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A PRIMER ON THE PHYSICS OF FREE-ELECTRON LASERS

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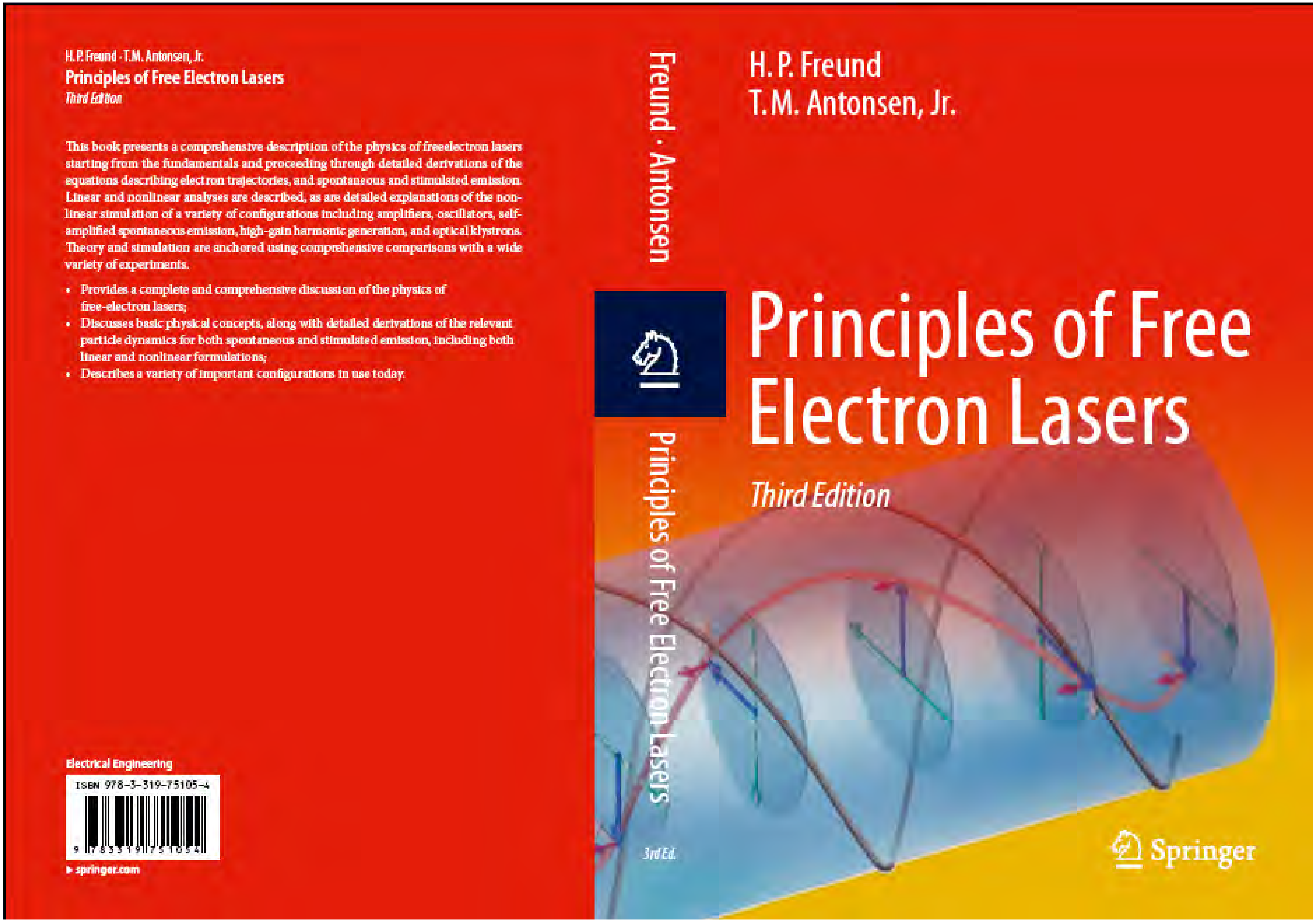
A LITTLE SELF-PROMOTION



IN THE BEGINNING….

The basic concept underlying the Free-Electron Laser was first described by Hans Motz in 1951

Journal of Applied Physics

Volume 22, Number 5 **May, 1951**

Applications of the Radiation from Fast Electron Beams

H. Motz

Microwave Laboratory, Stanford University, California

(Received July 3, 1950)

The radiation from fast electron beams passing through a succession of electric or magnetic fields of alternating polarity is examined. The radiation of maximum frequency is emitted in the forward direction. If the deflecting fields are not too large, a semiqualitative argument shows that the maximum frequency is the lowest possible harmonic. The frequencies emitted are determined by studying the Doppler effect, and the angular distribution of radiated energy as well as the total radiation are calculated in a simple straightforward manner with reference to well-known formulas of special relativity. The question of the coherence of the radiation is discussed. The spectral distribution of radiated energy is then calculated more exactly. It is concluded that several applications of the radiation appear possible. A scheme for obtaining millimeter-waves of considerable power is outlined. The upper limit of the power in a band extending down to a wave-length of 1 millimeter is calculated to be of the order of several kilowatts for a beam of one ampere and an energy of 1.5 megavolt. The use of the radiation for speed monitoring of beams with energies up to 1000 megavolt is discussed.

