e = 1.602\ 176\ 6341\ {\times}10^{-19}\mathrm{As}$	⇨	A

The new SI has seven defining constants





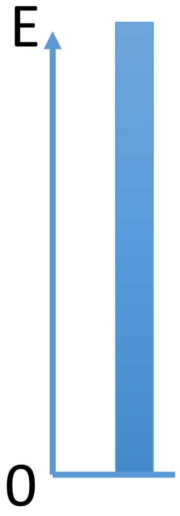
h

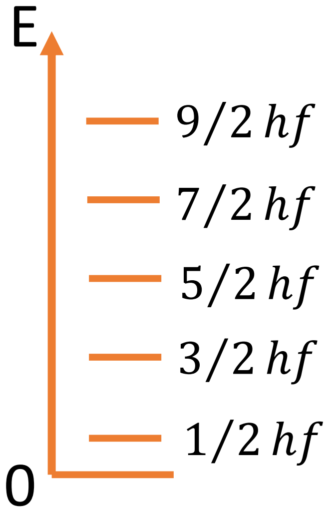
H. Kubbinga, A tribute to Max Planck, *Europhysics News* **49**, 27 (2018)

About *h*

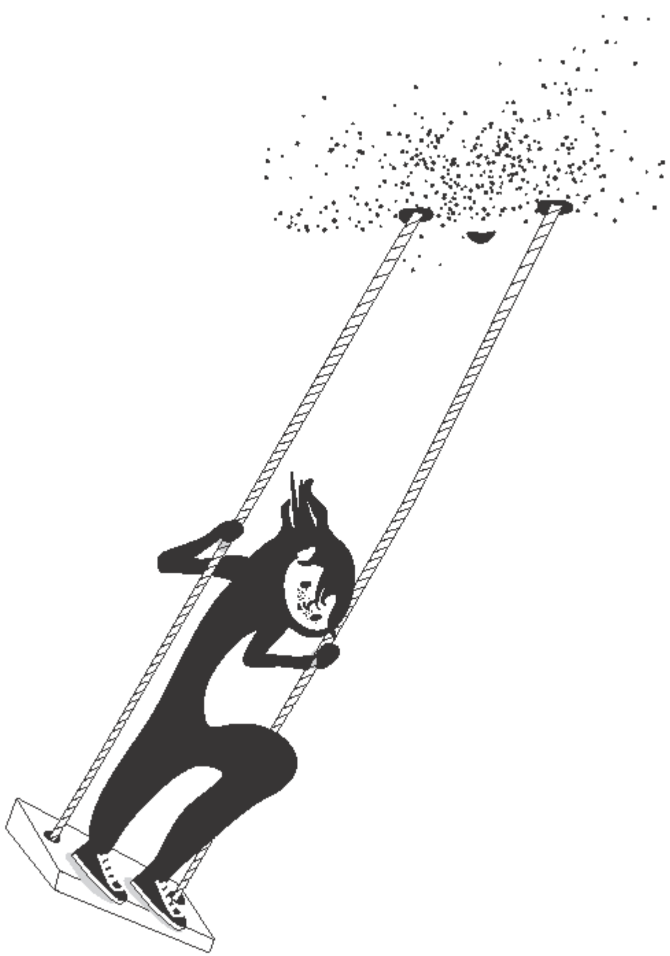
In a classical oscillator, the system's energy is continuous.

In a quantum oscillator, the system's energy is discrete.





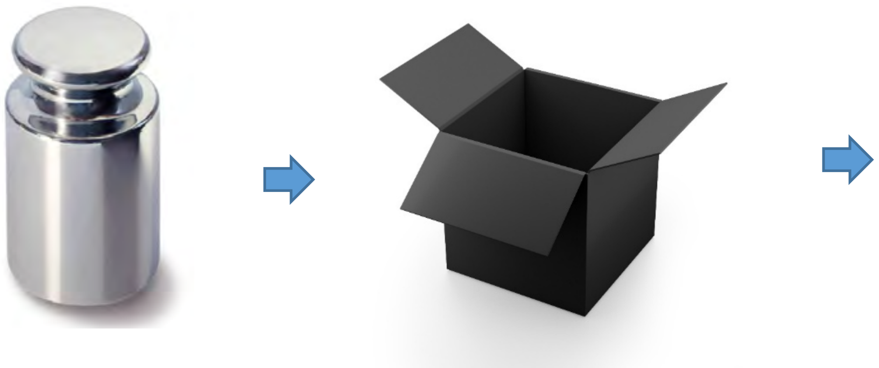
The Planck constant



The constant of quantum mechanics……

$$h = 6.626\ 070\ 15{\times}10^{-34}\ \mathrm{kg\ m^2 s^{-1}}$$

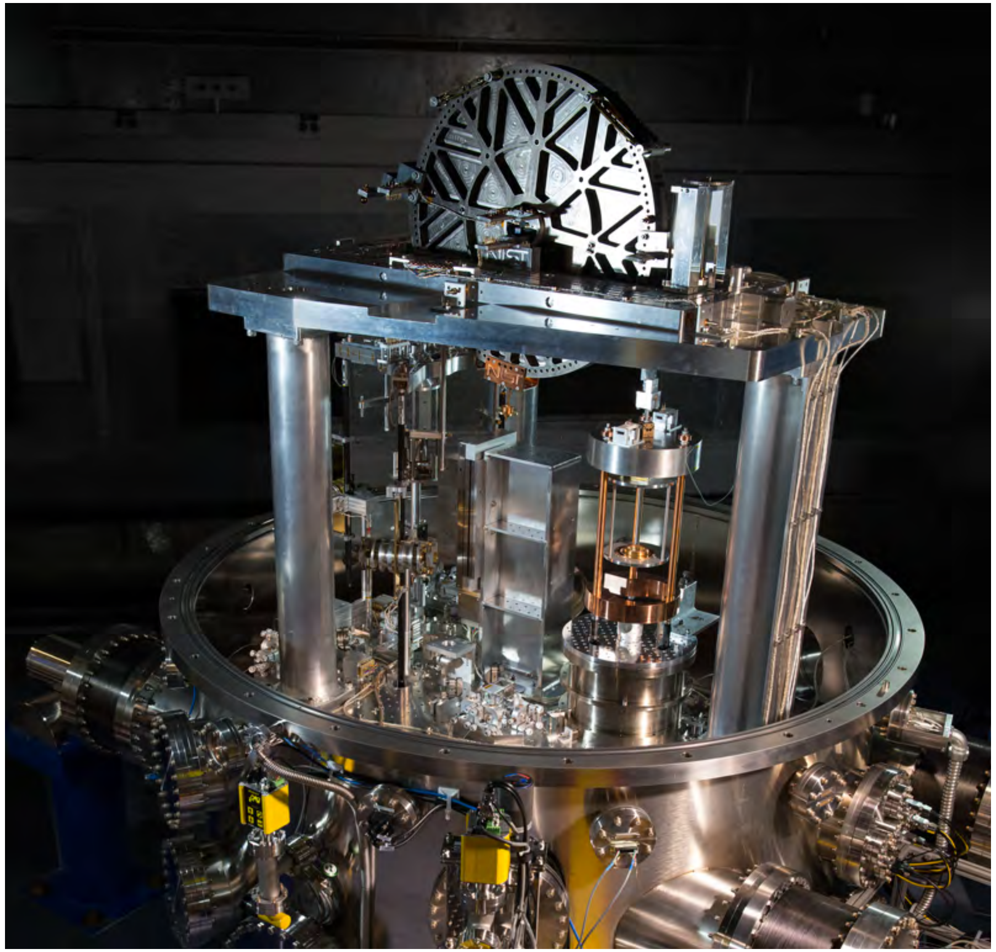
…..is measured in units kilogram (and m and s).



h

Two ways to connect *h* to the kilogram

X-**R**ay **C**rystal **D**ensity Method (XRCD)



Kibble balance

Photo: PTB

Two ways to connect *h* to the kilogram

- Use a small mass
- Scale
- $m \propto h$
- $M = r \cdot N \cdot m$

X-**R**ay **C**rystal **D**ensity Method (XRCD)

- Quantum electrical standards
- $R_K = \frac{h}{e^2}, K_J = \frac{h}{2e}$
- $\frac{K_J^2}{R_K} = \frac{h}{4}$
- $P_{el} = P_{mech}$

Kibble balance

X-Ray Crystal Density Method

$$E_n = \frac{1}{2} m_e c^2 \alpha^2 \frac{1}{n^2}$$

X-Ray Crystal Density Method

$$E_n = \frac{1}{2} m_e c^2 \alpha^2 \frac{1}{n^2} \quad \Rightarrow \quad m_e = 2 \frac{R_\infty}{\alpha^2 c} h$$

X-Ray Crystal Density Method

$$E_n = \frac{1}{2} m_e c^2 \alpha^2 \frac{1}{n^2} \quad \Longrightarrow \quad m_e = 2 \frac{R_\infty}{\alpha^2 c} \, h$$

$$m_{SI} = r \, m_e \qquad\qquad r \approx 51\,400$$

X-Ray Crystal Density Method

$$E_n = \frac{1}{2} m_e c^2 \alpha^2 \frac{1}{n^2} \quad \Longrightarrow \quad m_e = 2 \frac{R_\infty}{\alpha^2 c} h$$

$$m_{SI} = r \, m_e \qquad r \approx 51\,400$$

$$m_{macro} = N \, m_{SI} \qquad N \approx 2.15{\times}10^{25}$$

X-Ray Crystal Density Method

$$E_n = \frac{1}{2} m_e c^2 \alpha^2 \frac{1}{n^2} \quad \Rightarrow \quad m_e = 2 \frac{R_\infty}{\alpha^2 c} h$$

$$m_{SI} = r\, m_e \qquad r \approx 51\,400$$

$$m_{macro} = N\, m_{SI} \qquad N \approx 2.15 \times 10^{25}$$

$$rN \approx 1.1 \times 10^{30} \qquad \frac{\sigma_{rN}}{rN} \approx 1 \times 10^{-8}$$

How to count large numbers

Macroscopic volume,
volume of a molar mass

Microscopic volume,
volume of a unit cell:

$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}} \times \text{Bottles}$$

$$N_B = \frac{W \cdot L \cdot H}{w \cdot l \cdot h} \times 10$$

10 bottles per
unit cell

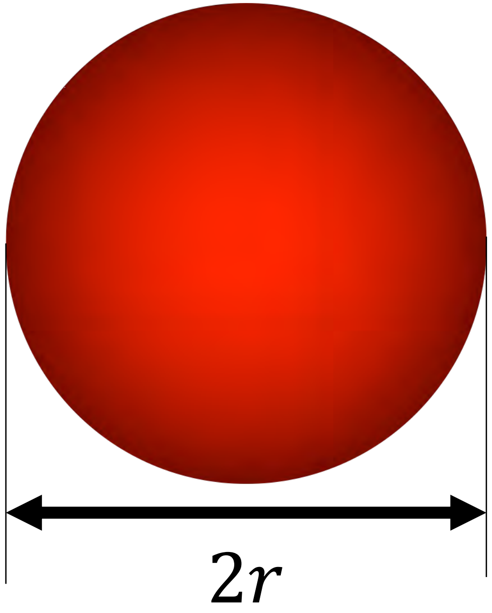
Pictures and pedagogic concept shamelessly stolen from Peter Becker, PTB

How to count large numbers

Macroscopic volume,
volume of a molar mass

Microscopic volume,
volume of a unit cell:

$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}} \times \text{Bottles}$$



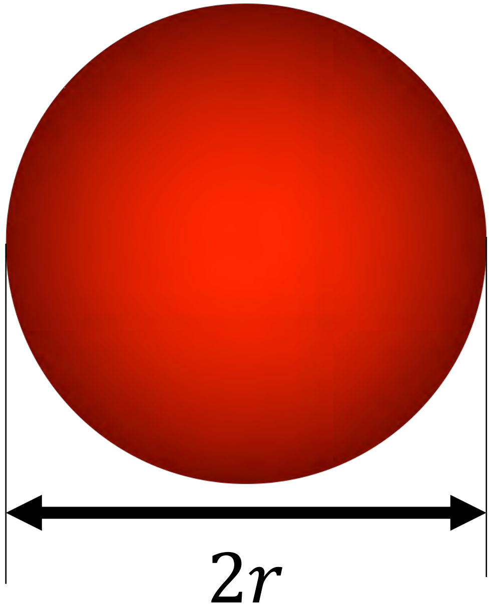
$$V = \frac{4}{3}\pi r^3$$

10 bottles per
unit cell

$$N_B = \frac{W \cdot L \cdot H}{w \cdot l \cdot h} \times 10$$

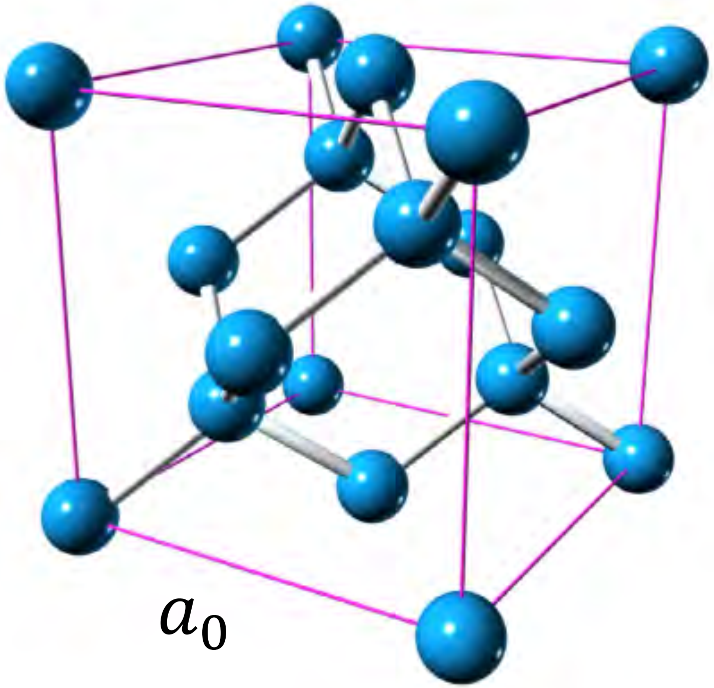
How to count large numbers

Macroscopic volume, volume of a molar mass



$$V = \frac{4}{3}\pi r^3$$

Microscopic volume, volume of a unit cell:



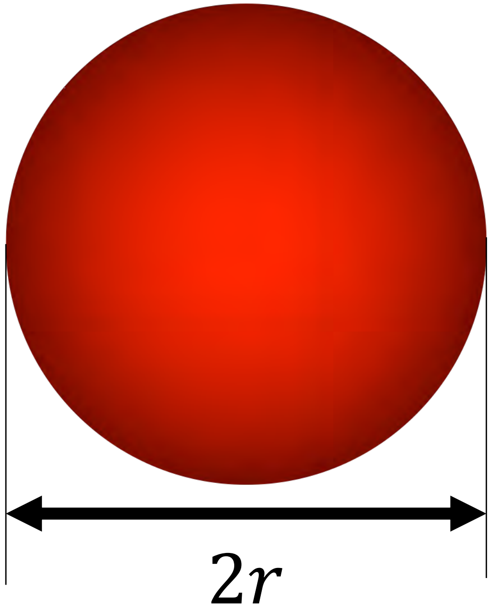
$$V_{uc} = a_0^3 = \left(\sqrt{8}d_{220}\right)^3$$

$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}} \times \text{Bottles}$$

$$N_B = \frac{W \cdot L \cdot H}{w \cdot l \cdot h} \times 10$$

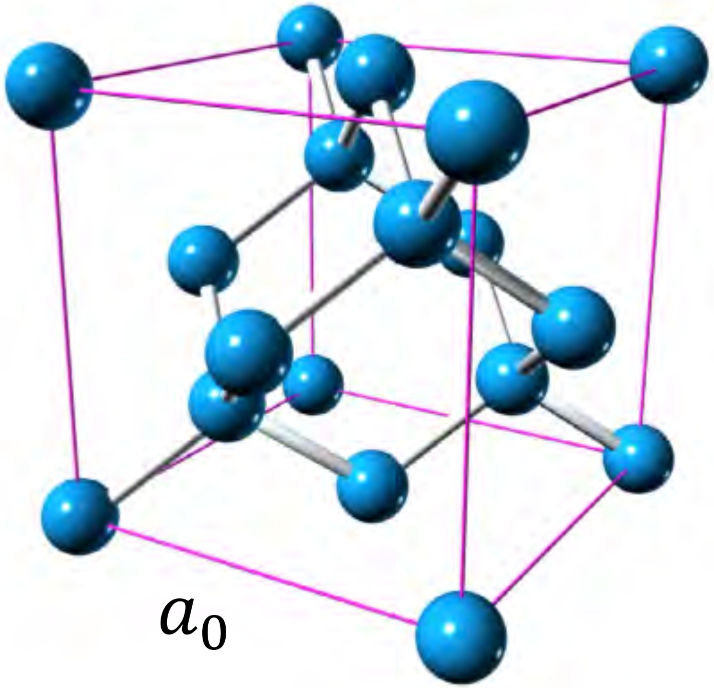
How to count large numbers

Macroscopic volume,
volume of a molar mass



$$V = \frac{4}{3}\pi r^3$$

Microscopic volume,
volume of a unit cell:



$$V_{uc} = a_0^3 = \left(\sqrt{8}d_{220}\right)^3$$

$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}} \times \text{Bottles}$$

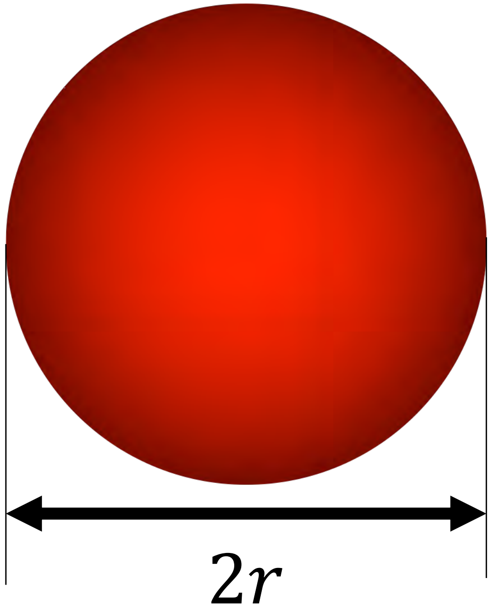
$$N_B = \frac{W \cdot L \cdot H}{w \cdot l \cdot h} \times 10$$

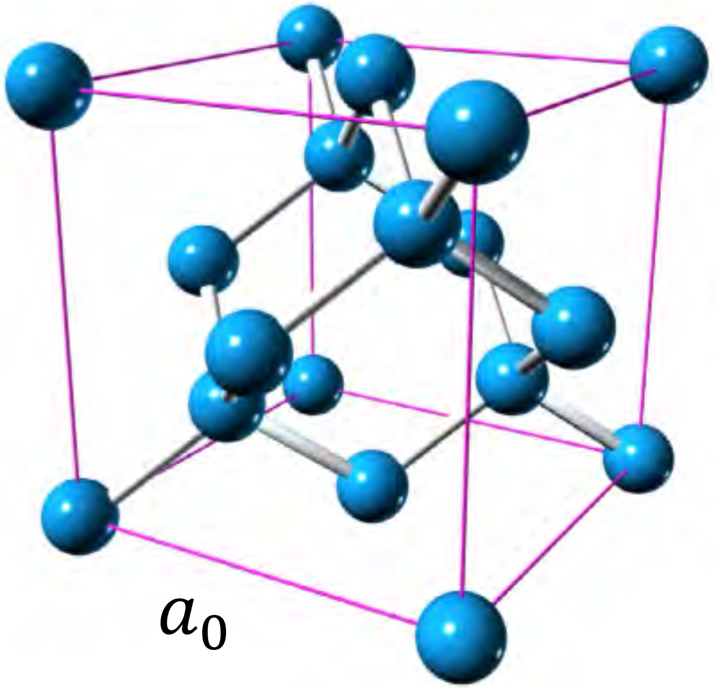
How to count large numbers

Macroscopic volume,
volume of a molar mass

Microscopic volume,
volume of a unit cell:

$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}} \times \text{atoms}$$





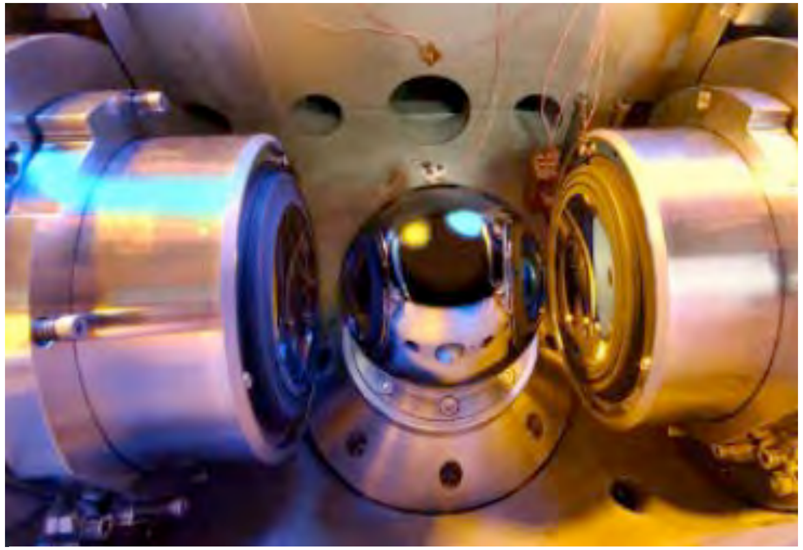
$$N = \frac{\frac{4}{3}\pi r^3}{16\sqrt{2}d_{220}^3} \times 8$$

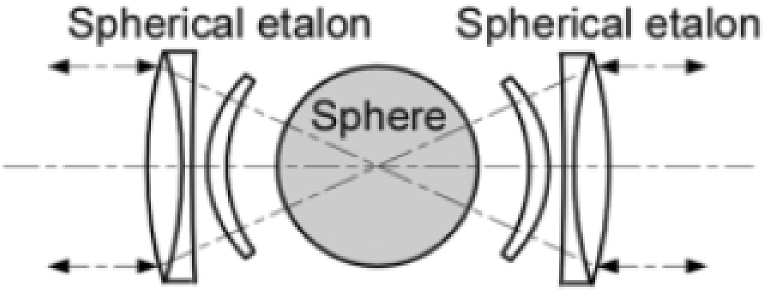
$$V = \frac{4}{3}\pi r^3$$

$$V_{uc} = a_0^3 = \left(\sqrt{8}d_{220}\right)^3$$

How to count large numbers

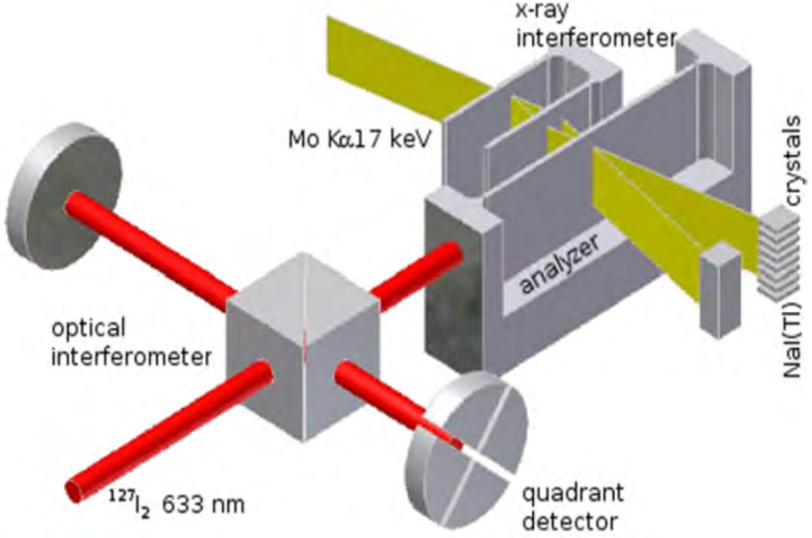
Macroscopic volume,
volume of a molar mass





Pictures: PTB

Microscopic volume,
volume of a unit cell:



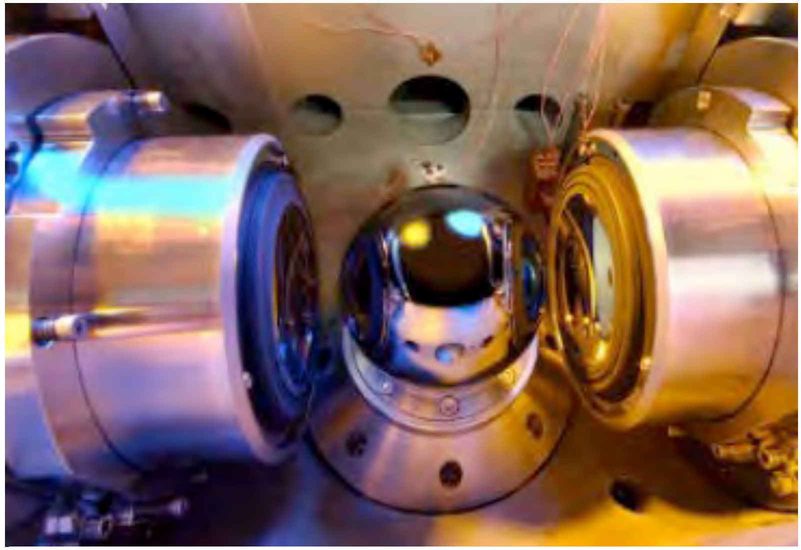
$$V_{uc} = a_0^3 = \left(\sqrt{8}d_{220}\right)^3$$

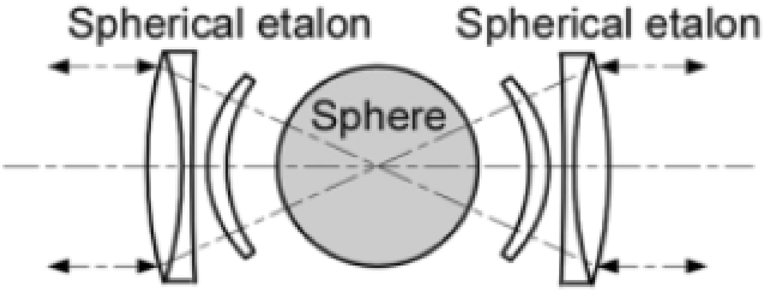
$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}} \times \text{atoms}$$

$$N = \frac{\frac{4}{3}\pi r^3}{16\sqrt{2}d_{220}^3} \times 8$$

How to count large numbers

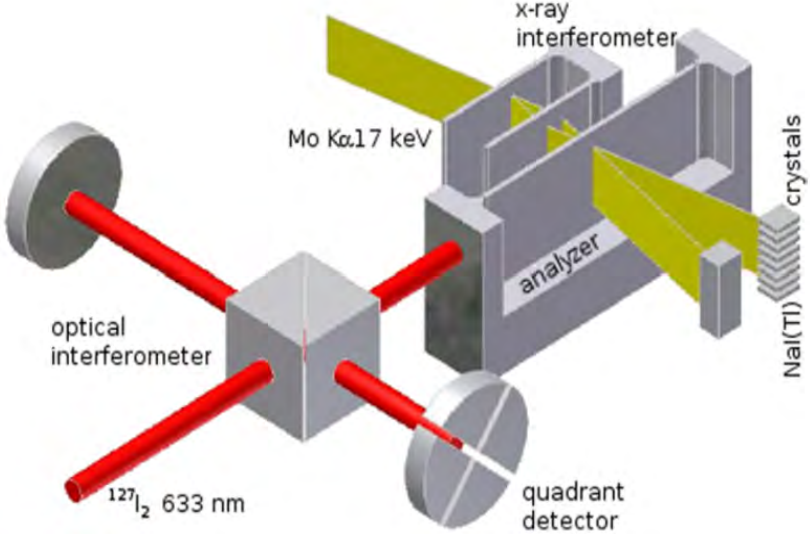
Macroscopic volume,
volume of a molar mass

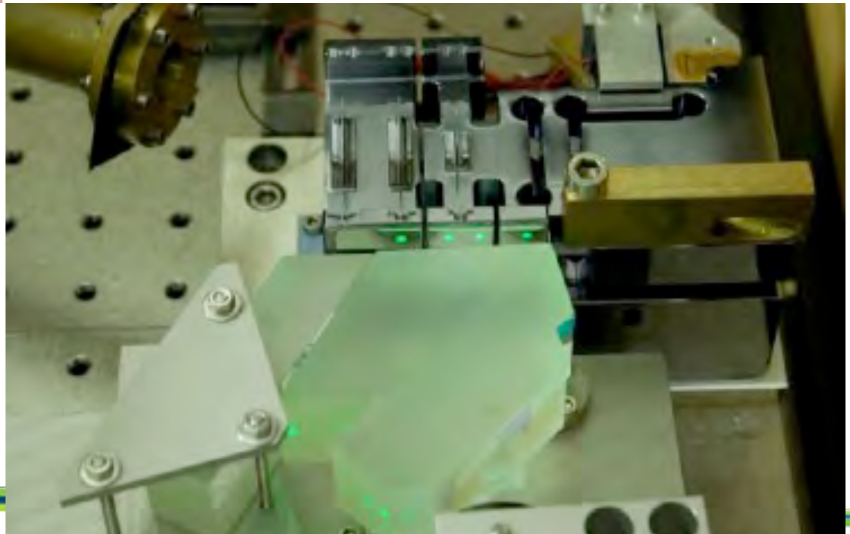




Pictures: PTB

Microscopic volume,
volume of a unit cell:





$$\frac{\text{Macroscopic Volume}}{\text{Microscopic Volume}}\times\text{atoms}$$

$$N = \frac{\frac{4}{3}\pi r^3}{16\sqrt{2}d_{220}^3}\times 8$$

Two points to emphasize

$$N = \frac{\pi}{12\sqrt{2}}\underbrace{\left(\frac{d}{d_{220}}\right)^3}_{\text{Unitless number}}$$

Two points to emphasize

Measured in Germany and Japan in m

$d \approx 0.1\ \mathrm{m}$

$$N = \frac{\pi}{12\sqrt{2}}\underbrace{\left(\frac{d}{d_{220}}\right)}_{\text{Unitless number}}^{3}$$

Measured in Italy in m

$d_{220} \approx 2.0\times10^{-10}\ \mathrm{m}$

$$\frac{d}{d_{220}} \approx 5\times10^{8}$$

Different Isotopes of Silicon

$$m_{SI} = r\, m_e$$

$$m_{SI} = \underbrace{f_{28} r_{28}\, m_e}_{91.8\,\%} + \underbrace{f_{29} r_{29}\, m_e}_{4.8\,\%} + \underbrace{f_{30} r_{30}\, m_e}_{3.3\,\%}$$

Natural Si	
f_{28}	92.2 %
f_{29}	4.7 %
f_{30}	3.1 %

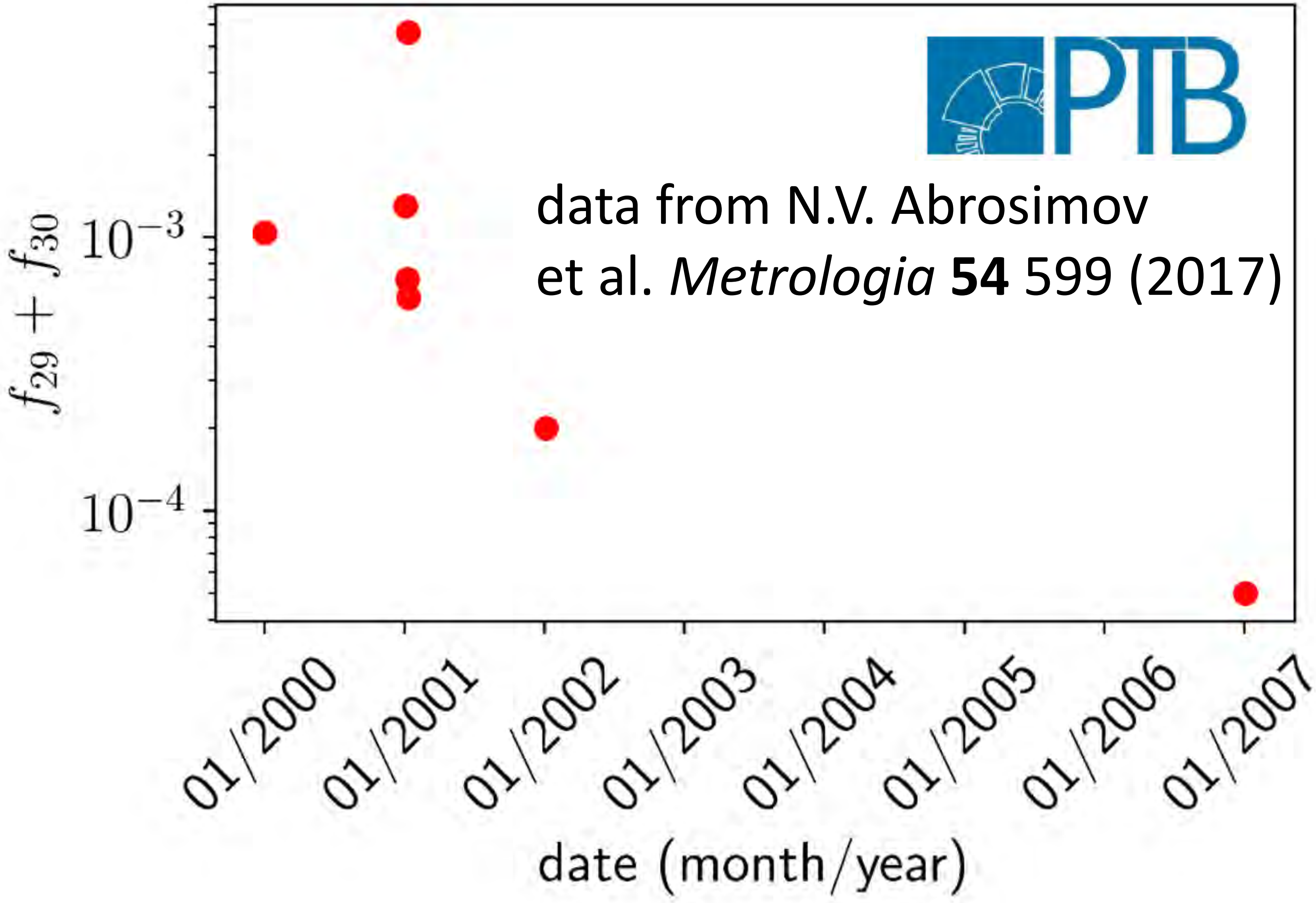
Enriched Si	
f_{28}	99.9958 %
f_{29}	4.1×10^{-5}
f_{30}	1.1×10^{-6}

99.9956 % $\quad 4.2\times10^{-5} \quad 1.1\times10^{-6}$

Different Isotopes of Silicon

Natural Si	
f_{28}	92.2 %
f_{29}	4.7 %
f_{30}	3.1 %

Enriched Si	
f_{28}	99.9958 %
f_{29}	4.1×10^{-5}
f_{30}	1.1×10^{-6}



data from N.V. Abrosimov et al. *Metrologia* **54** 599 (2017)

XRCD: Putting it all together

$$m = h\frac{R_\infty}{\alpha^2 c}\frac{\pi}{6\sqrt{2}}\left(\frac{d}{d_{220}}\right)^3 \sum_{i=28,29.30} f_i\, r_i - m_{deficit} + m_{surface}$$

The quantities printed in black are known.
For each crystal the following quantities need to be measured:

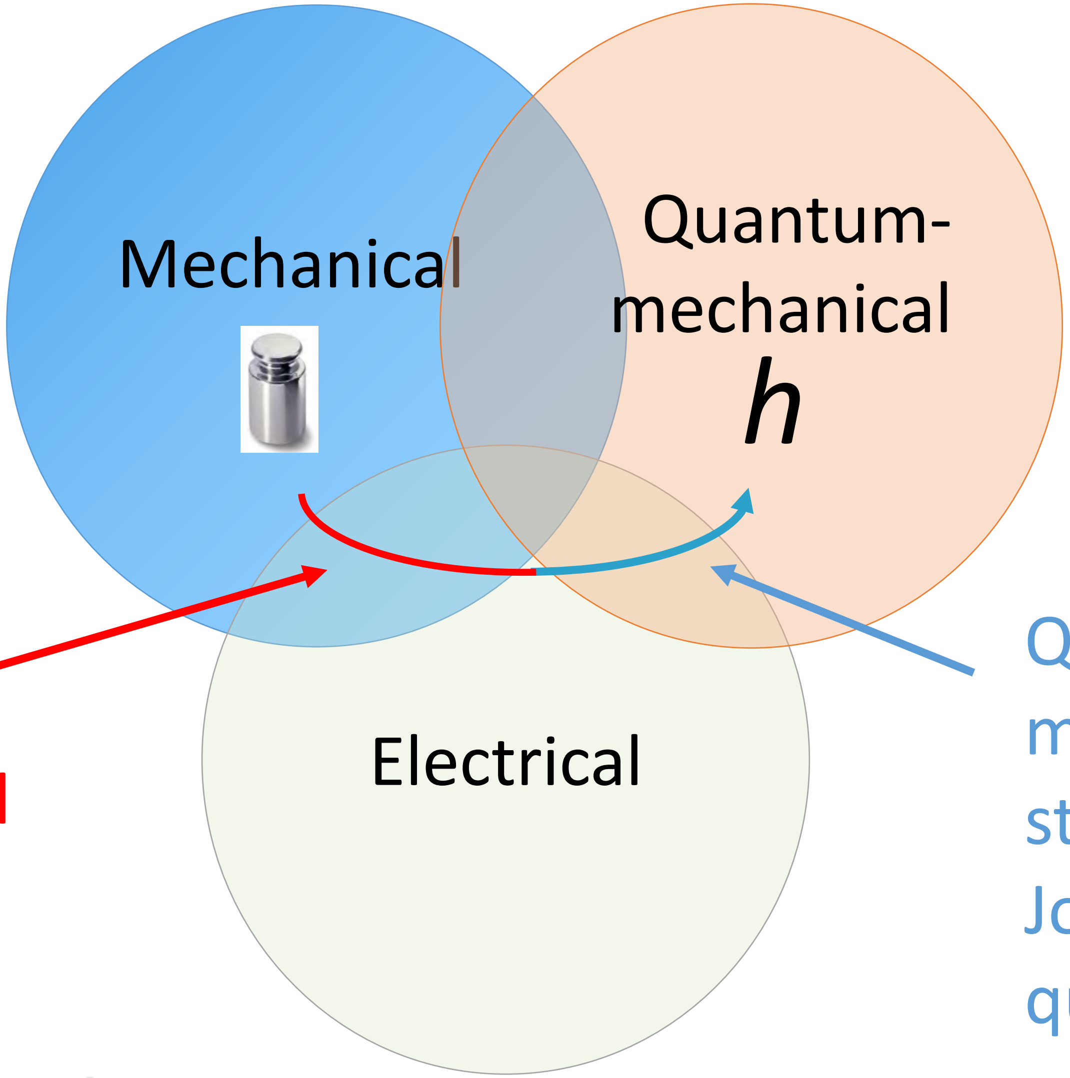
• Diameter d	sphere interferometer
• Lattice parameter d_{220}	absolute, relative, modelled
• Isotopic abundances f_{28}, f_{29}	mass spectrometry)
• Impurities, vacancies $m_{deficit}$	infrared absorption spectroscopy
• Surface layer $m_{surface}$	XPS, XRF, ellipsometry

Kibble balance formerly known as watt balance

- Principle was invented in 1975 by Bryan Kibble.
- Was used to realize the ampere.
- After the discovery of the quantum Hall effect by Klaus von Klitzing in 1980 it could be used to measure h.
- From May 20th 2019 forward it can be used to realize the unit of mass.

Bryan Kibble
1938 - 2016

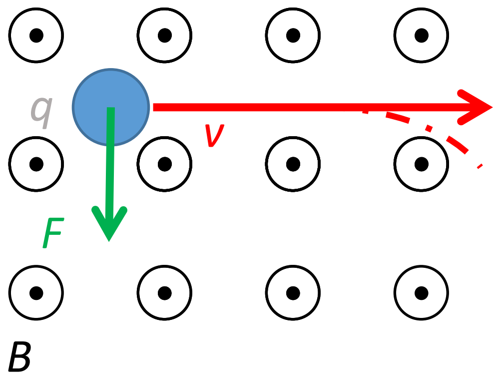
Mechanics, E & M, Quantummechanics



Kibble balance
Precision mechanical device that links mechanical to electrical quantities

Quantum electrical measurement standards, i.e., Josephson Effect quantum Hall effect.

The Lorentz force



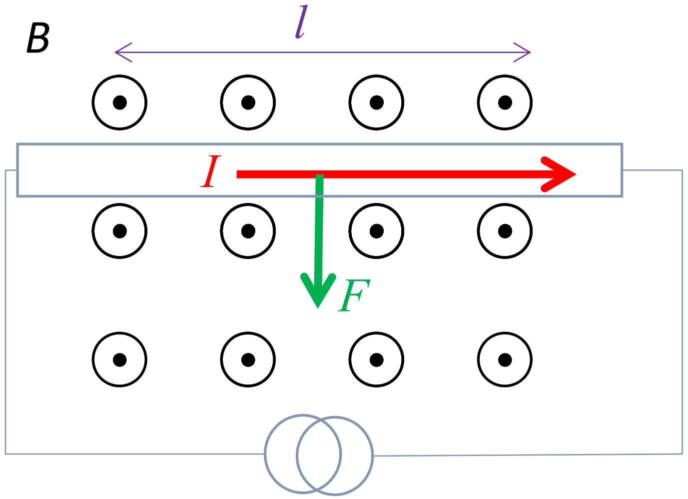
A charge moving in a magnetic field experiences a force:

$$\vec{F} = q\vec{v}\times\vec{B}$$

Facts on the Lorentz force:

- Perpendicular to *B*.
- Perpendicular to *v*.
- Proportional to *B*, i.e. more *B* more *F*.

For a current in a wire:



$$\vec{F} = I\vec{l}\times\vec{B}$$

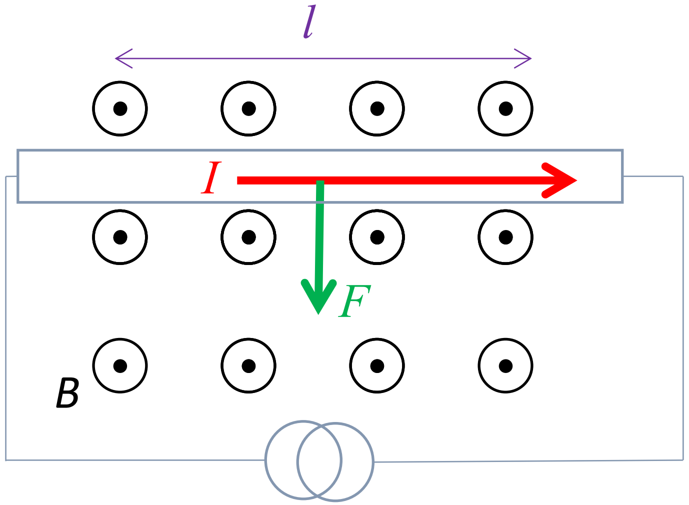
simplified:

$$F = IBl$$

- Perpendicular to wire.
- Perpendicular to *B*.
- Proportional to *B*.
- Proportional to *I*.
- Proportional to *l*.

A Motor is a Generator

Motor (force mode):

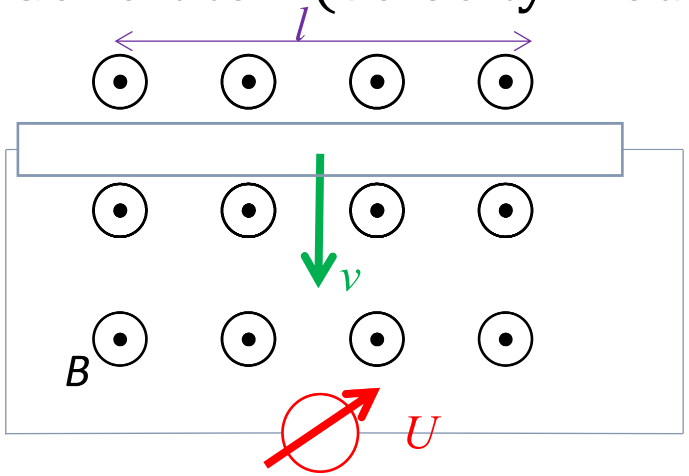


$$F = IBl$$

Cause: Current, I
Effect: Force, F

Circuit is completed,
current can flow.