BASIC INTERACTION

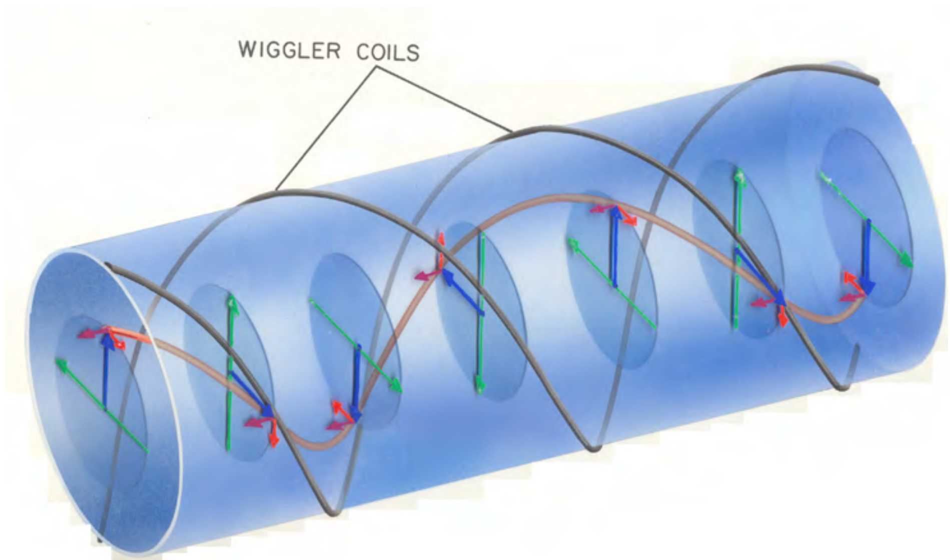
In general, an FEL consists of an electron beam propagating through a wiggler.

The axial ponderomotive force is $(\mathbf{v}_w \times \mathbf{B}_R)_z$ acts to decelerate the beam. The energy lost by the beam in this way acts to amplify the electromagnetic wave.

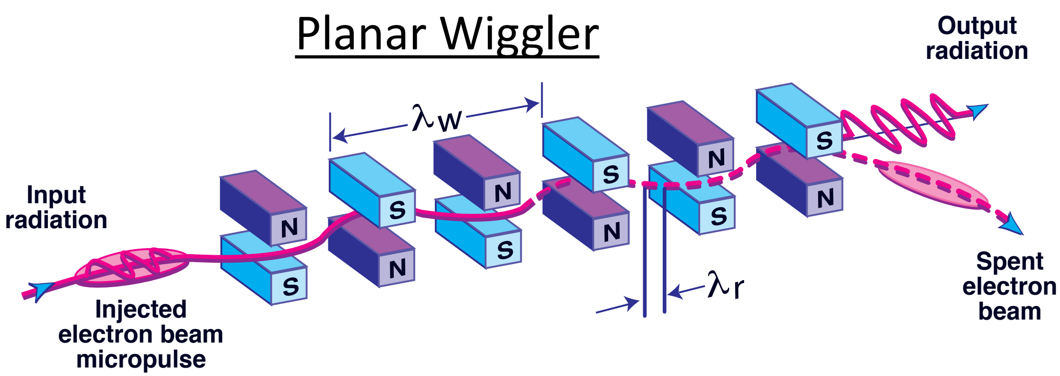
FEL INTERACTION

- **Electron beam undulates in wiggler and bunches at optical wavelength**
- **Optical radiation is amplified at the double-Doppler-shifted wavelength of the wiggler**

Helical Wiggler



Planar Wiggler



The effective wiggler strength is reduced to the rms wiggler field in a planar wiggler. Harmonic behavior is also different.

Resonance occurs when the phase velocity of the ponderomotive wave matches the electron velocity → $\omega = (k + k_w)\upsilon_{\|}$

IS IT AN FEL OR A UBITRON?

LASER OR TRAVELING-WAVE TUBE?