Generator (velocity mode):



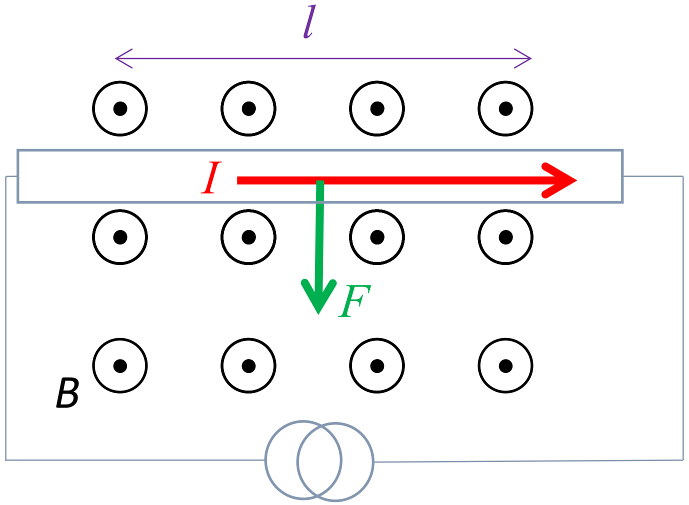
$$U = vBl$$

Cause: velocity, v
Effect: Voltage, U

Circuit is open,
current can NOT flow.
Voltage is called the induced
voltage or EMF.

A Motor is a Generator

Motor (force mode):

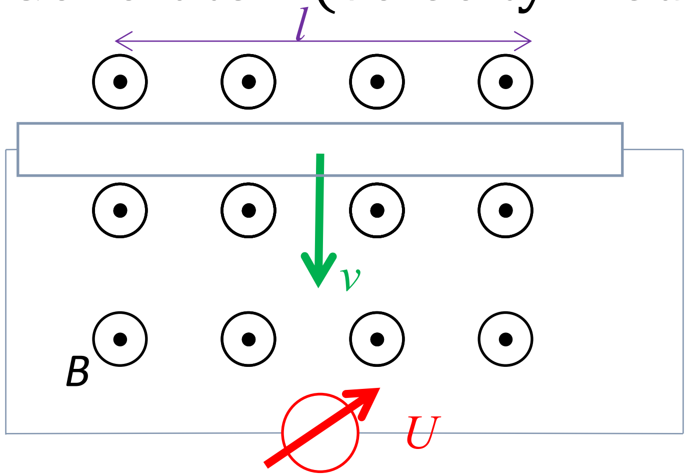


$$F = IBl$$

:

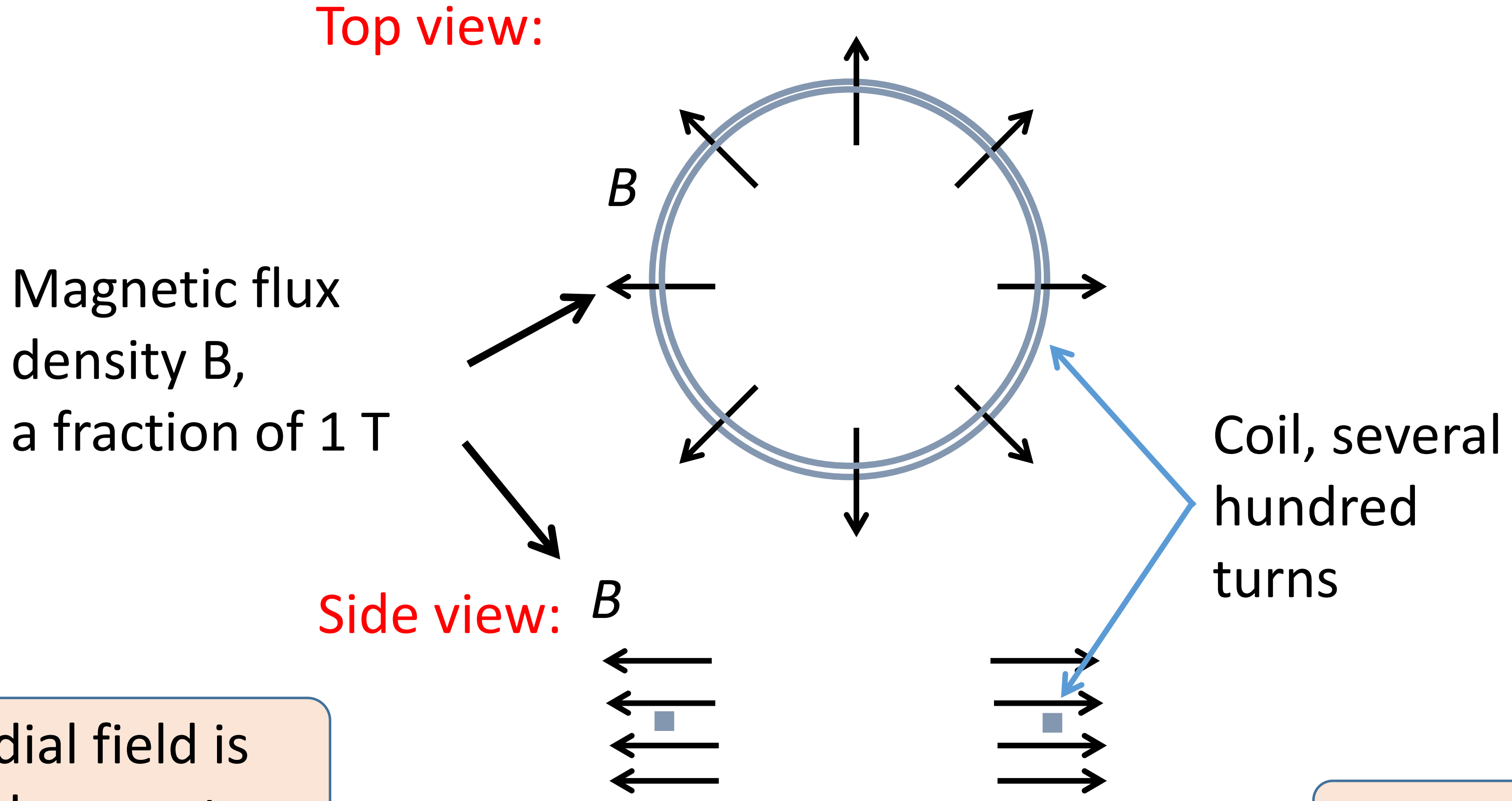
$$\frac{F}{U} = \frac{IBl}{vBl} = \frac{I}{v}$$

Generator (velocity mode):



$$U = vBl$$

Better Geometry



A coil in a radial field is the preferred geometry for modern watt balances.

Even better: $B_r \propto \frac{1}{r}$

Get a feel for the numbers

- B = 0.5 T
- l = 1,000 m
- Force mode: A current of 0.01 A is flowing through the wire. Calculate the Force produced by the coil.

- $F =$

Get a feel for the numbers

- B = 0.5 T
- l = 1,000 m
- Force mode: A current of 0.01 A is flowing through the wire. Calculate the Force produced by the coil.

- $F = I\,Bl = 0.01\ \mathrm{A}\quad 0.5\ \mathrm{T}\quad 1000\ \mathrm{m}$ = 5 N

Get a feel for the numbers

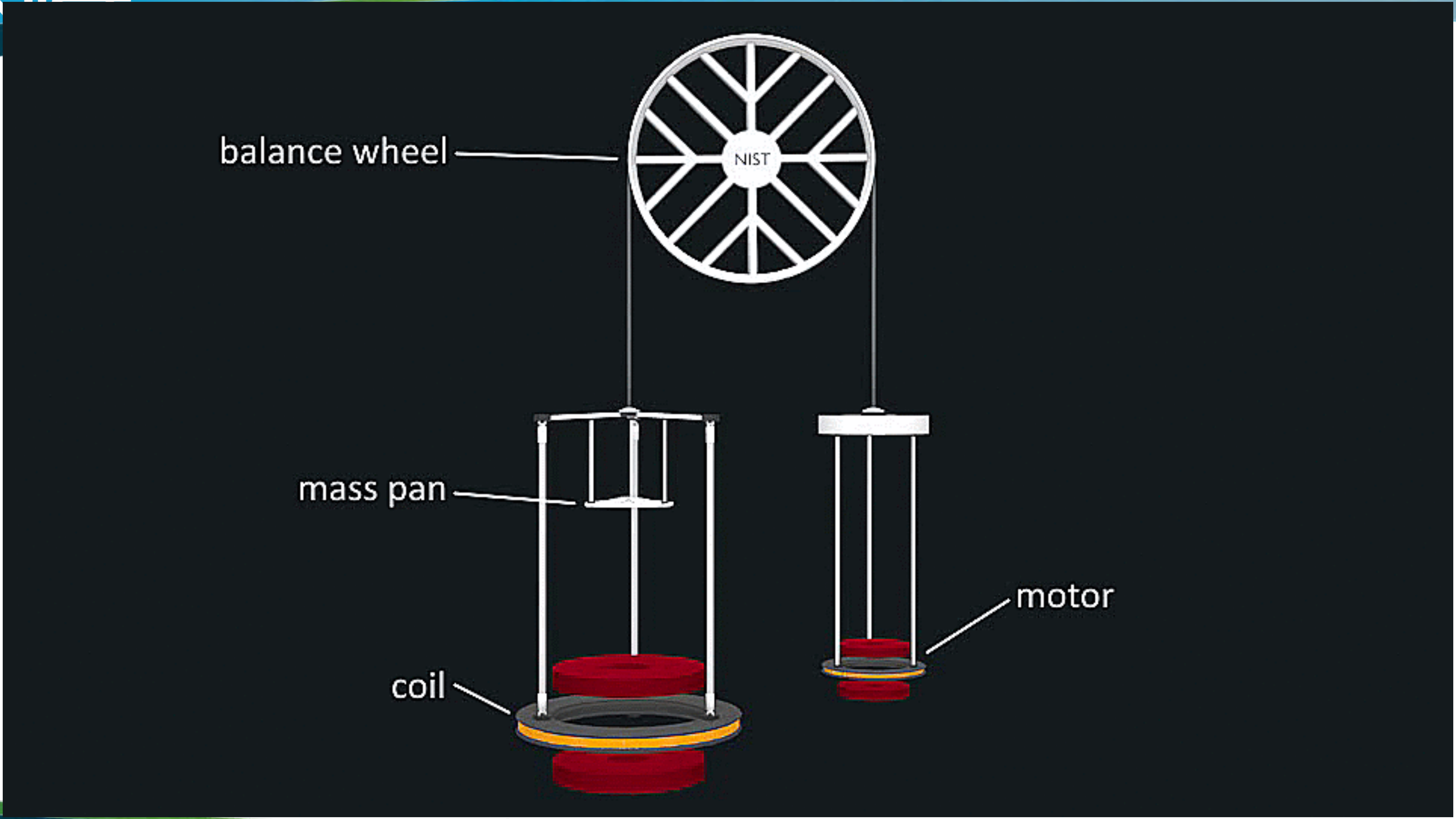
- B = 0.5 T
- l = 1,000 m
- Velocity mode: The coil is moving with 2 mm/s through the magnetic field. Calculate the induced voltage

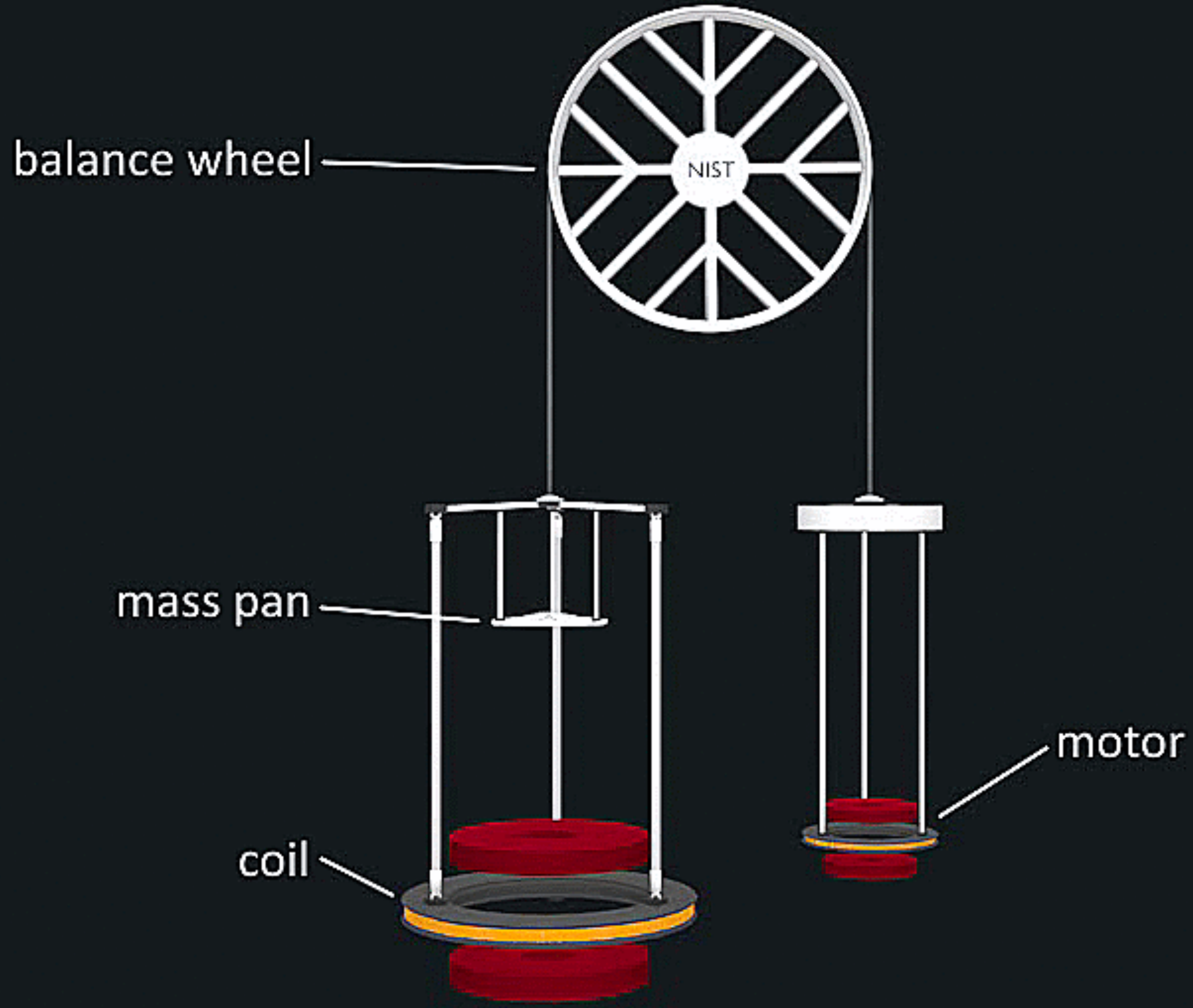
- $U =$

Get a feel for the numbers

- B = 0.5 T
- l = 1,000 m
- Velocity mode: The coil is moving with 2 mm/s through the magnetic field. Calculate the induced voltage

- $U = vBl = 0.002\frac{\mathrm{m}}{\mathrm{s}}\ 0.5\ \mathrm{T}\ 1000\ \mathrm{m}$ = 1 V