John Madey

Bob Phillips

BACKGROUND

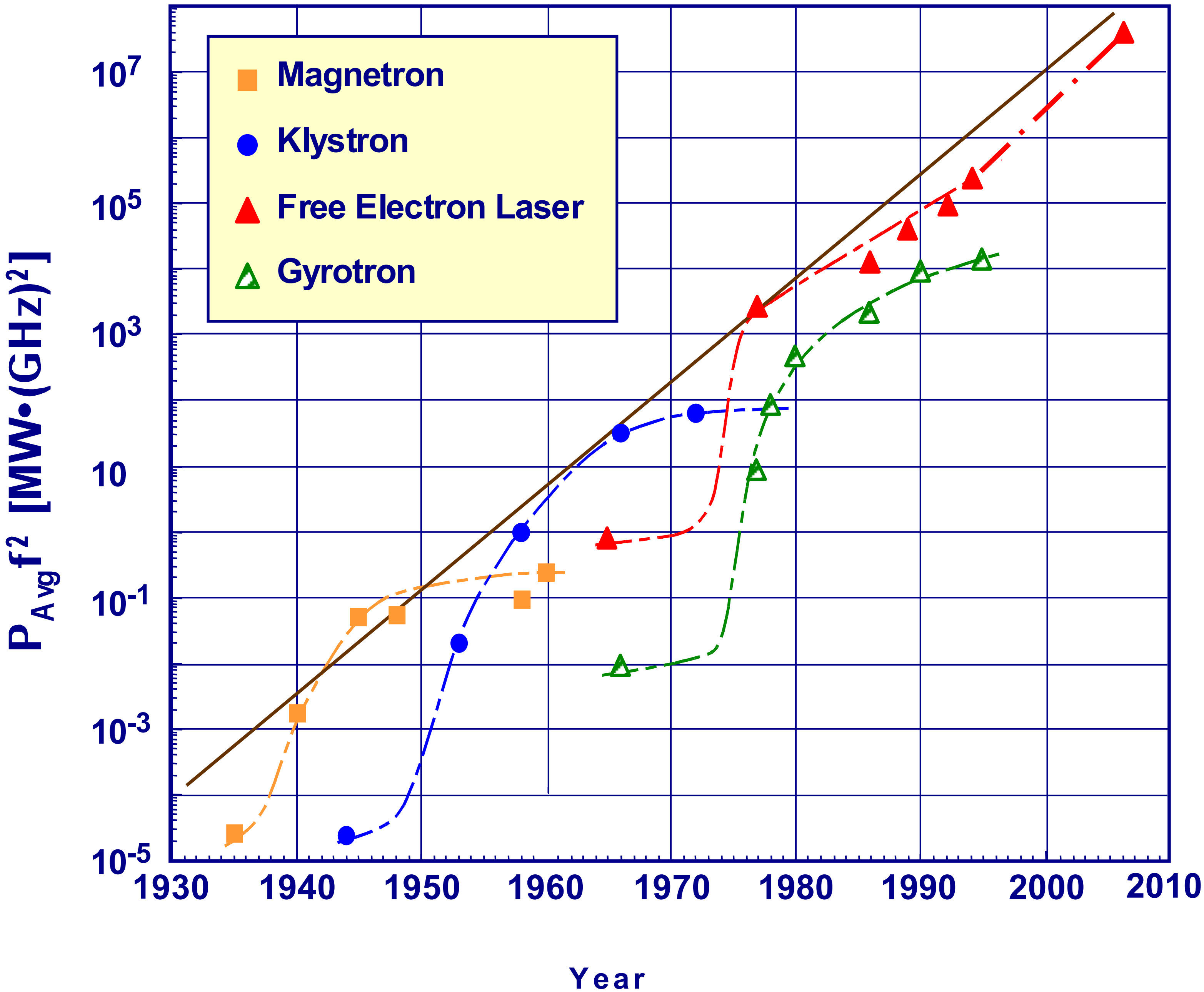
Question: Is an FEL a laser?

Answer: No, it's a classical device akin to a TWT

Quibble: Quantum mechanical effects do come into play when the spreading of the electron wave packet over the length of the wiggler is comparable to the radiated wavelength, but this is not an issue even for the currently envisioned X-FELs

Implication: The numerical techniques pioneered at the MIT Radiation Lab during WWII and further developed for treating TWTs can be adapted to treat FELs.

PROGRESS IN VACUUM ELECTRON DEVICES



WIGGLER OR UNDULATOR?

- The term *wiggler* was coined in the early days of FEL research, and *undulator* was once used interchangeably

- The convention now is to use *wiggler* for strong oscillatory motion and *undulator* for weaker oscillatory motion

- The distinction is codified using the *wiggler strength parameter*

$$K \equiv \frac{eB_w}{m_e c^2 k_w} = 0.09337 B_w[kG]\lambda_w[cm]]$$

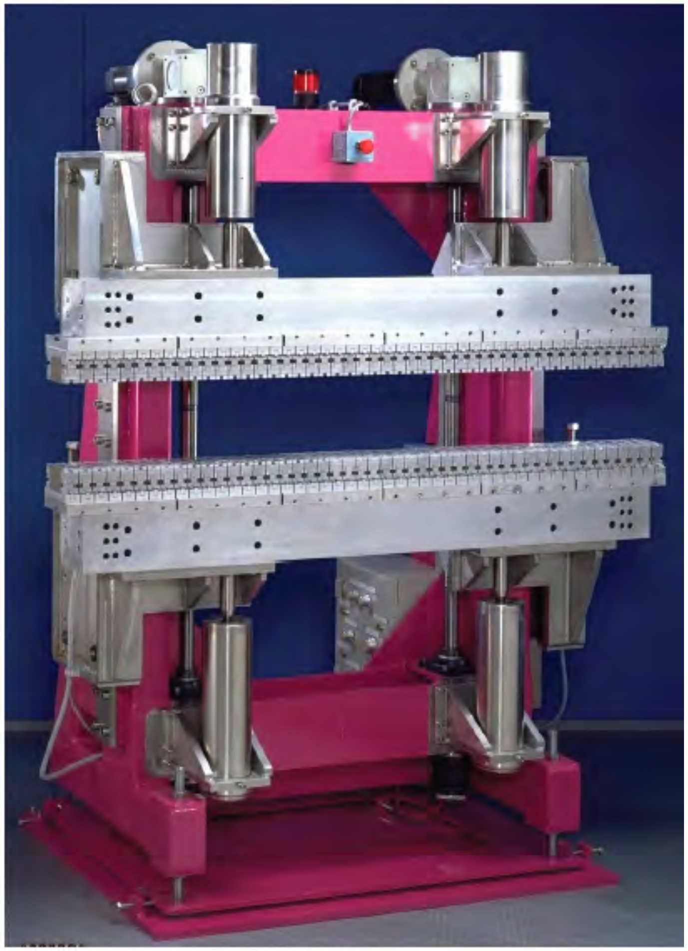
- When K is less than or of the order of unity, we use *undulator*
  - This includes most FELs

- When $K >> 1$, we use the *wiggler*
  - Used for many insertion devices in synchrotron light sources

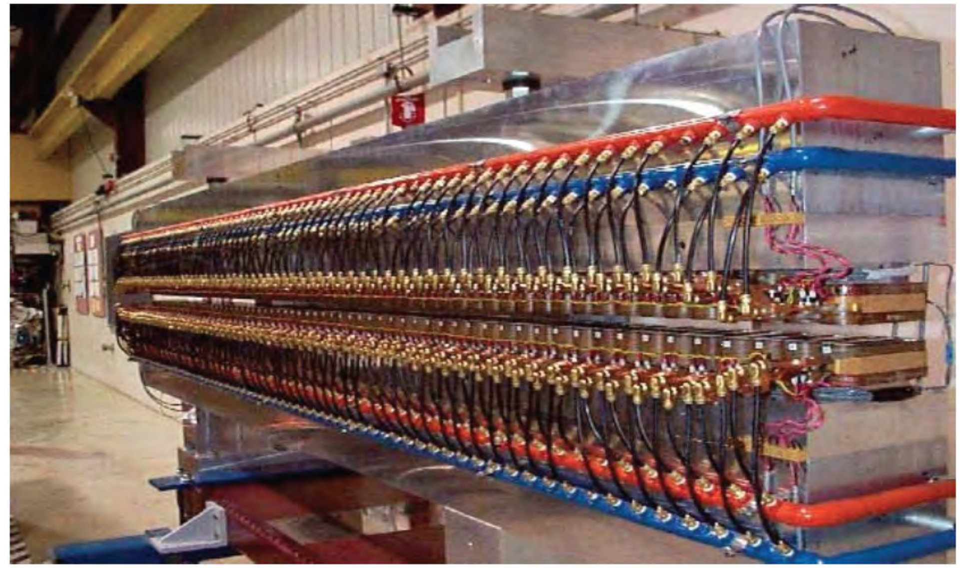
- I will use the terms interchangeably however