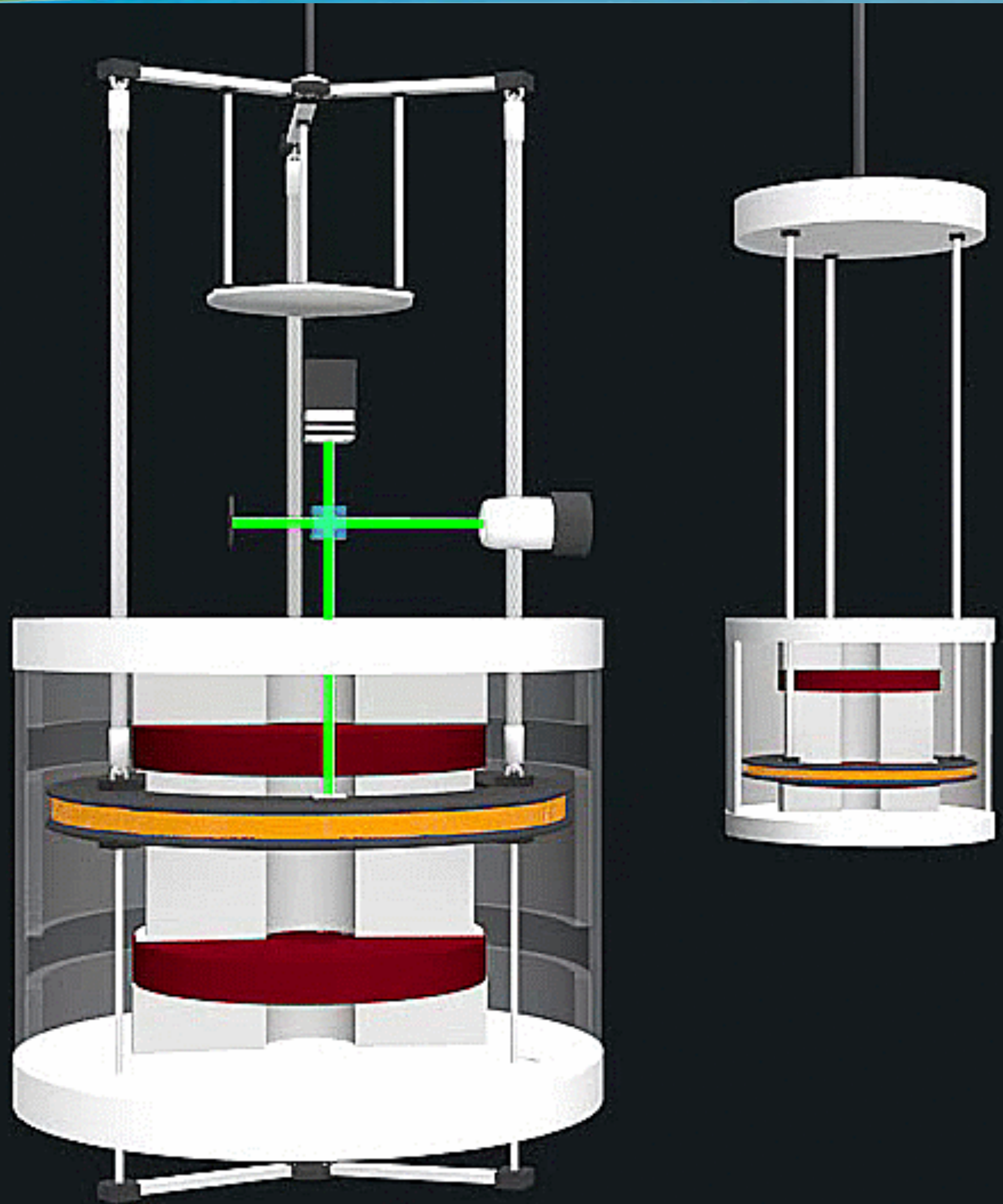

balance wheel
NIST
mass pan
motor
coil

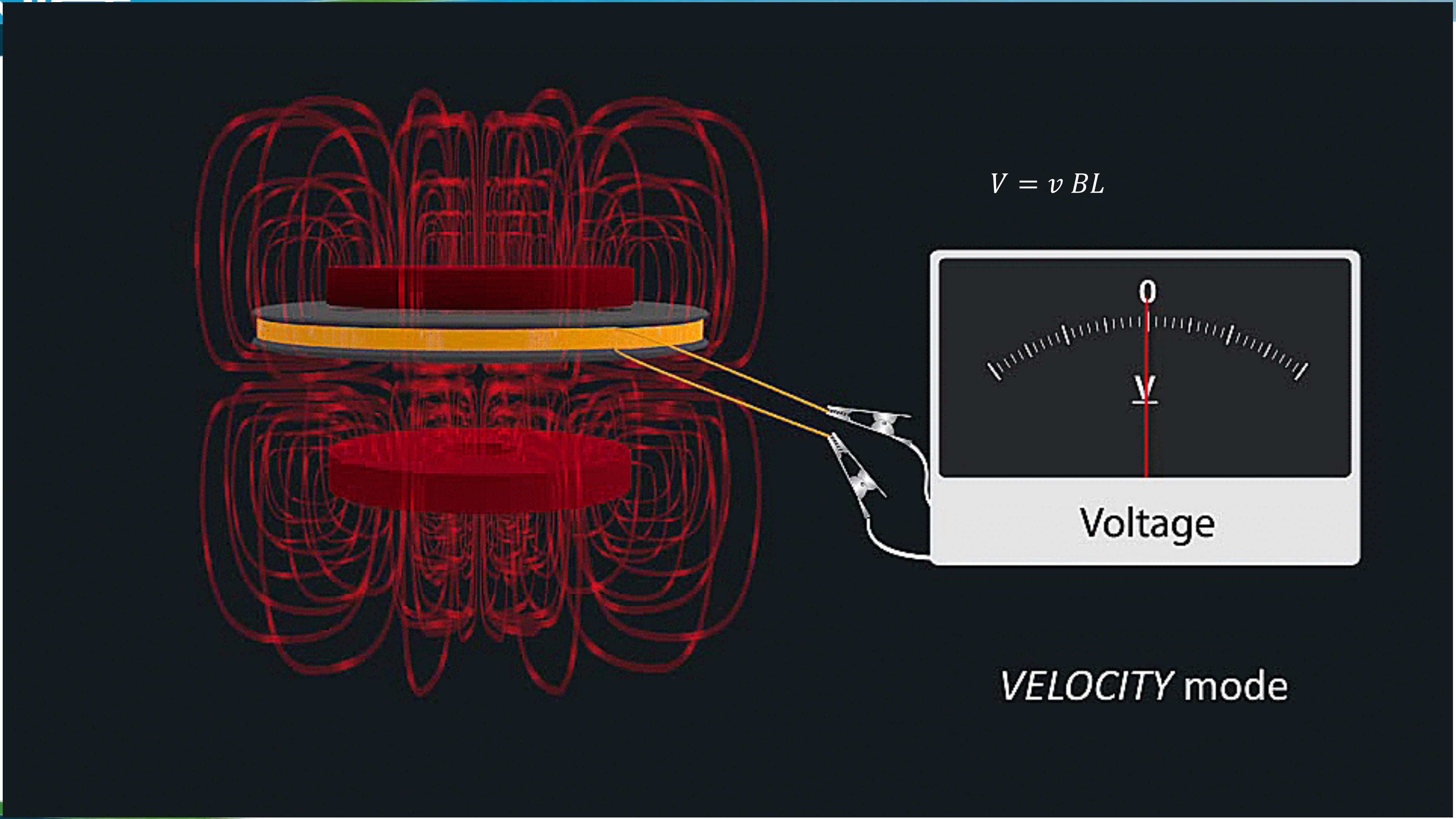

balance wheel
NIST
mass pan
coil
motor

VELOCITY mode

$$F = I\,BL$$

magnet flux





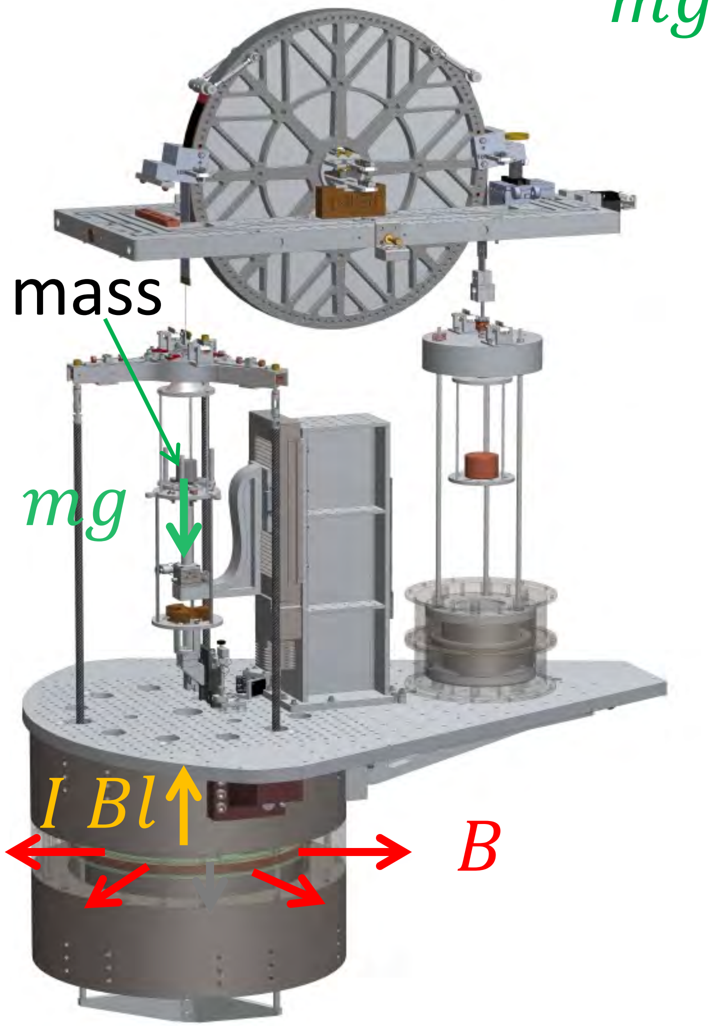
$$V = v\,BL$$

VELOCITY mode

Two modes of the Kibble balance

Force mode

$mg = I\,Bl$



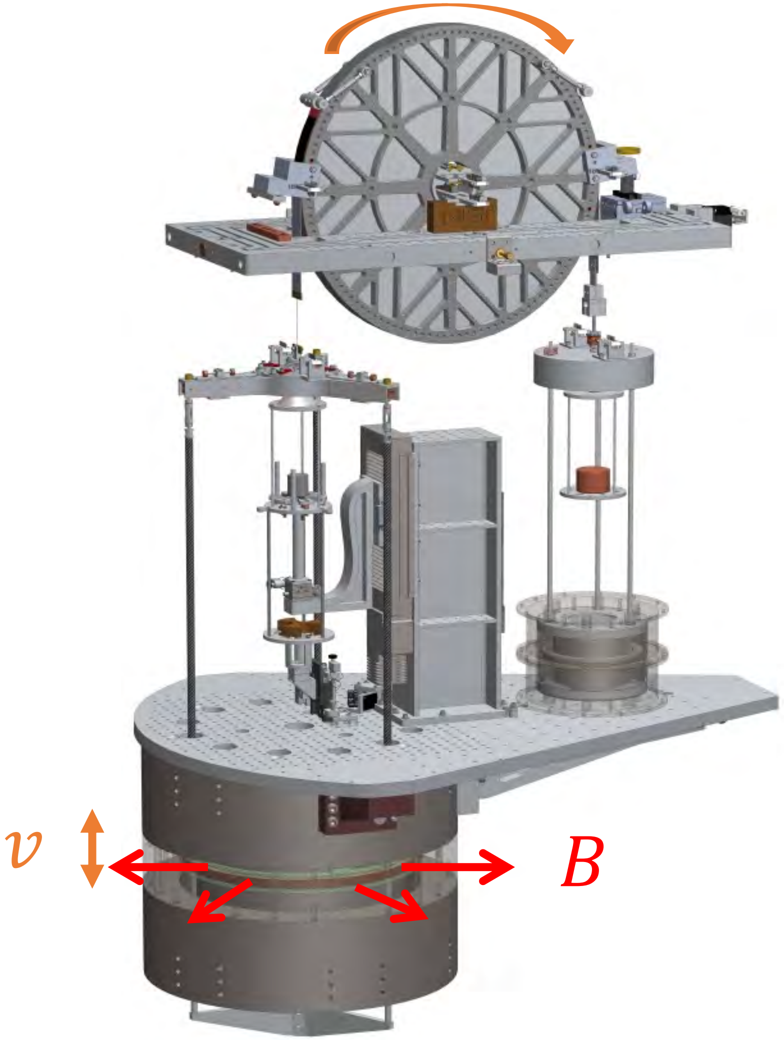
Velocity mode

$U = v\,Bl$

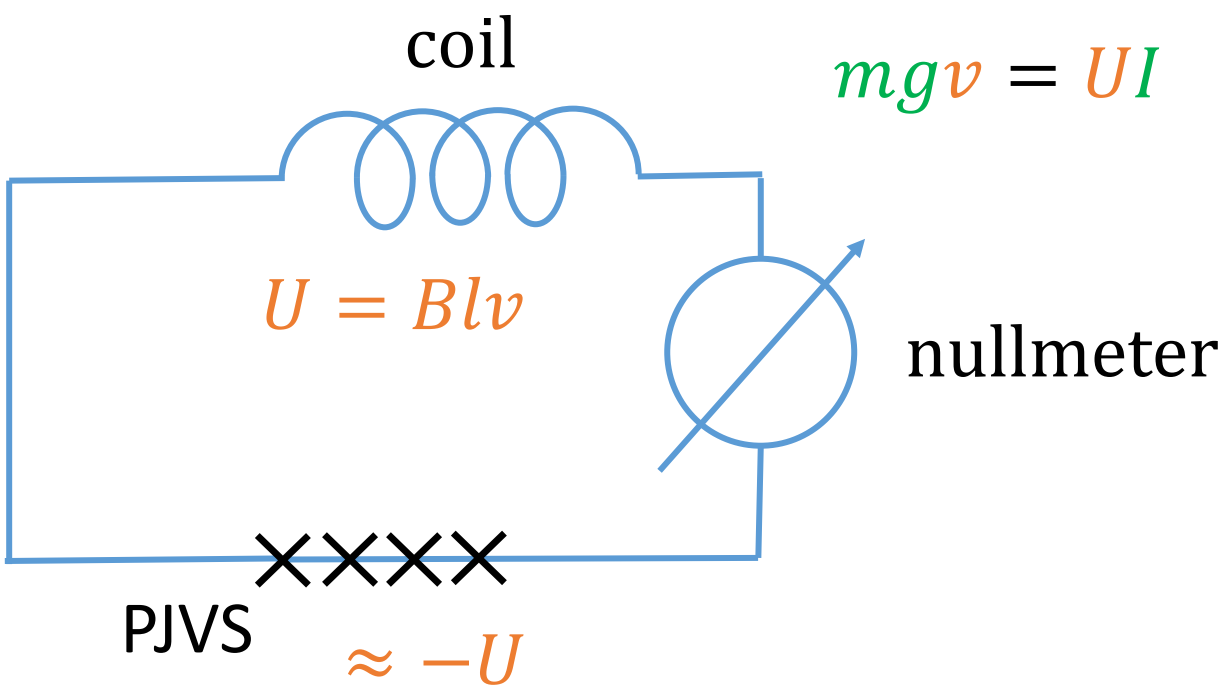
$$\frac{mg}{U} = \frac{I}{v}$$

$$mgv = UI$$

$$m = \frac{UI}{gv}$$



Measurement of the induced voltage



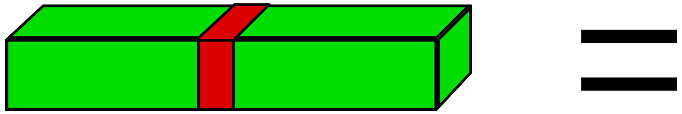
$mgv = UI$

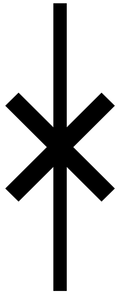
$U = Blv$

$\approx -U$

Josephson effect

$\Psi_1 = A_1 e^{i\theta_1}$ $\Psi_2 = A_2 e^{i\theta_2}$





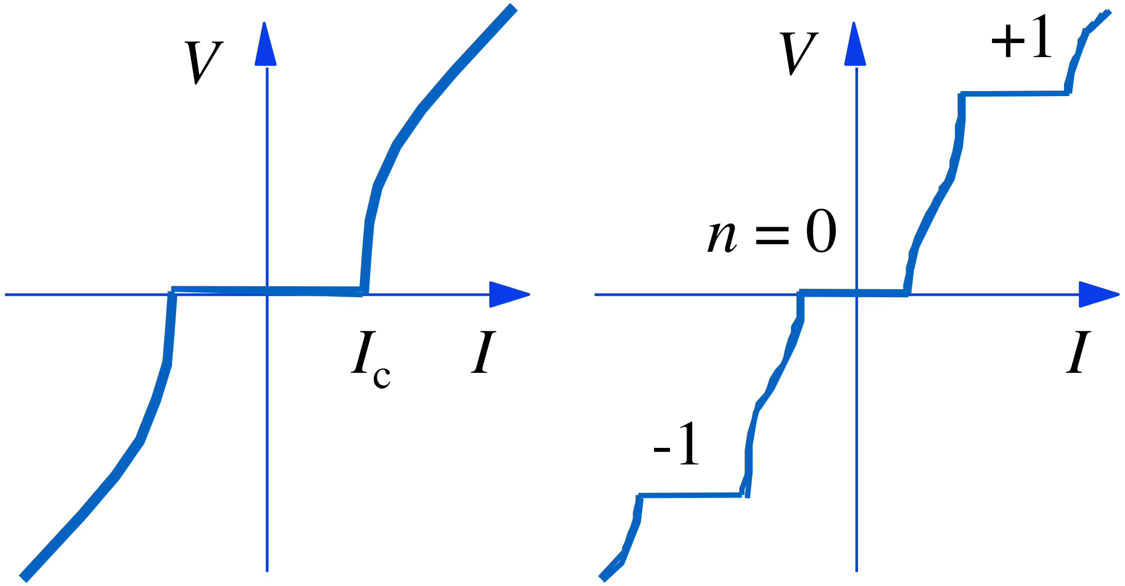
$$\phi = \theta_2 - \theta_1$$

$$V = \frac{h}{2e}\frac{1}{2\pi}\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