PLANAR WIGGLERS/UNDULATORS

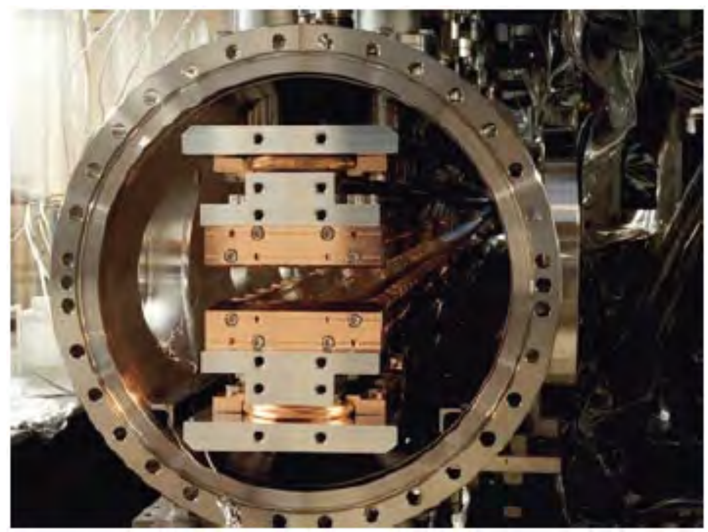
ESRF Undulator



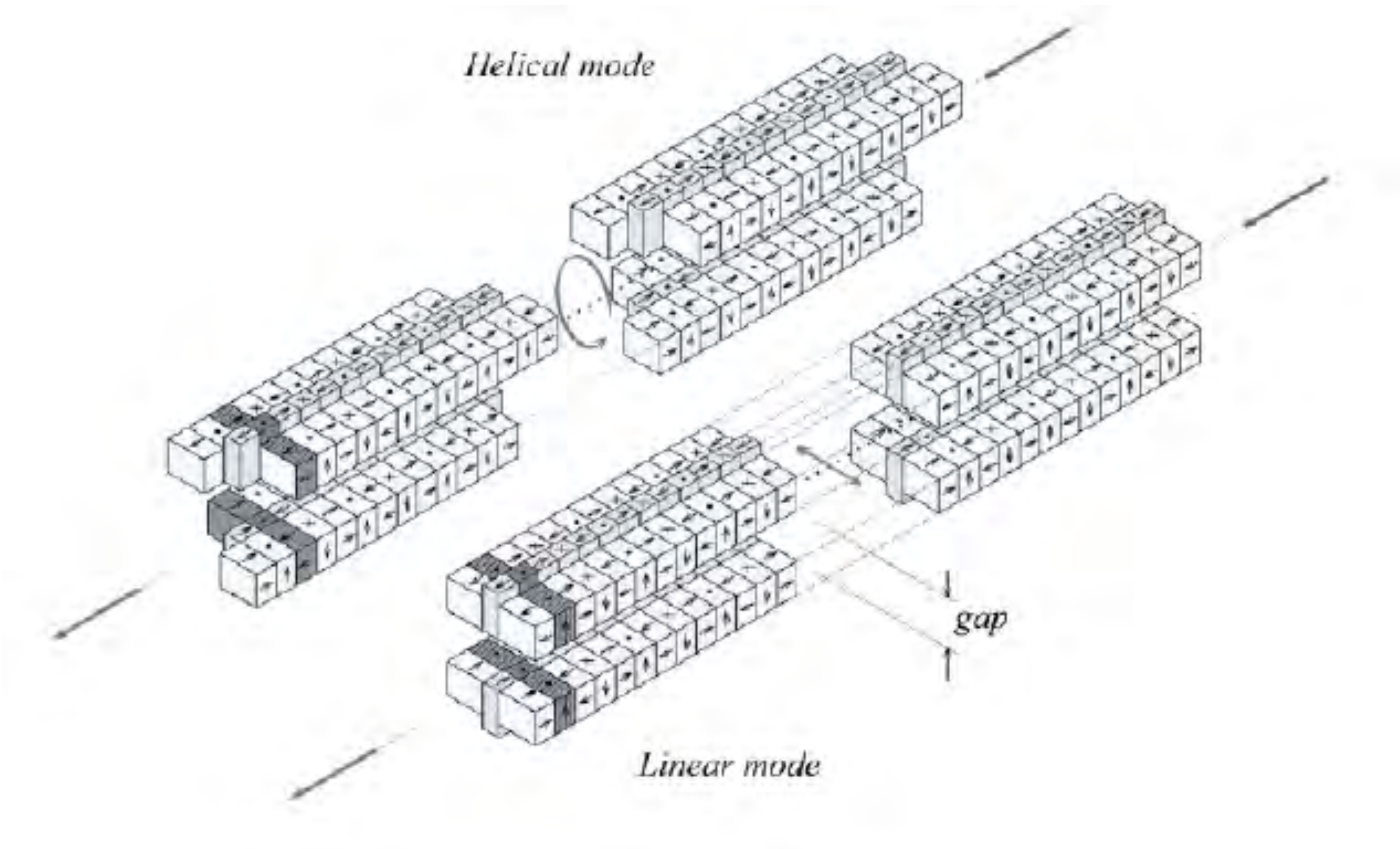
U. Wisconsin Electromanet Undulator



ESRF In-Vacuum Undulator



VARIABLE POLARIZATION WIGGLERS



From ASTeC (Daresbury Lab) Conceptual Design

CURRENT ACTIVITY

- **XFELs**
  - USA: SLAC/LCLS, LCLS-II
  - Germany: DESY
  - Switzerland: Paul Scherer Institute
  - Japan: Spring-8
  - South Korea: Pohang
  - China: Shanghai
- **UV/EUV**
  - Germany: DESY
  - Italy: Fermi-Trieste
- **IR**
  - UK: Daresbury
  - Germany: Rossendorf
  - Netherlands: Nijmegen
- **THz**
  - Germany: Fritz Haber Institute
  - Netherlands: Nijmegen
  - Japan: KEK
  - India: Delhi
  - Russia: Novosibirsk