or

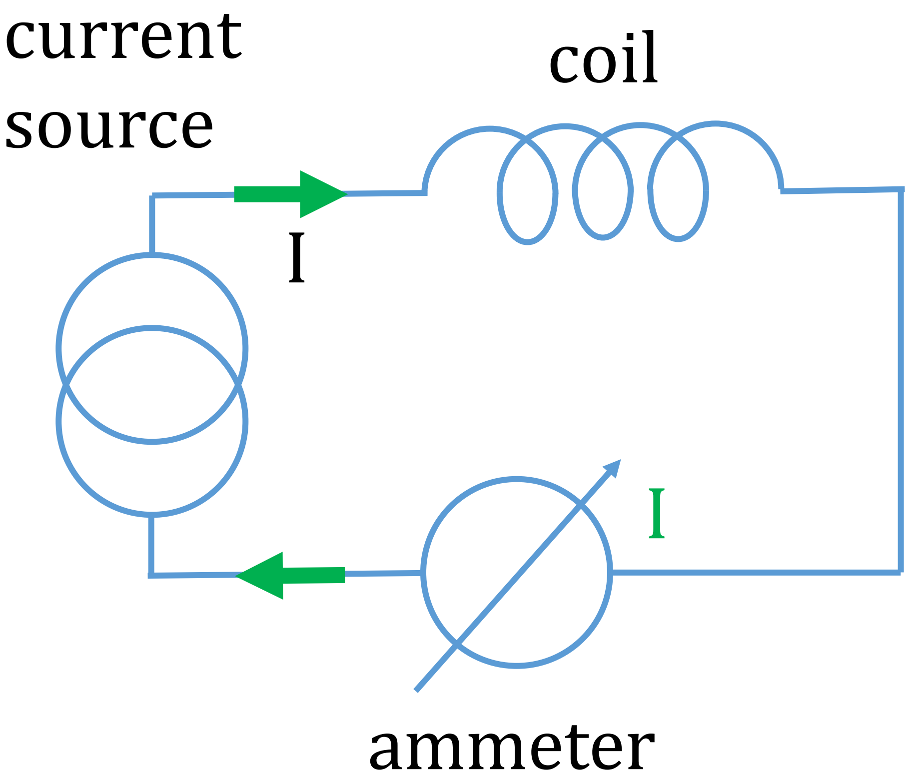
$$V = \frac{h}{2e}f$$

- Predicted in 1962 by Brian Josephson
- Weak link between two superconductors



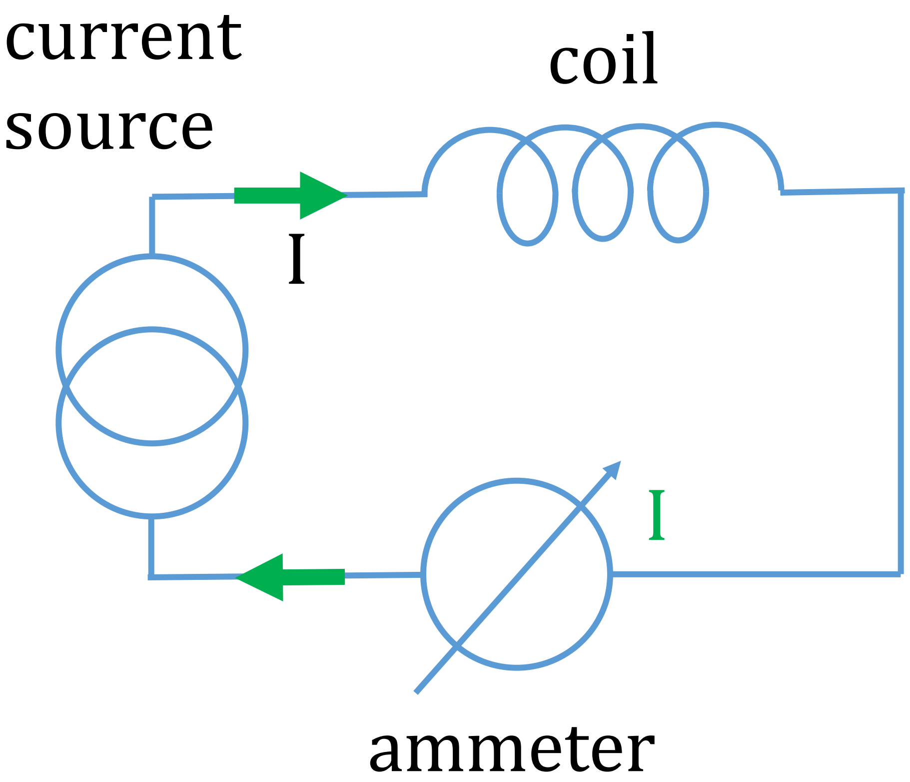
DC only with AC

Measurement of the electrical current



$$mgv = UI$$

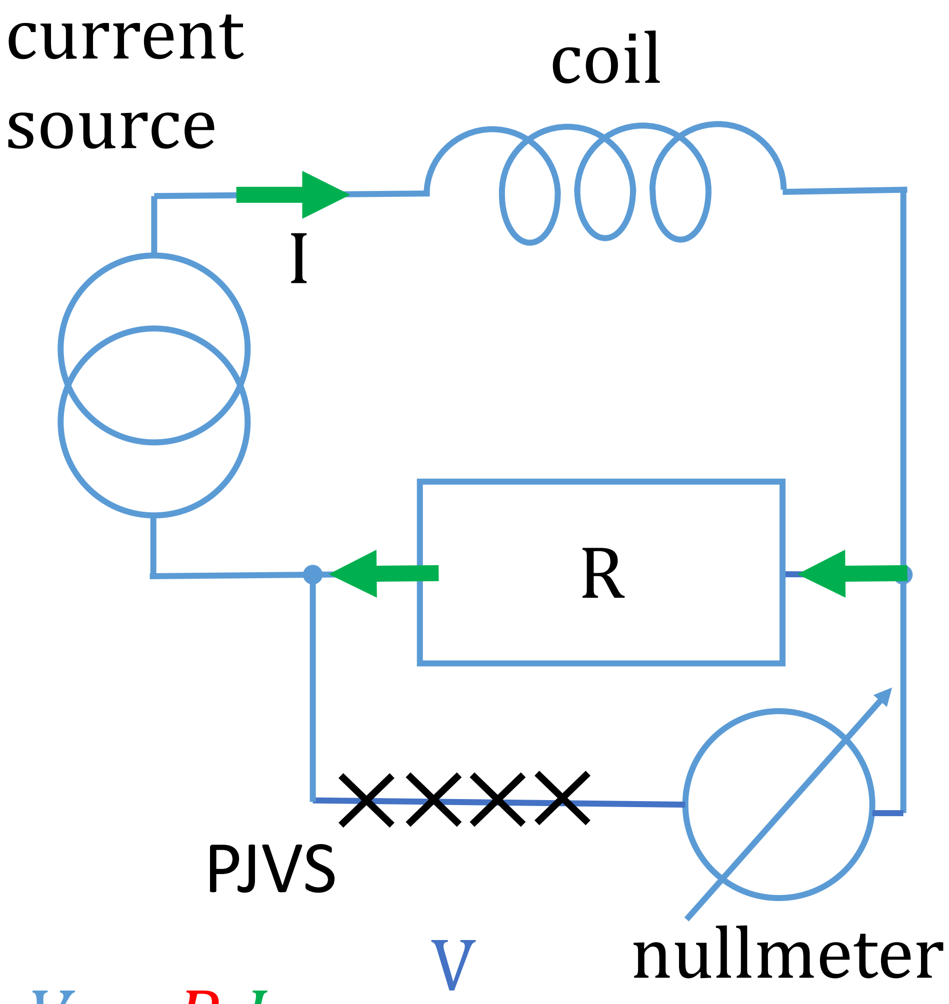
Measurement of the electrical current



$$mgv = UI$$

$$mgv = U\frac{V}{R}$$

$$m = \frac{UV}{Rgv}$$



$$V = R\,I$$

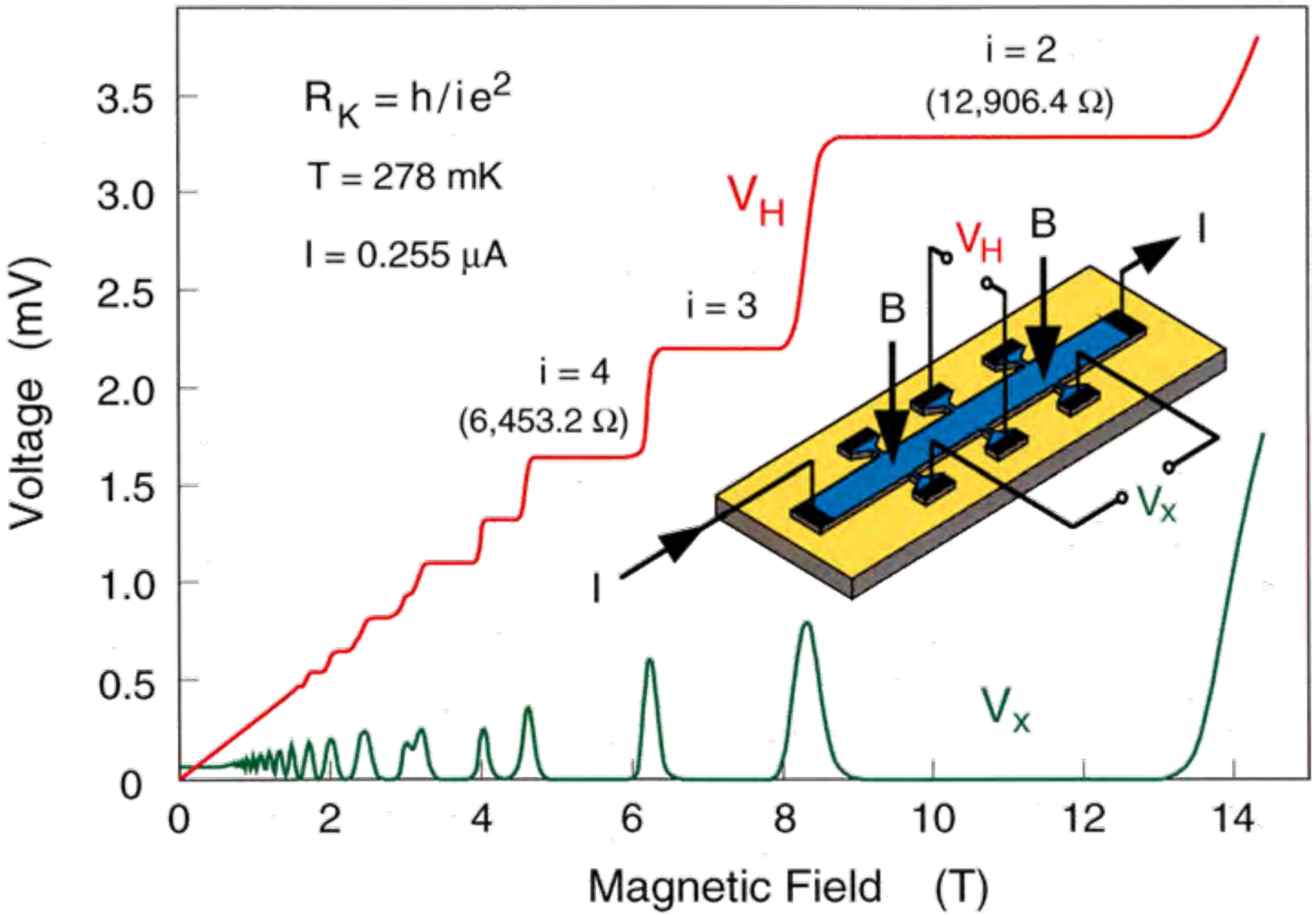
Quantum Hall effect

$$R_H = \frac{V_H}{I} = \frac{B}{eN_s}$$

$$N_S = i\,B\,\frac{e}{h}$$

$$R_H = \frac{1}{i}\underbrace{\frac{h}{e^2}}_{R_K \approx 25.8\,k\Omega}$$

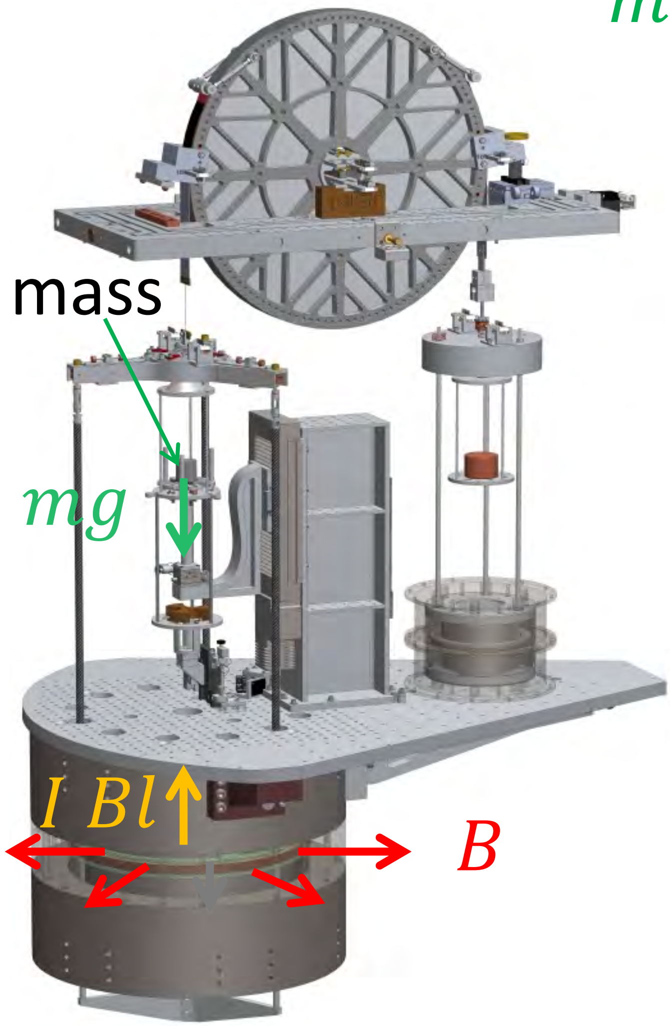
Discovered by Klaus v. Klitzing 1980



Two modes of the Kibble balance

Force mode

$$mg = I\,Bl$$



$$\frac{mg}{U} = \frac{I}{v}$$

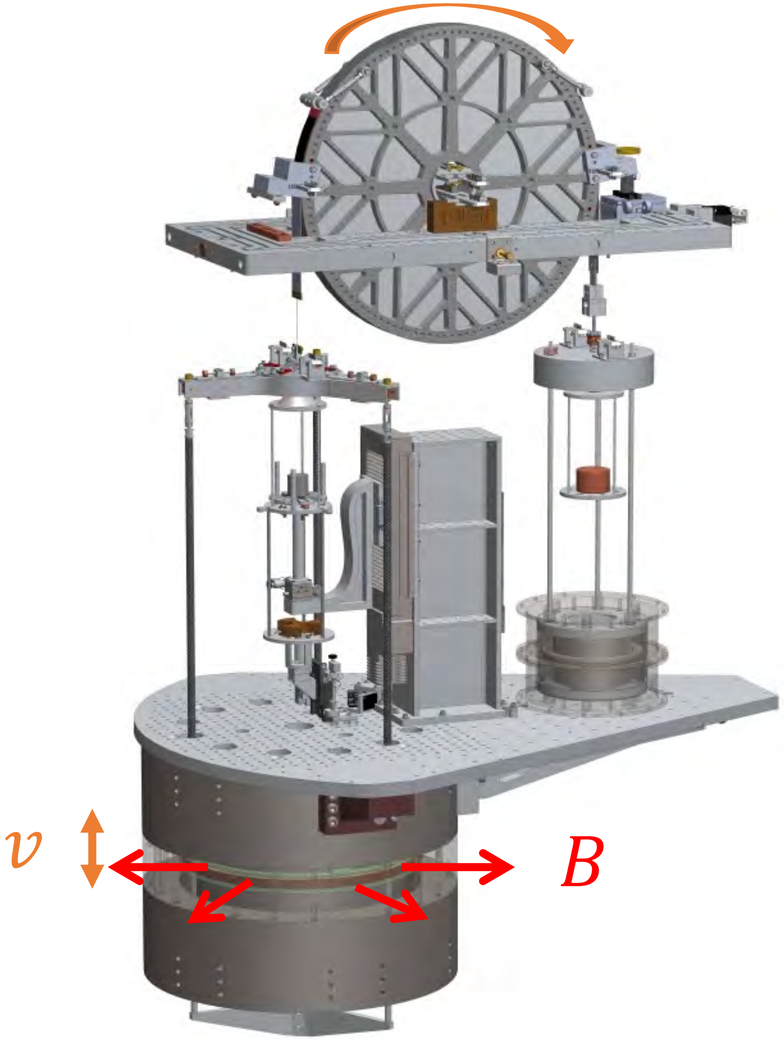
$$mgv = UI$$

$$mgv = \frac{r}{4} h f_1 f_2$$

$$m = \frac{r}{4} h \frac{f_1 f_2}{gv}$$

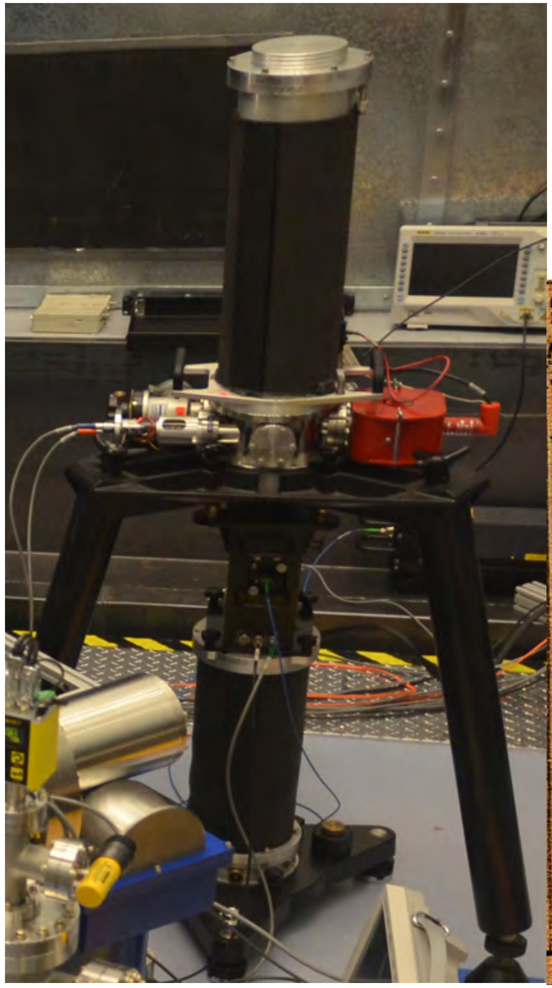
Velocity mode

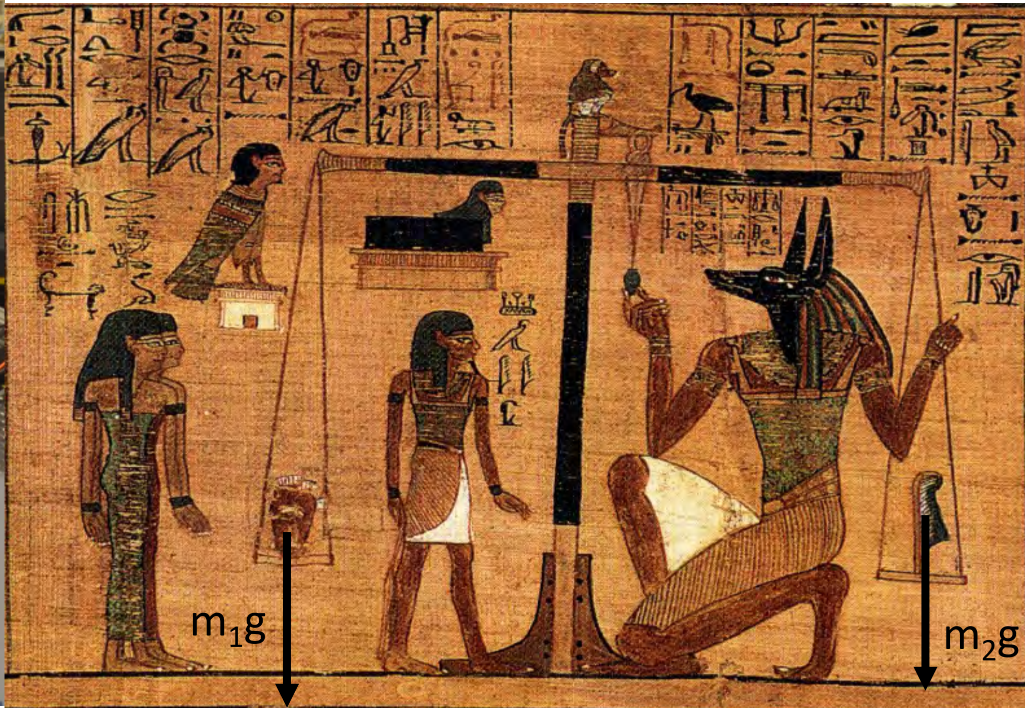
$$U = v\,Bl$$

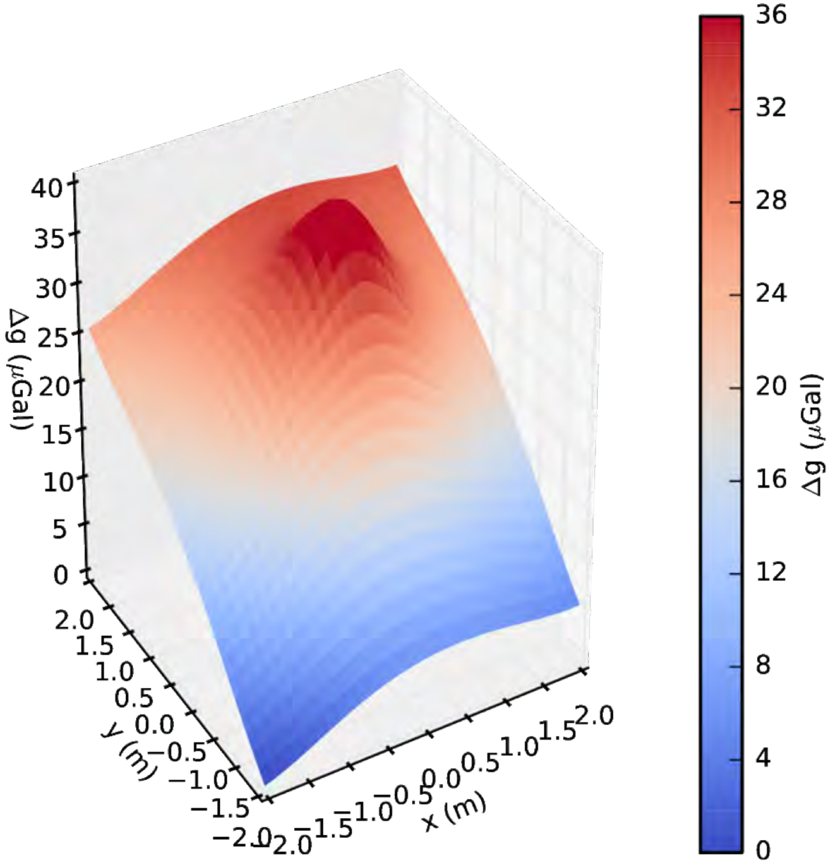


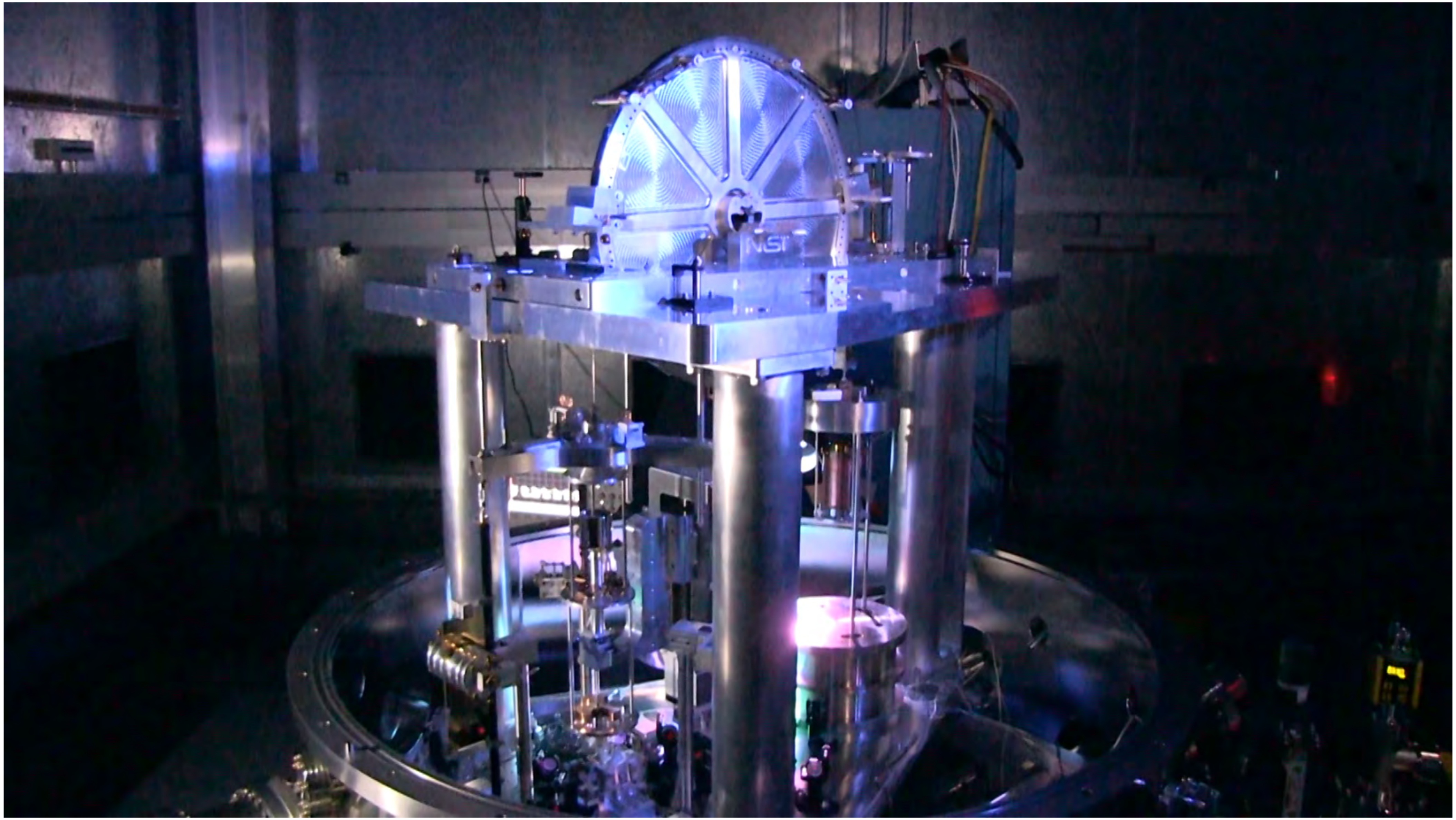
The Kibble balance requires g

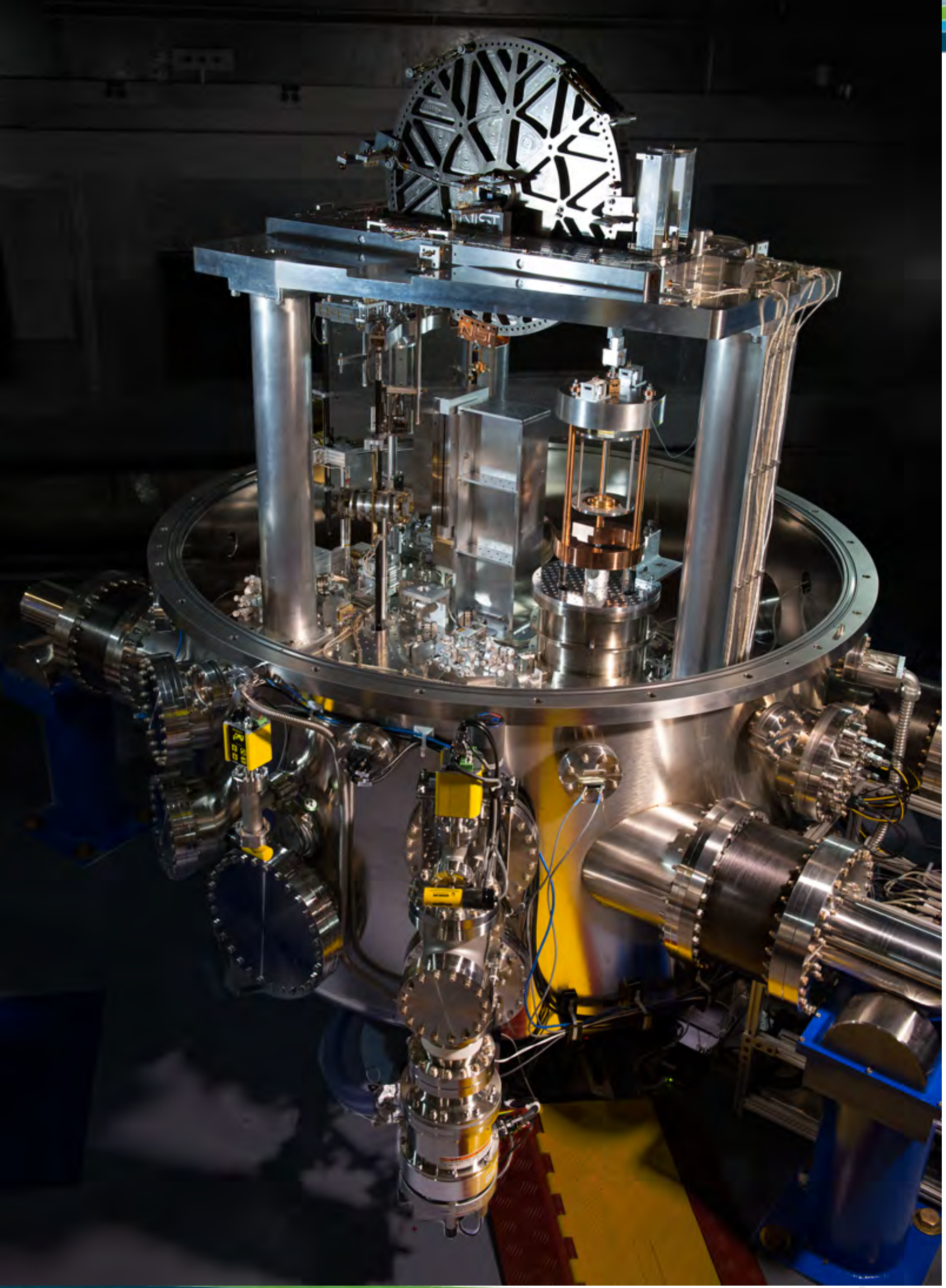
$$\frac{\sigma_g}{g} = 4.4\times10^{-9}$$



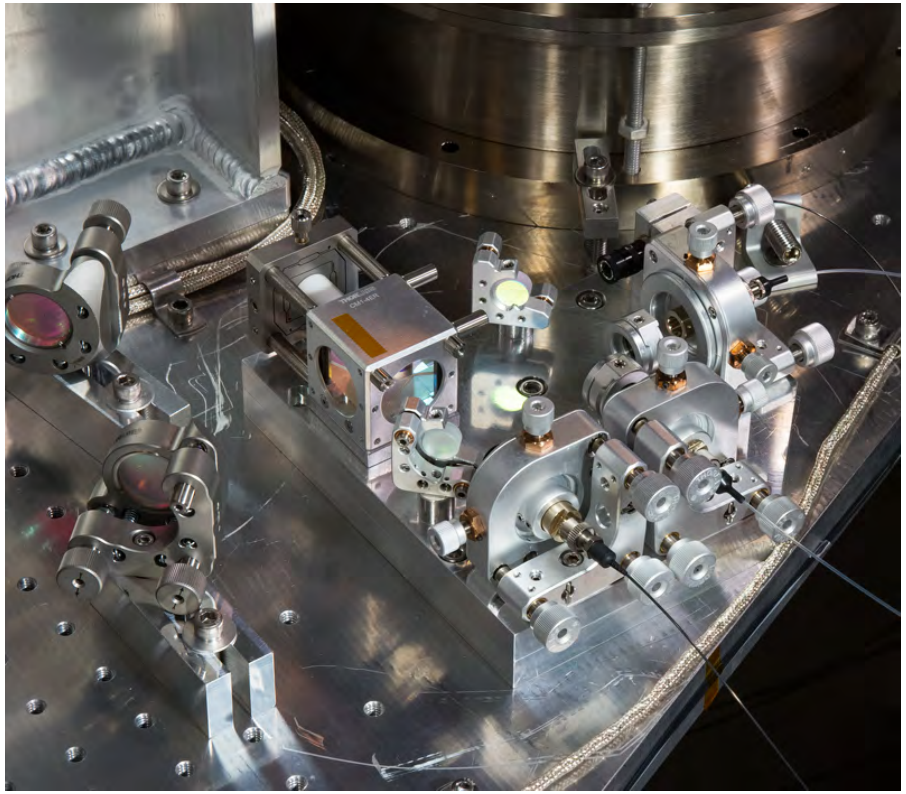




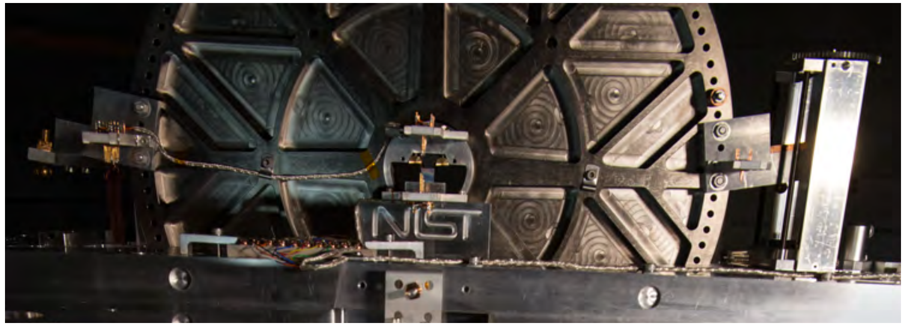

NIST

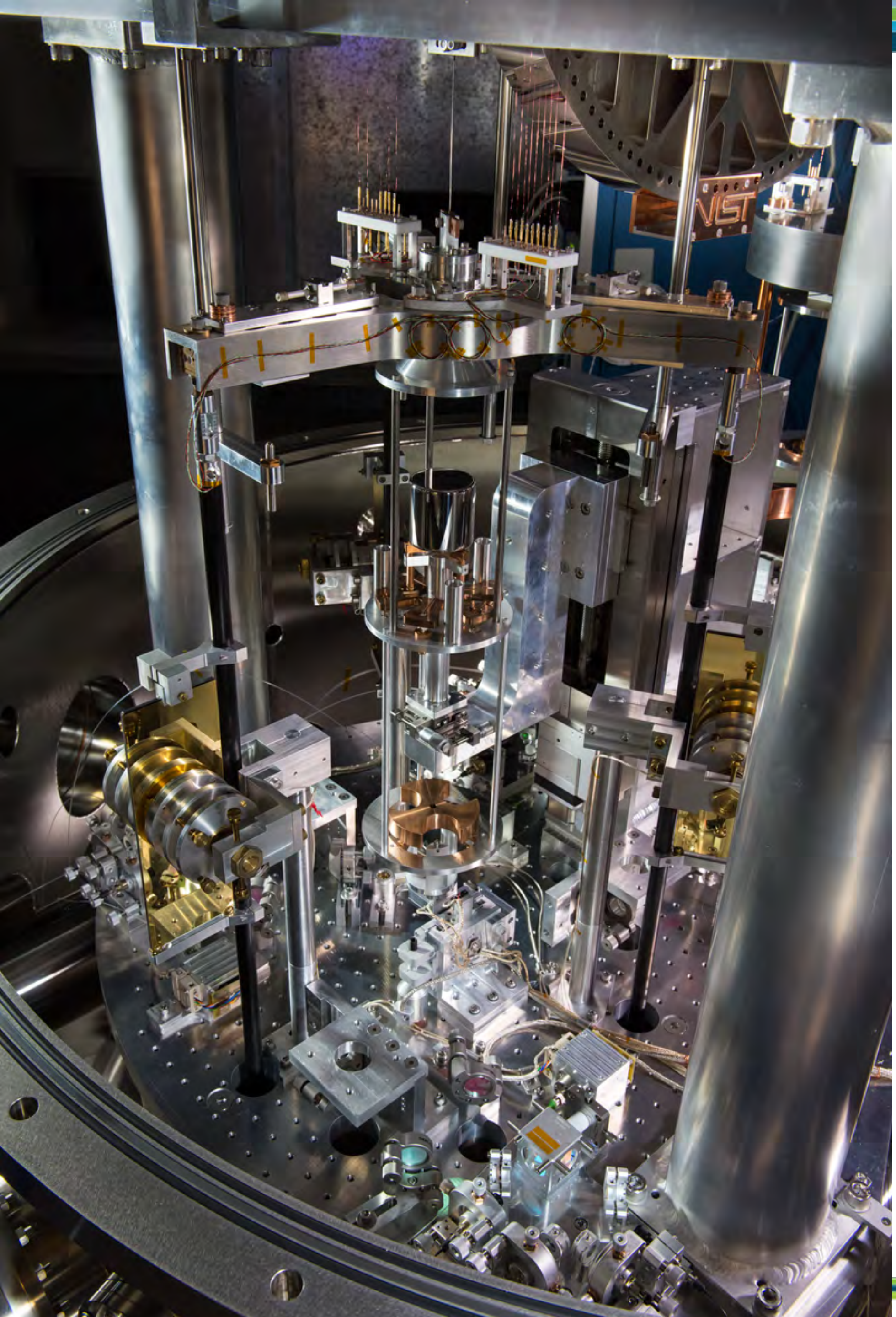


1 of 4 interferometer



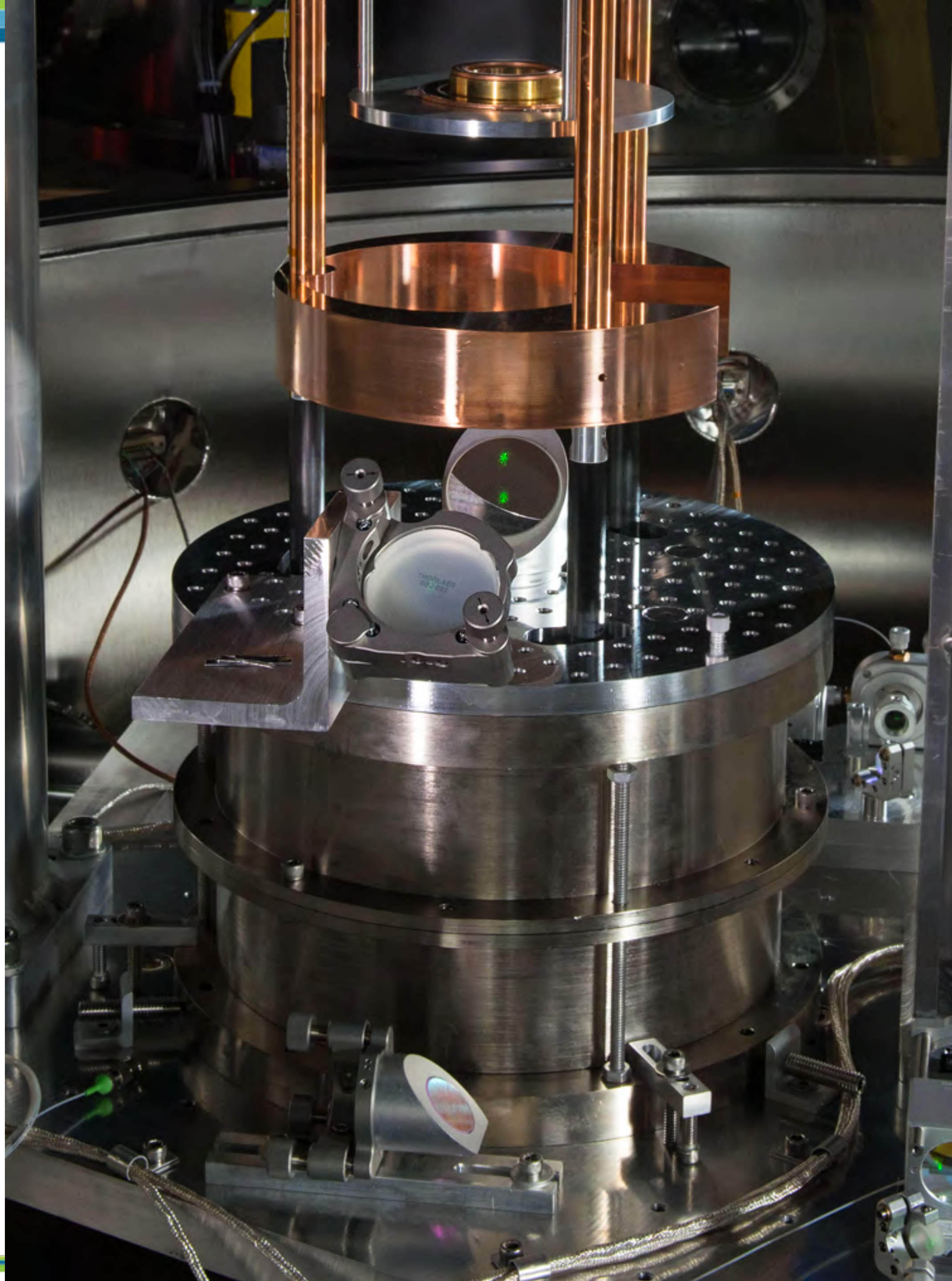
Knife edge pivot

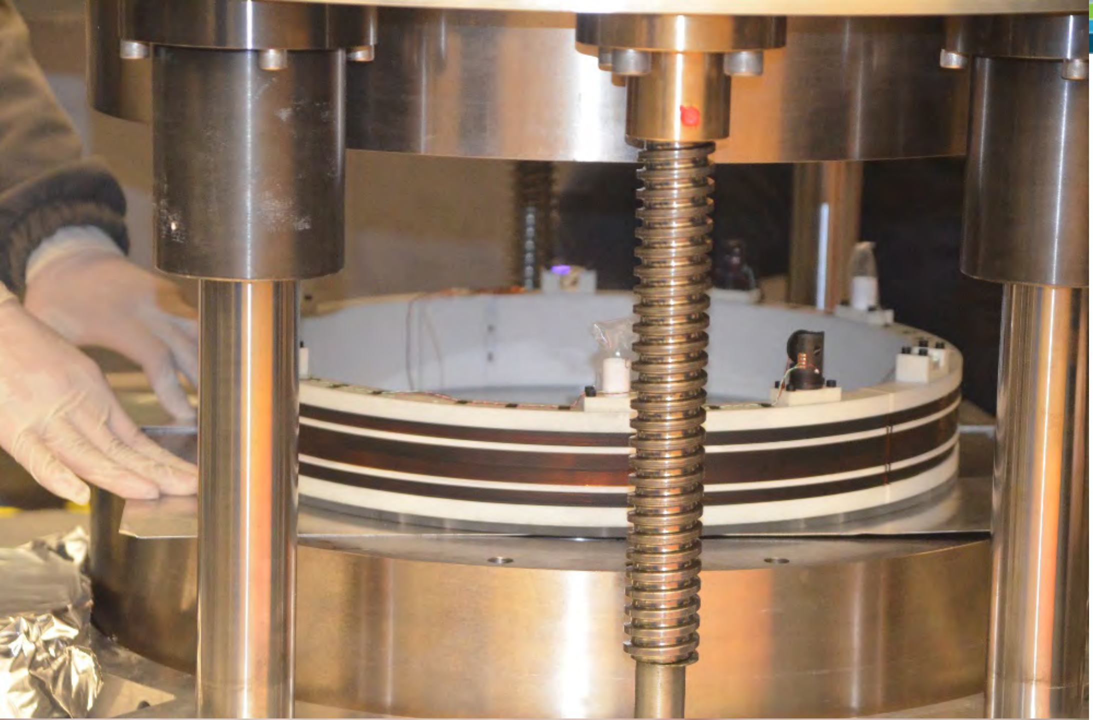




Busy main mass side

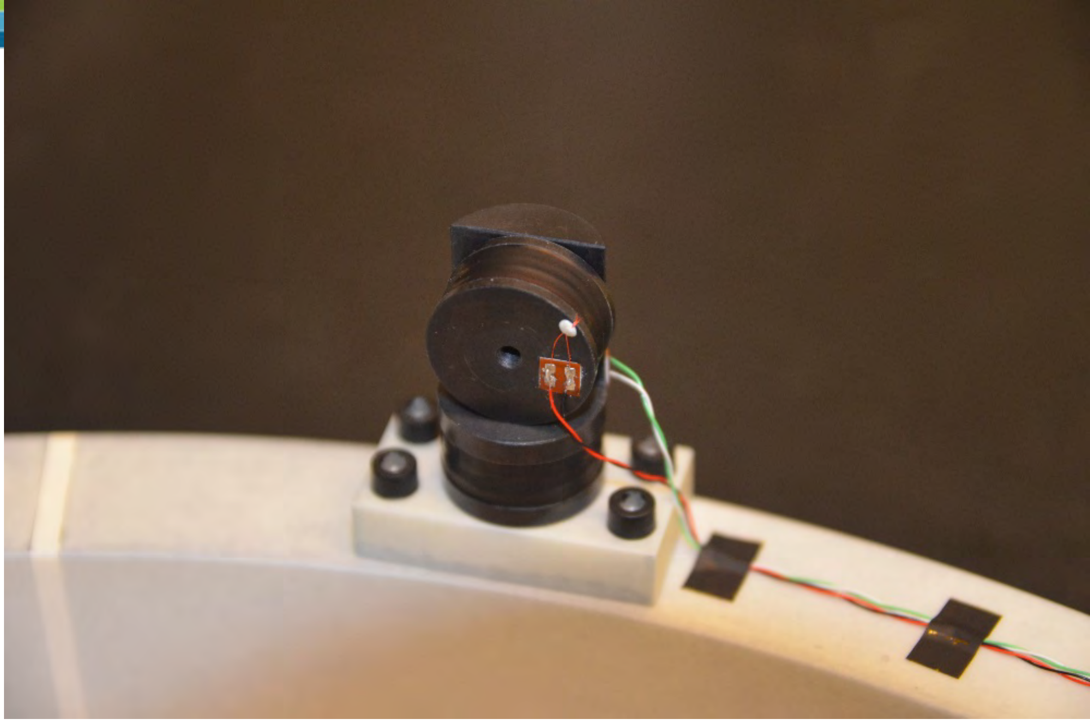
Counter mass side with dead weight

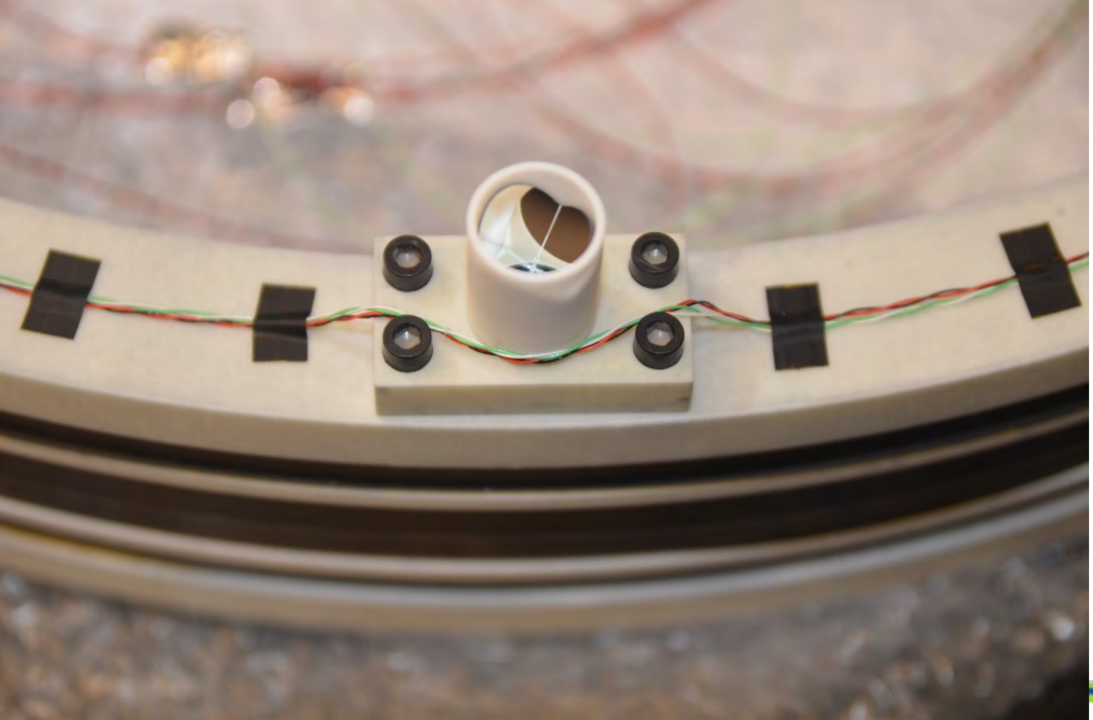




Installation of the coil

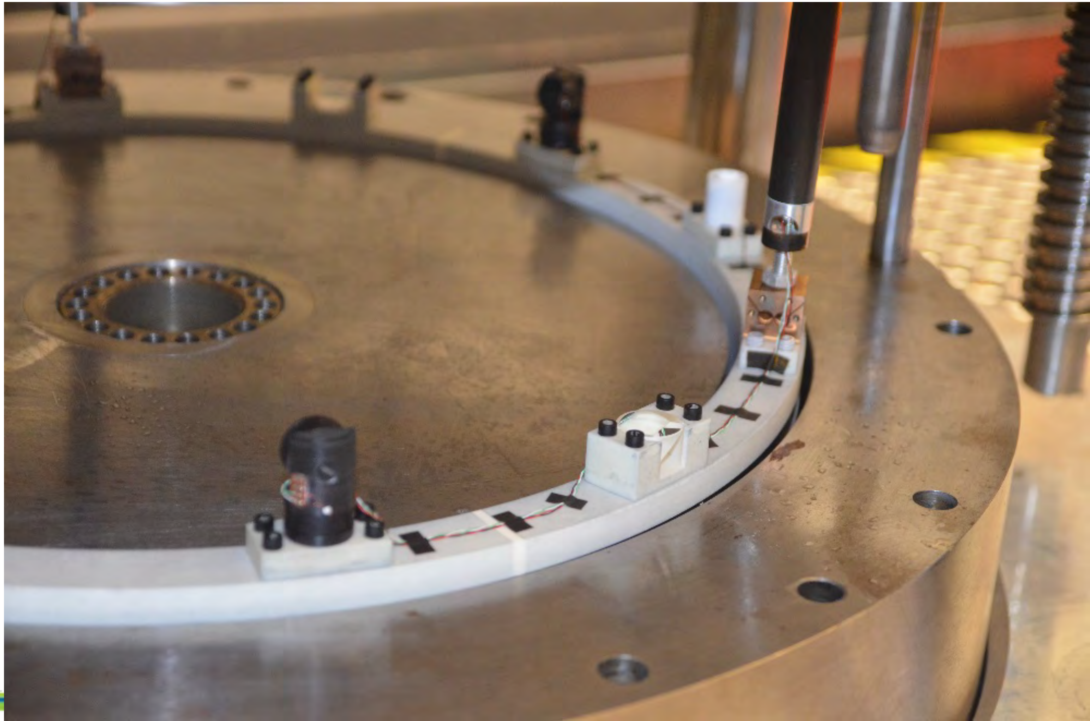
2 of 6 coils for x/y and Θ_x/Θ_y damping