This represents only a limited list

THE FEL RESONANCE IN FREE-SPACE

- The interaction occurs at the intersection of the forward-propagating free-space mode and the ponderomotive wave

$$\omega = ck \rightarrow \text{Free-space mode}$$

$$\omega = (k + k_w)\upsilon_{||} \rightarrow \text{Ponderomotive Wave}$$

- Upon substitution

$$\omega = \left(\frac{\omega}{c} + k_w\right)\upsilon_{||}$$

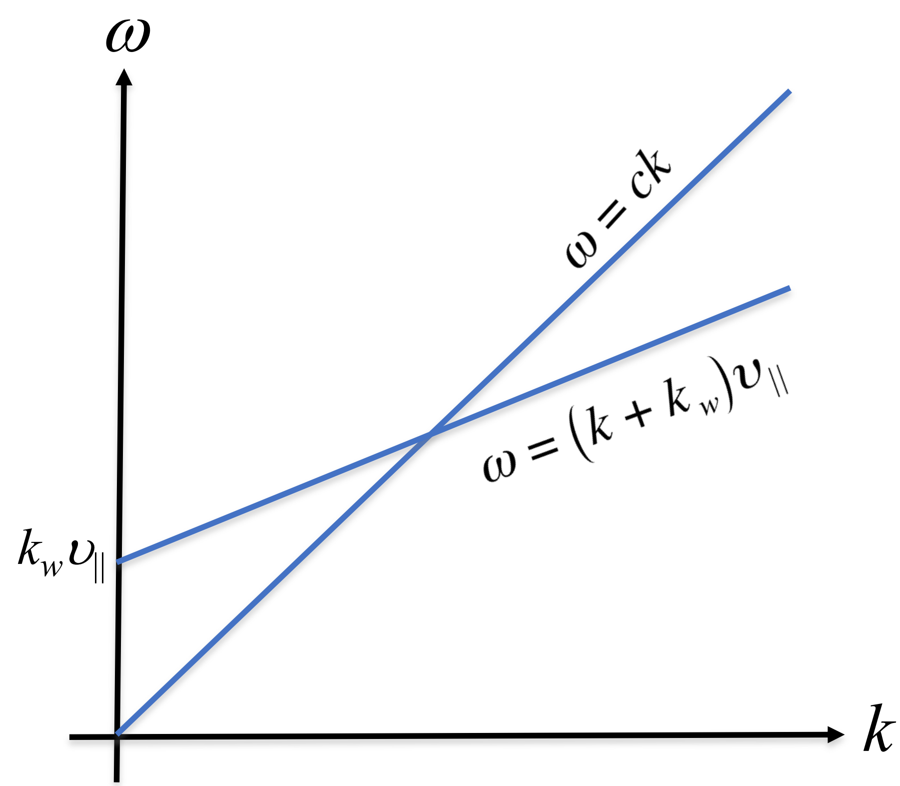
- And rearrangement

$$\left(1 - \beta_{||}\right)\omega = k_w \upsilon_{||}$$

- Hence

$$\omega = \left(1 + \beta_{||}\right)\gamma_{||}^2 k_w \upsilon_{||} \approx 2\gamma_{||}^2 k_w \upsilon_{||}$$

$$\gamma_{||}^2 = \frac{1}{1 - \beta_{||}^2}$$



THE PENDULUM EQUATION

- Electron energy transfer dynamics is governed by the nonlinear pendulum equation

$$\frac{d^2\psi}{dz^2} = -\mu^2 \sin\psi \quad \longrightarrow \quad \mu^2 \propto B_w B_{rad}$$

- Where the ponderomotive phase is defined as

$$\psi(z,t_0) = -\omega t_0 + \int_0^z dz' \left[k + k_w - \frac{\omega}{\upsilon_z(z',t_0)} \right]$$

- This equation can be integrated once to obtain

$$\frac{d\psi}{dz}\frac{d^2\psi}{dz^2} = -\mu^2 \frac{d\psi}{dz}\sin\psi \quad \longrightarrow \quad \frac{1}{2}\frac{d}{dz}\left(\frac{d\psi}{dz}\right)^2 = \mu^2 \frac{d}{dz}\cos\psi$$

$$\longrightarrow \quad \frac{1}{2}\left(\frac{d\psi}{dz}\right)^2 - \mu^2 \cos\psi = H$$