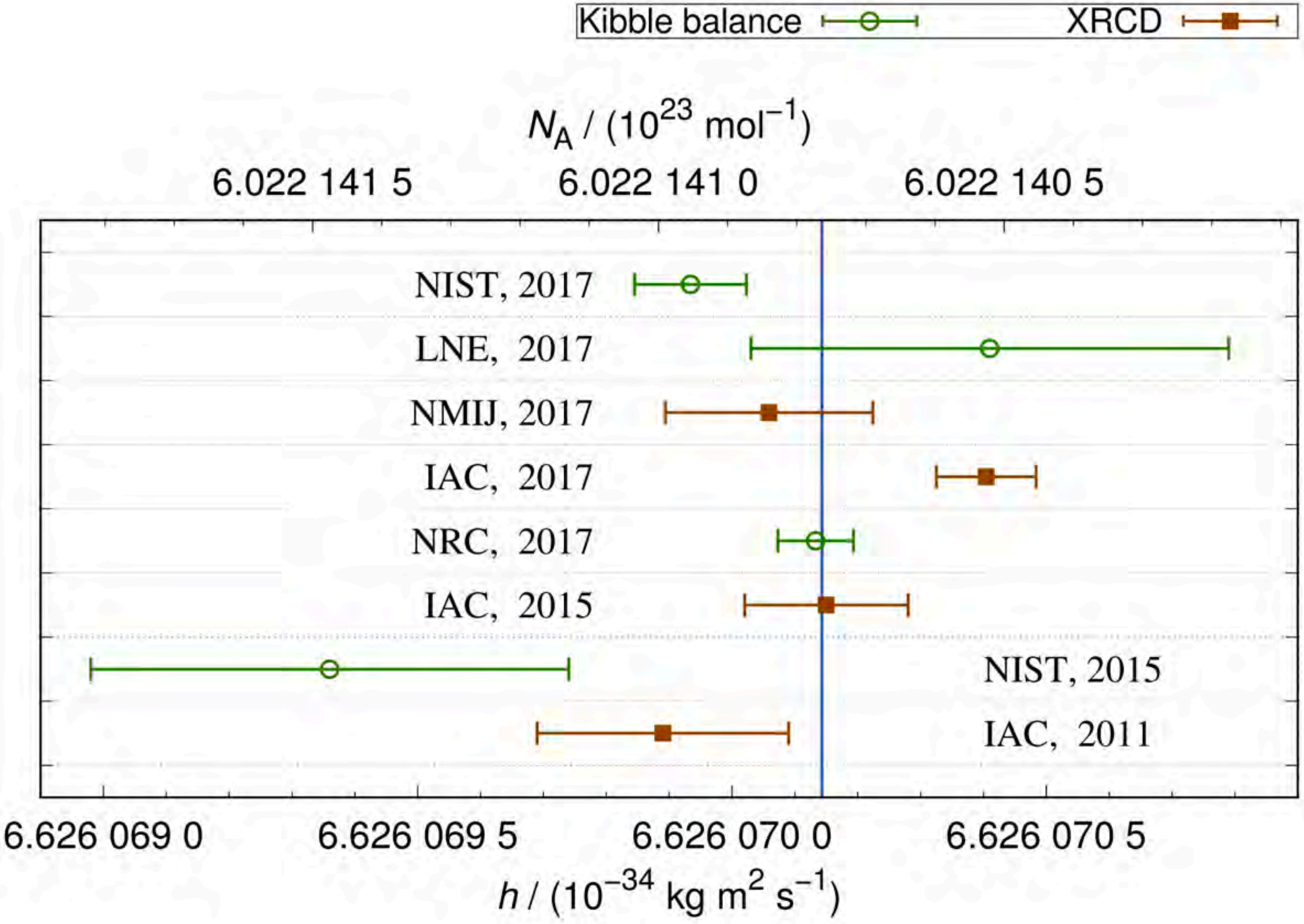


1 of 3 corner cubes for interferometer

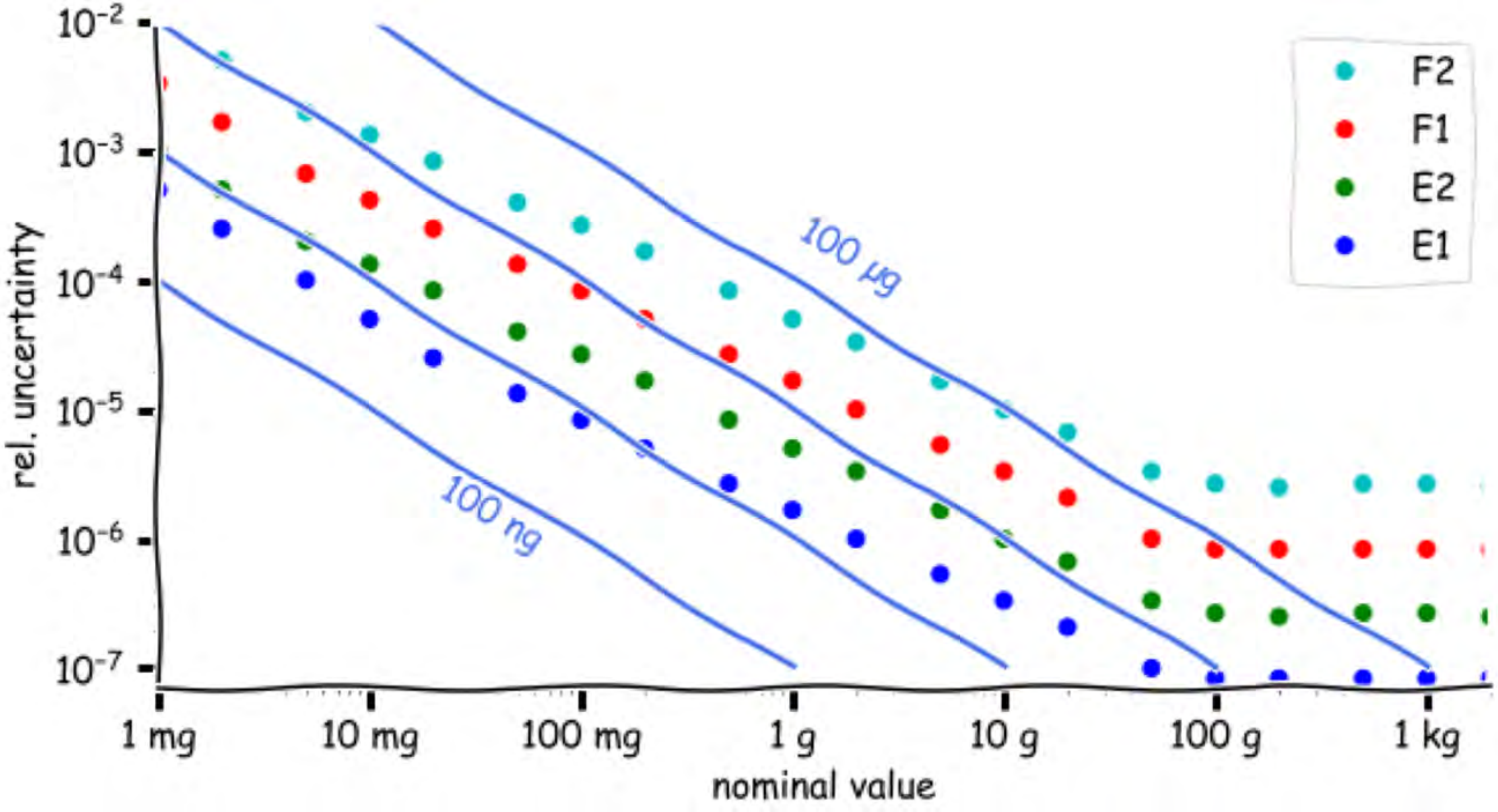
Coil inside air gap



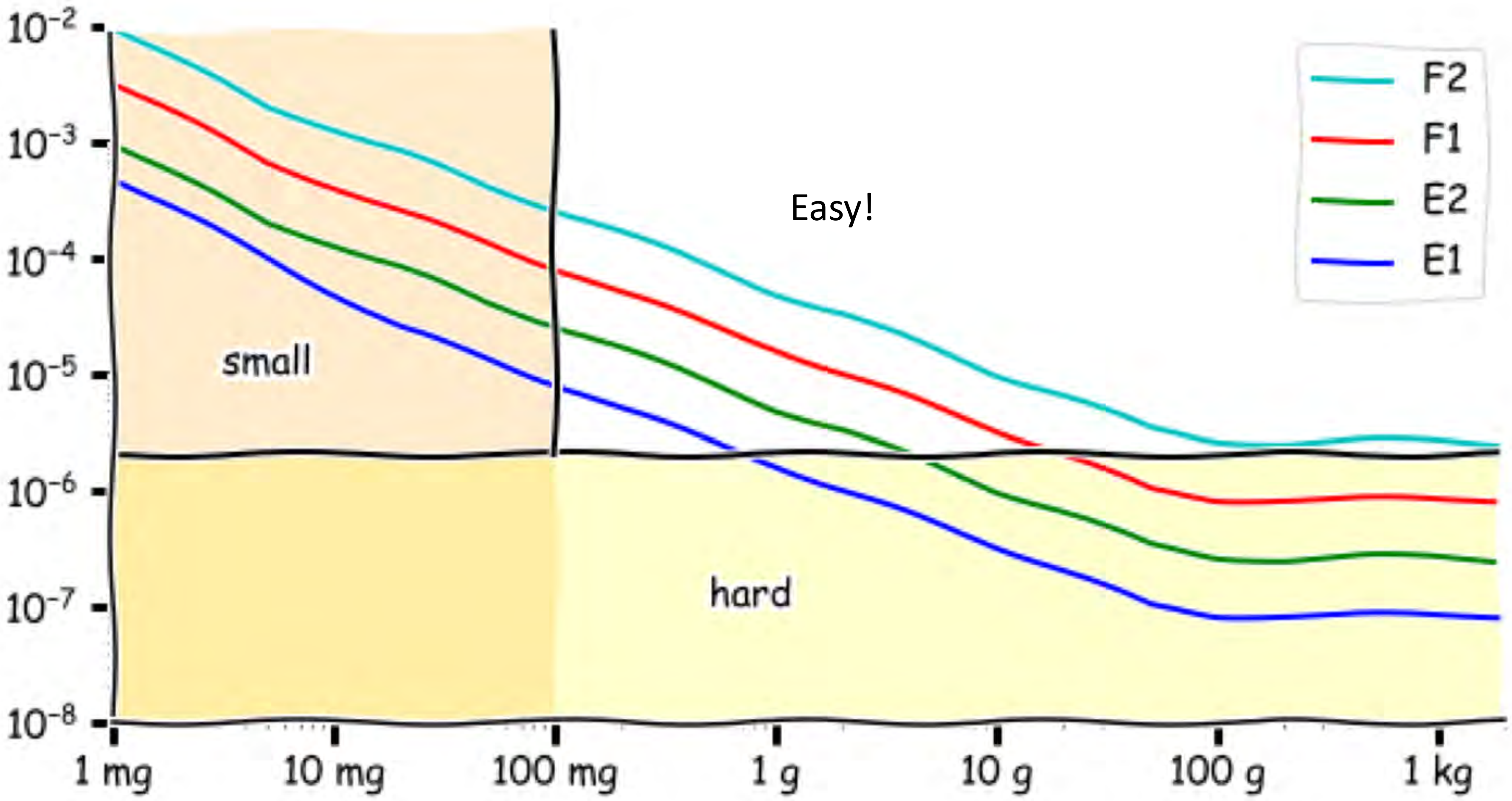
How well do we know *h*?



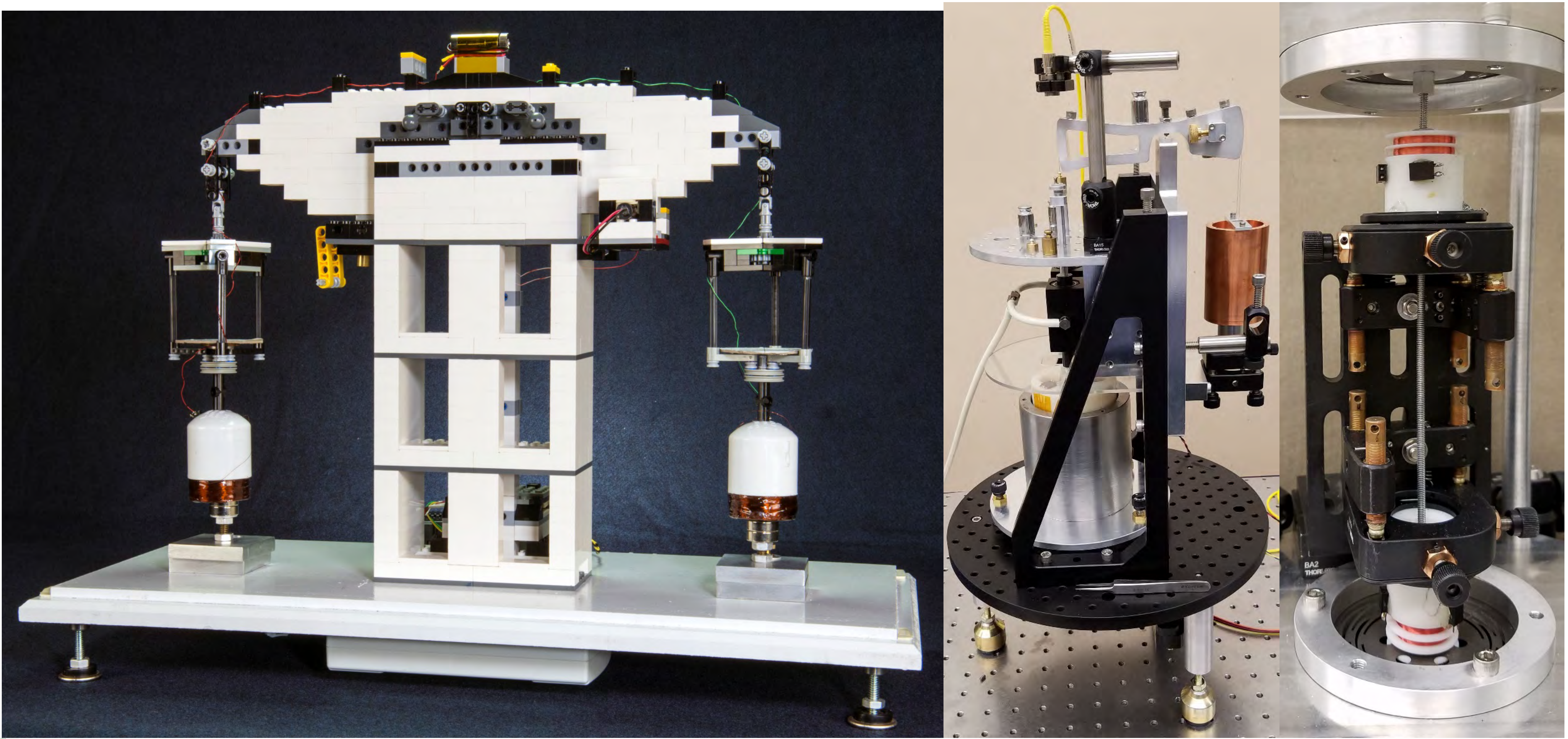
Kibble balances work at 1kg, but don't have to



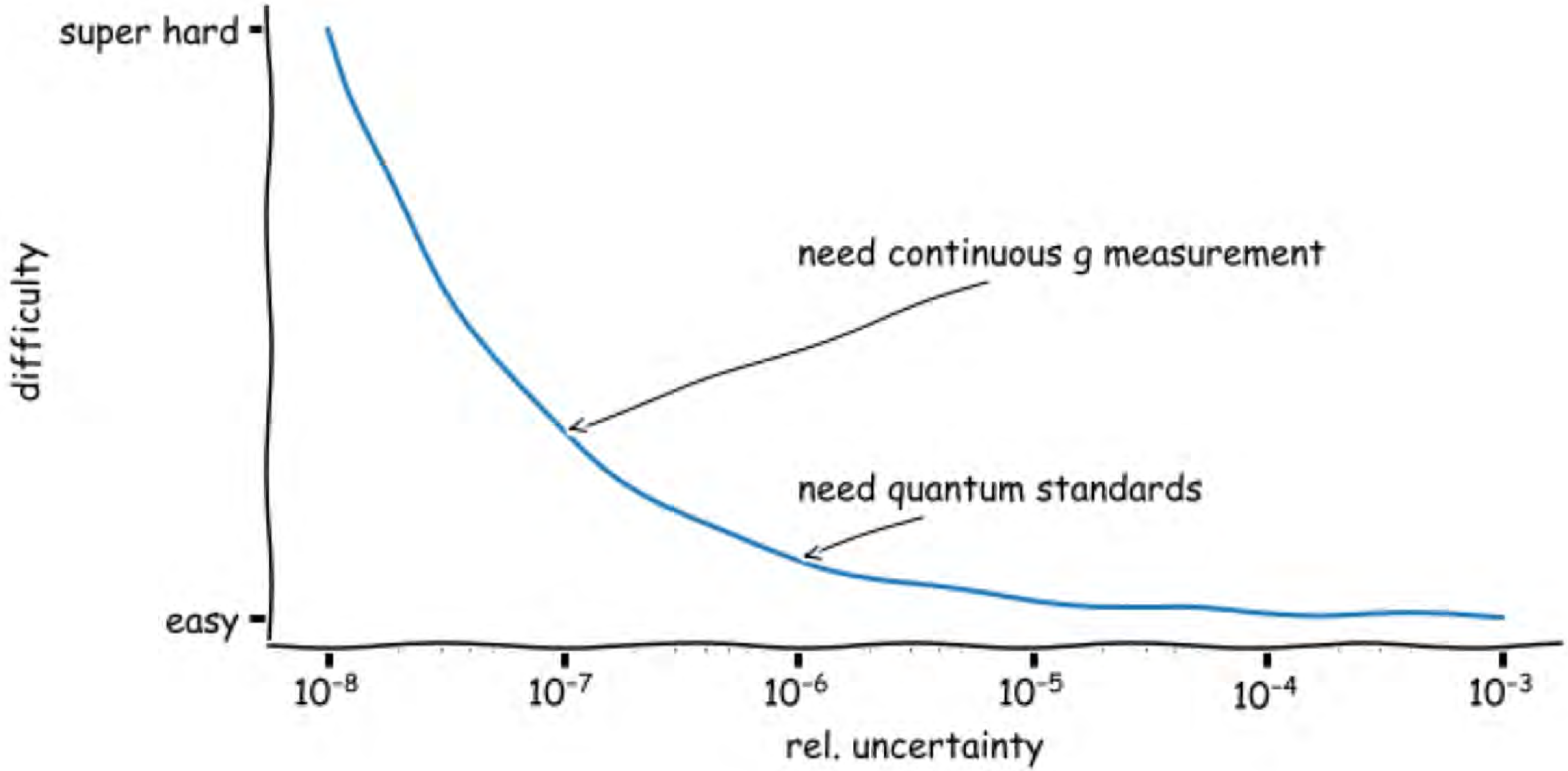
The sweet spot



A table top Kibble balance is a great research project



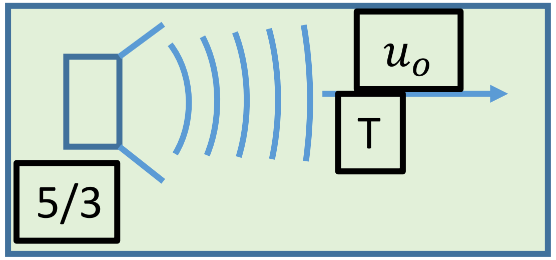
Difficulty of building a Kibble balance

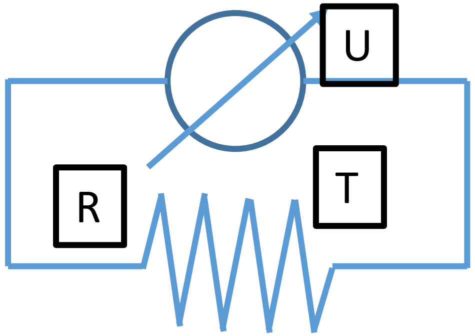


The Boltzmann constant

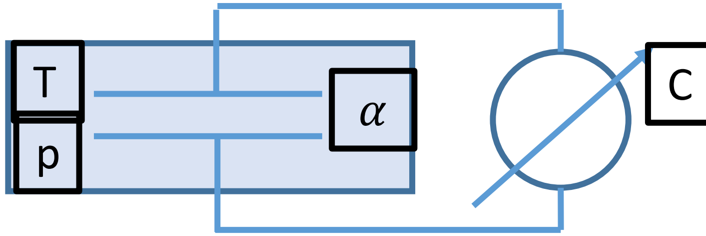
Three methods, capable of $\frac{\sigma_k}{k} = 2 \times 10^{-6}$

1. Acoustic Gas thermometry -- Speed of sound $u_o = \sqrt{\frac{5}{3}\frac{kT}{m}}$ for a monoatomic gas





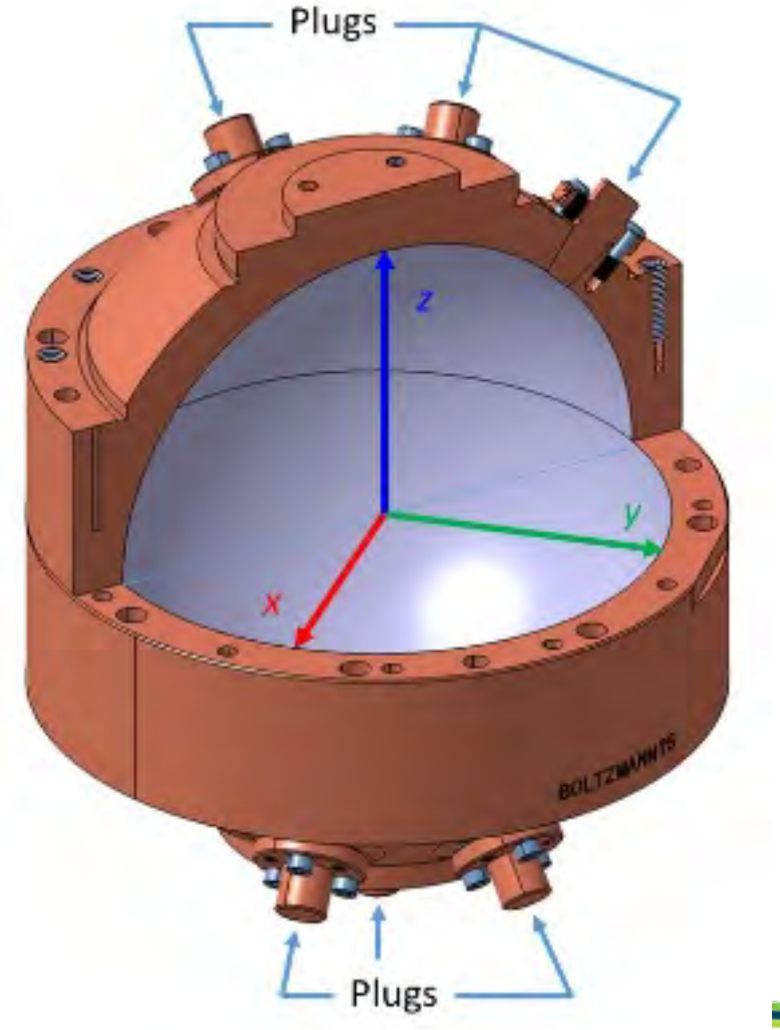
2. Johnson Noise thermometry – Noise voltage $\langle U^2 \rangle = 4kTR\Delta\upsilon$



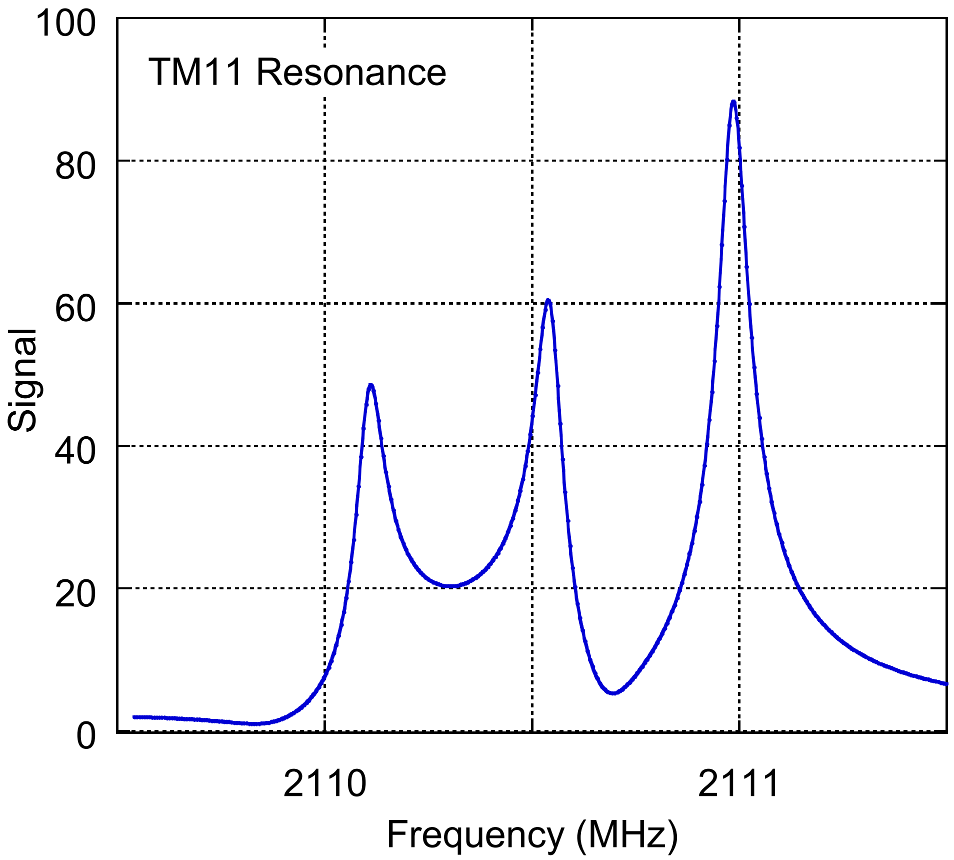
3. Dielectric gas constant thermometry -- $\varepsilon = \varepsilon_o + \frac{\alpha p}{kT}$

Acoustic Gas Thermometry

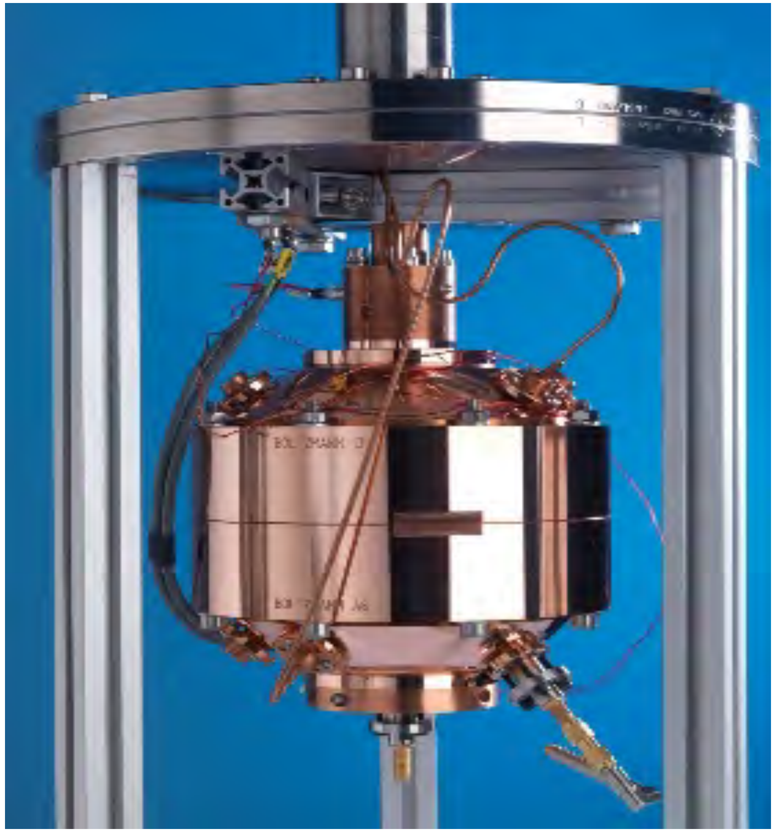
Triaxial elliptical resonator

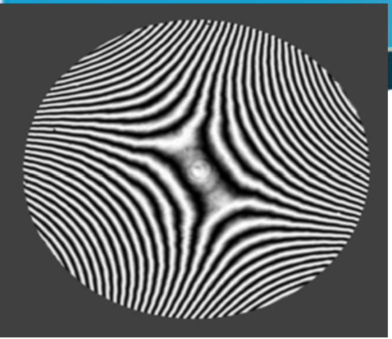


Microwave resonance to infer the volume

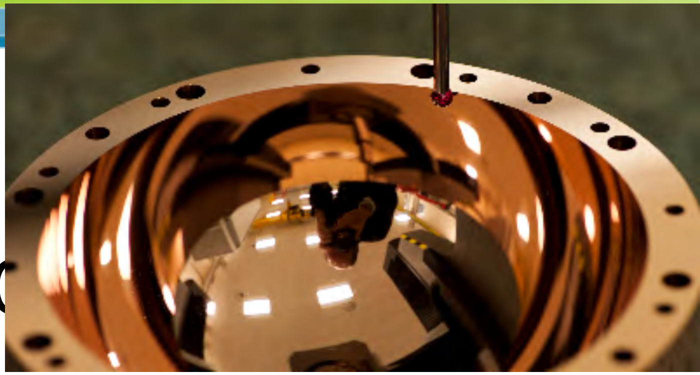


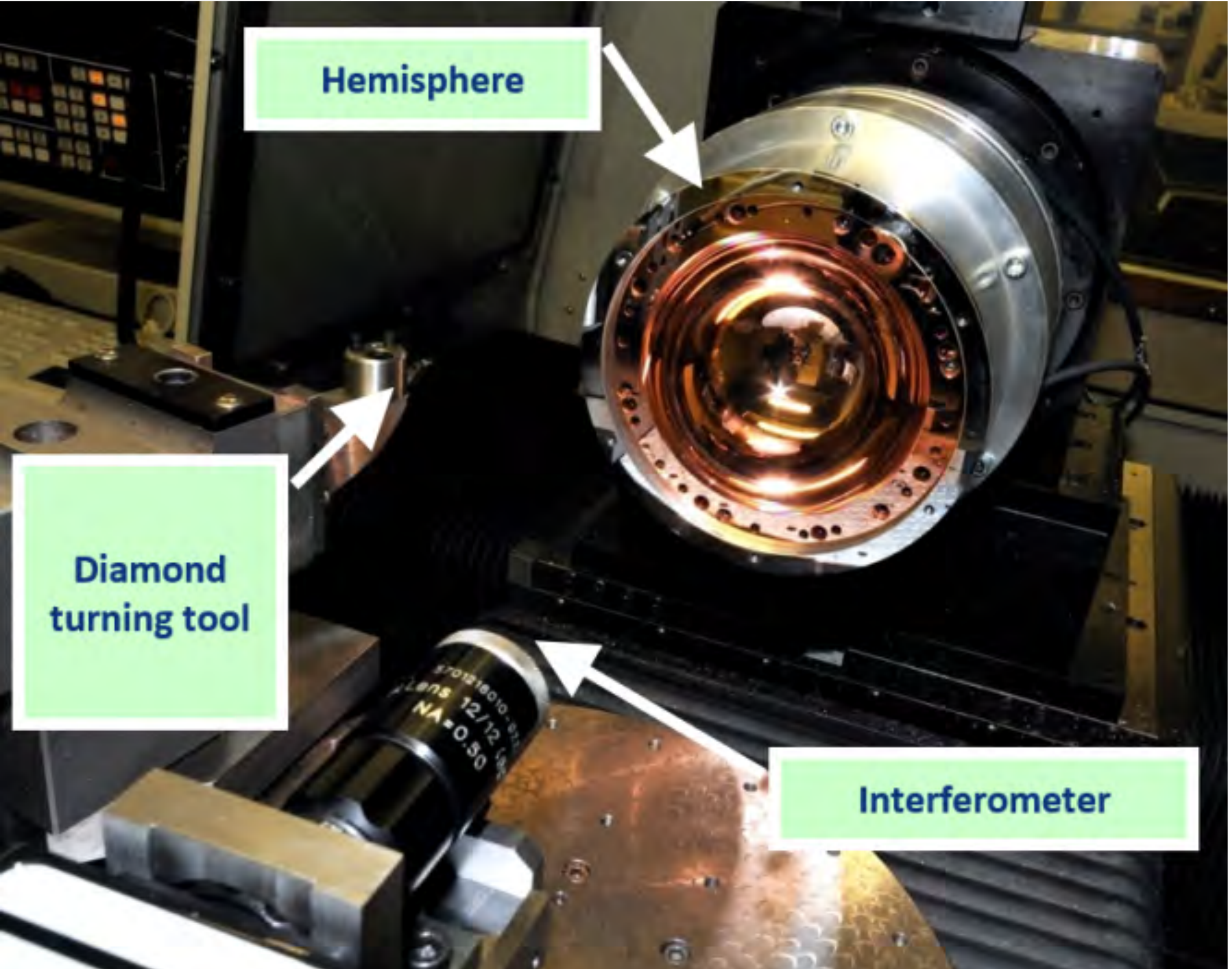
Acoustic resonance
to get acoustic wavelength





Resonator is precisely machined

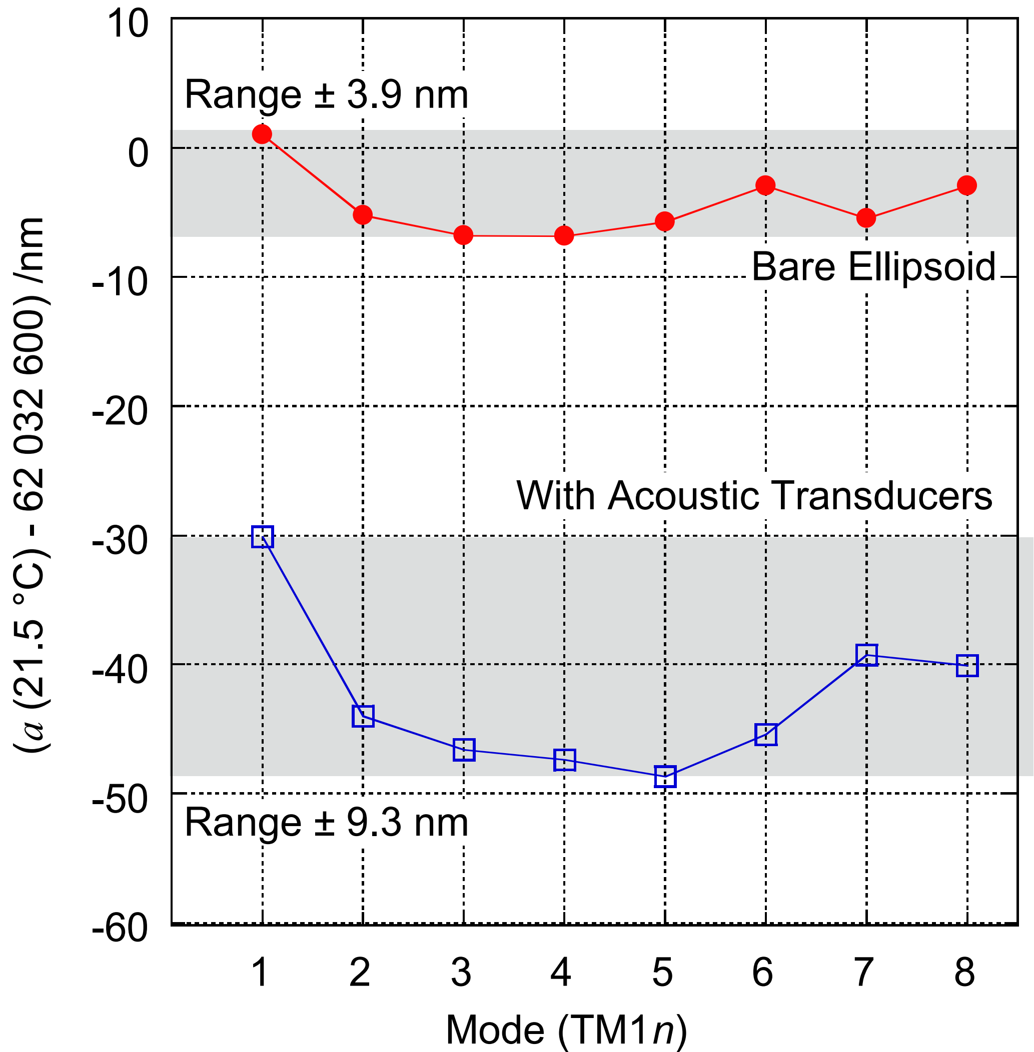




Hemisphere

Diamond turning tool

Interferometer

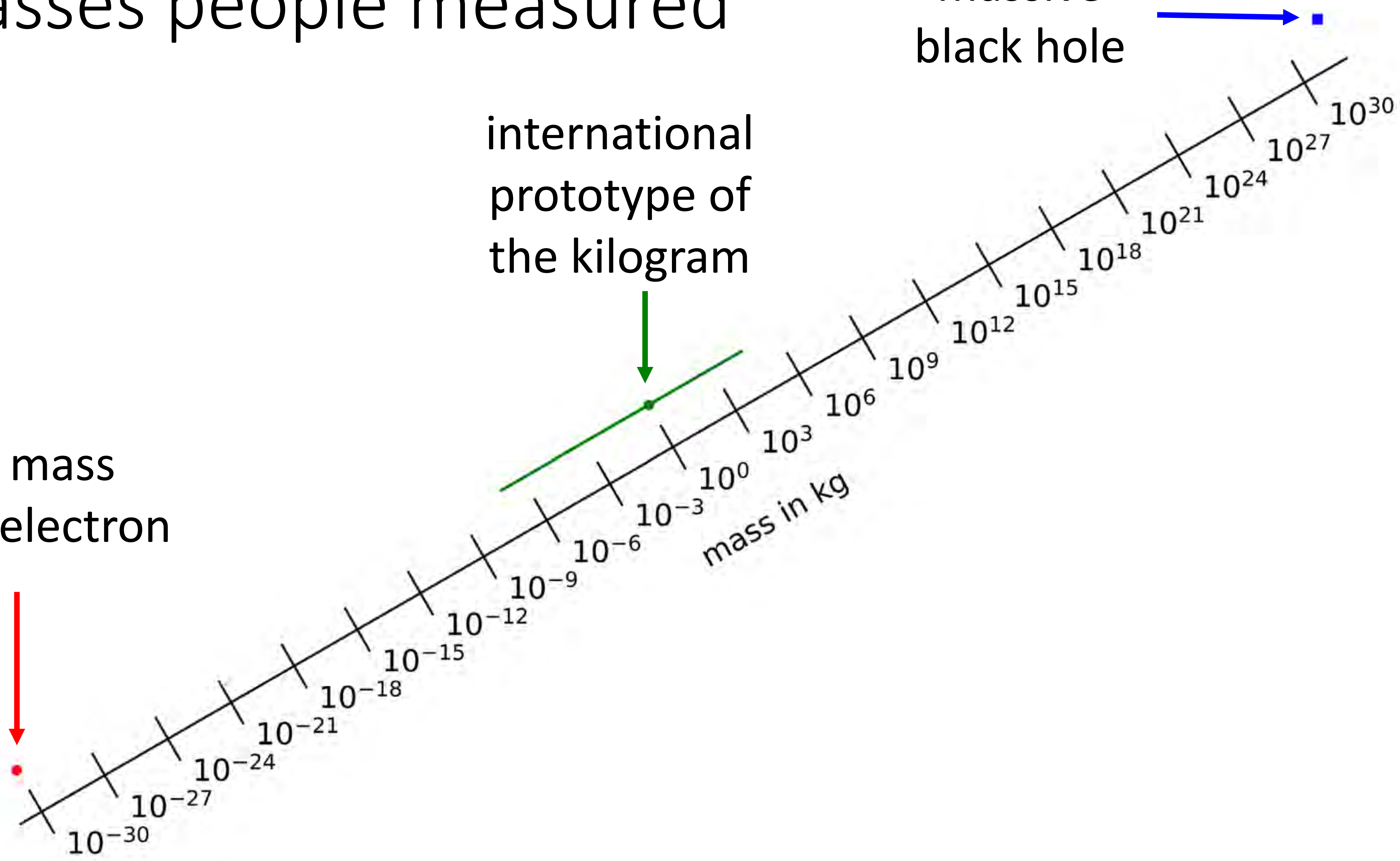


Next steps

The General conference of weights and measures (CGPM) will vote on the revision of the new SI on November 16th.

If accepted the new SI will come in effect on May 20 2019.

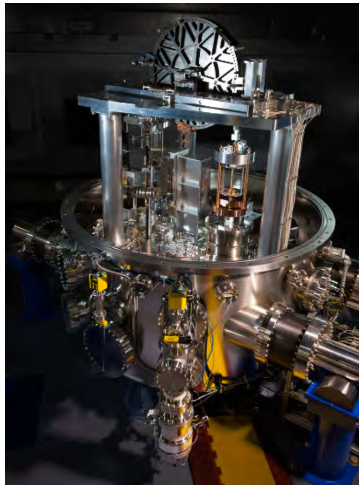
Masses people measured



Summary

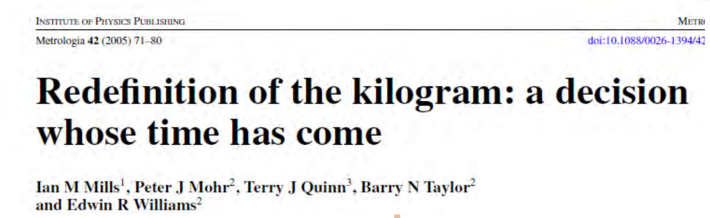
- Transition from 7 base units to 7 defining constant.
- Principle of the X-ray Crystal Density Method
- Principle of the Kibble balance

Thank you for your attention



Thank you to my wonderful colleagues

- Darine Haddad
- Leon Chao
- Frank Seifert
- David Newell
- Jon Pratt



Institute of Physics Publishing Metr

Metrologia 42 (2005) 71–80 doi:10.1088/0026-1394/42

Redefinition of the kilogram: a decision whose time has come

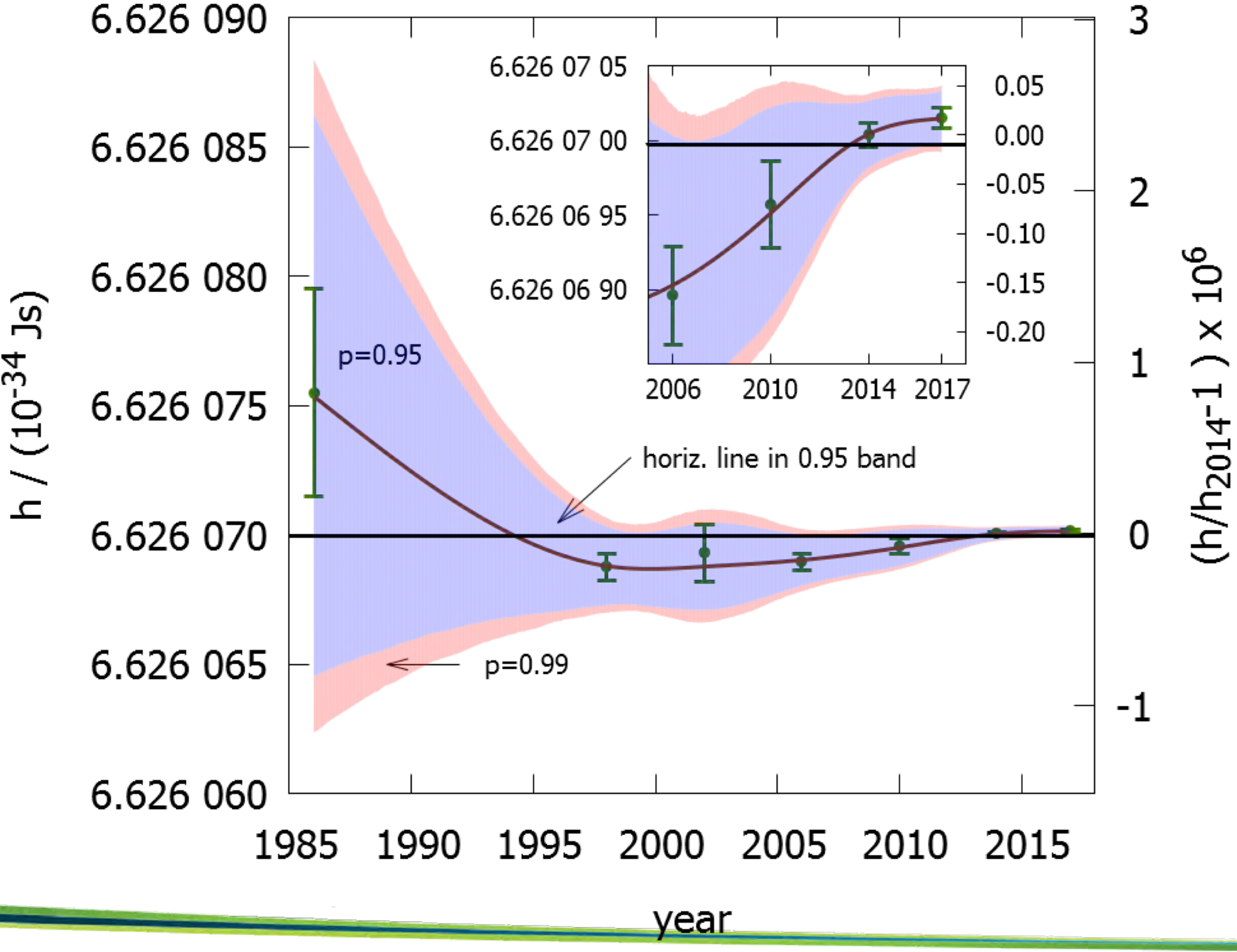
Ian M Mills[1], Peter J Mohr[2], Terry J Quinn[3], Barry N Taylor[2] and Edwin R Williams[2]

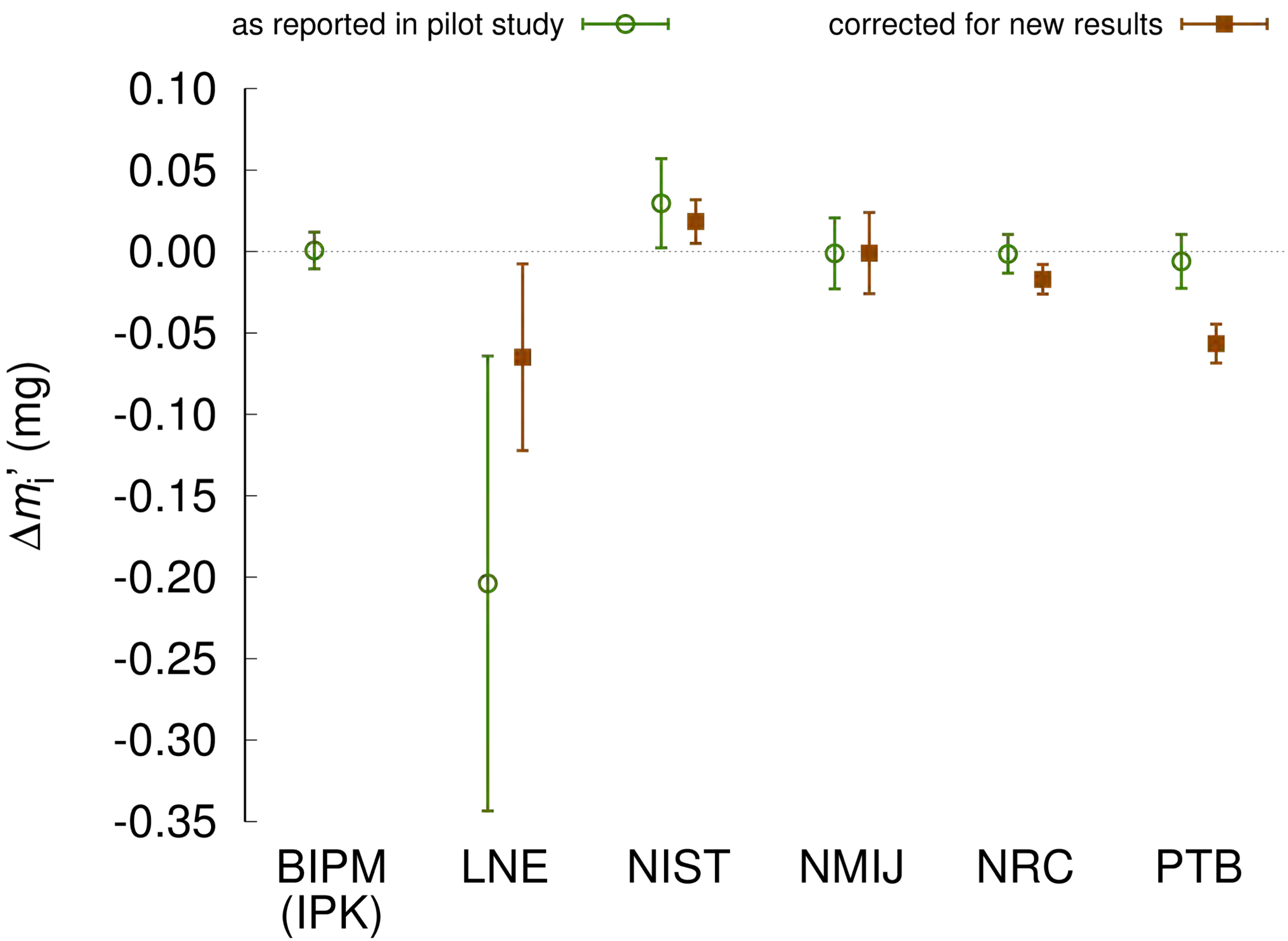
CCM G-1,
Three < 5 x 10^{-8},
One < 2 x 10^{-8}

Extraordinary
comparison
reveals
35 μg shift.

5 x 10^{-8}

2 x 10^{-8}


6.626 090
6.626 085
6.626 080
6.626 075
6.626 070
6.626 065
6.626 060
$h / (10^{-34}$ Js)
1985
1990
1995
2000
2005
2010
2015
year
3
2
1
0
-1
$(h/h_{2014}-1) \times 10^6$
p=0.95
p=0.99
horiz. line in 0.95 band
6.626 07 05
6.626 07 00
6.626 06 95
6.626 06 90
0.05
0.00
-0.05
-0.10
-0.15
-0.20
2006
2010
2014
2017


as reported in pilot study
corrected for new results
$\Delta m_i'$ (mg)
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
BIPM (IPK)
LNE
NIST
NMIJ
NRC
PTB