- The **separatrix** is found for $H = \mu^2$ and delineates the boundary between trapped & untrapped orbits

$$\frac{d\psi}{dz} = \pm\, 2\mu \cos\left(\psi/2\right)$$

THE MEANING OF $d\psi/dz$

- The interaction is strong if the phase is nearly constant

$$\frac{d\psi}{dz} = k + k_w - \frac{\omega}{\upsilon_{||}} \approx 0$$

  - On-resonance $d\psi/dz = 0$

- It can also be thought of as an energy mis-match parameter. Define:

$$\upsilon_{res} = \frac{\omega}{k + k_w}$$

$$\frac{d\psi}{dz} = \frac{\omega}{c}\left(\frac{1}{\beta_{res}} - \frac{1}{\beta_{||}}\right) = \frac{\omega}{c}\left(\frac{1}{\left(1 - 1/\gamma_{res}^2\right)^{1/2}} - \frac{1}{\left(1 - 1/\gamma_{||}^2\right)^{1/2}}\right)$$

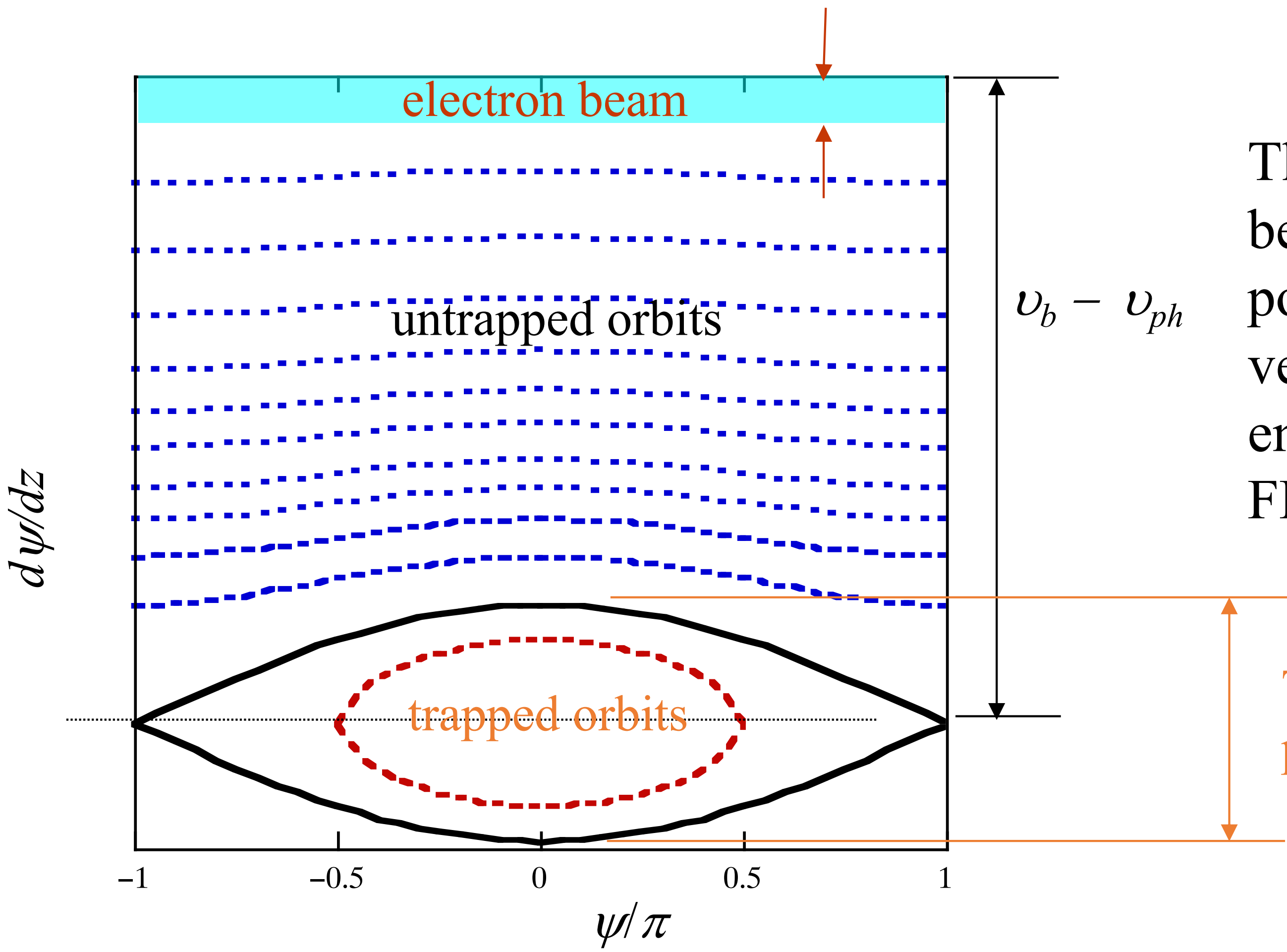
$$\approx \frac{\omega}{c}\left(\frac{1}{2\gamma_{res}^2} - \frac{1}{2\gamma_{||}^2}\right) = \frac{\omega}{2c}\frac{\left(\gamma_{||}^2 - \gamma_{res}^2\right)}{\gamma_{||}^2\gamma_{res}^2} \approx \frac{\omega}{2c}\frac{\left(\gamma_{||} - \gamma_{res}\right)\left(\gamma_{||} + \gamma_{res}\right)}{\gamma_{||}^2\gamma_{res}^2}$$

$$\approx \frac{\omega}{\gamma_{res}^2 c}\frac{\left(\gamma_{||} - \gamma_{res}\right)}{\gamma_{||}} = 2k_w\frac{\left(\gamma_{||} - \gamma_{res}\right)}{\gamma_{||}} \qquad \gamma_{||} \approx \gamma_{res} >> 1$$

PHASE SPACE DYNAMICS

• The FEL operates best when the axial energy spread of the electron beam, $\Delta\upsilon_b$, is much less than the velocity difference between the beam and the ponderomotive wave.

$\Delta\upsilon_b << \upsilon_b - \upsilon_{ph}$ allows for a large part of the beam to be in resonance and trapped

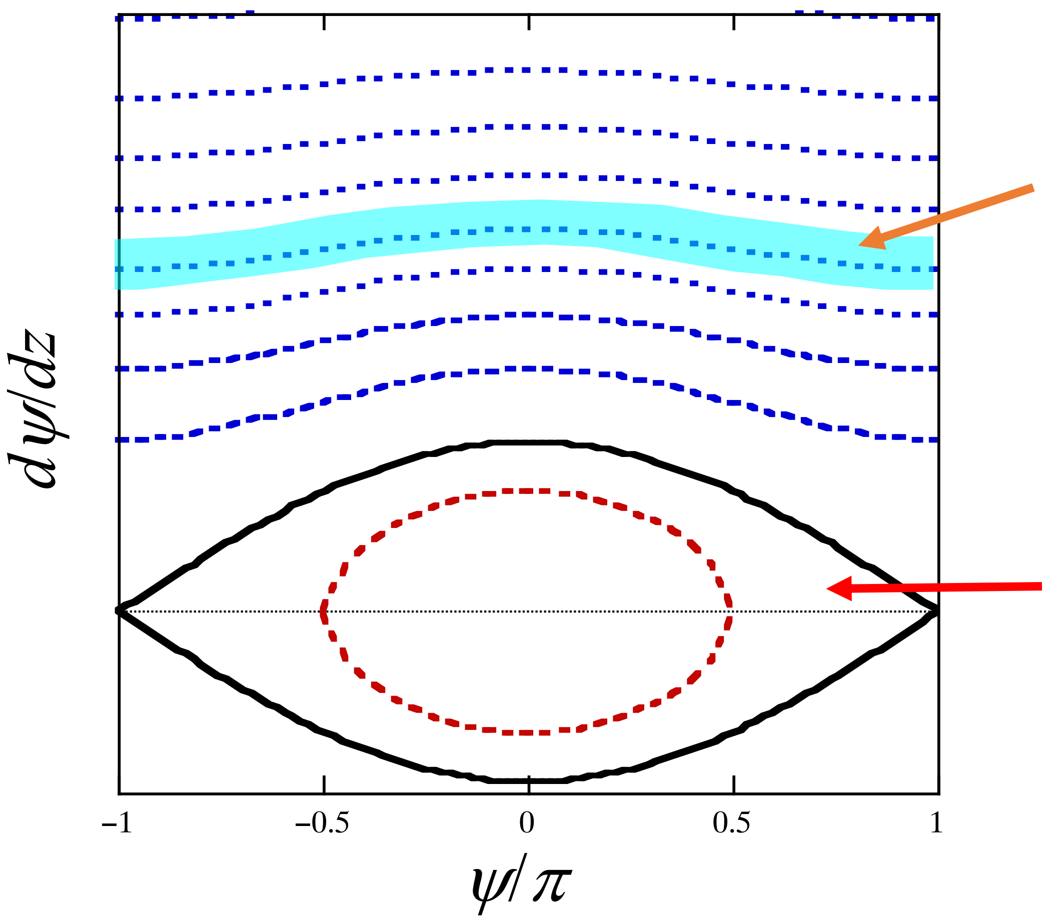


The velocity difference between the beam and ponderomotive phase velocities defines the free energy available from the FEL

The separatrix depth is proportional to the Power$^{1/4}$

PHASE SPACE - LINEAR REGIME

The linear regime exists when the electron beam is executing untrapped trajectories in phase space. Wave growth may be exponential if the wiggler is longer than an exponentiation length.

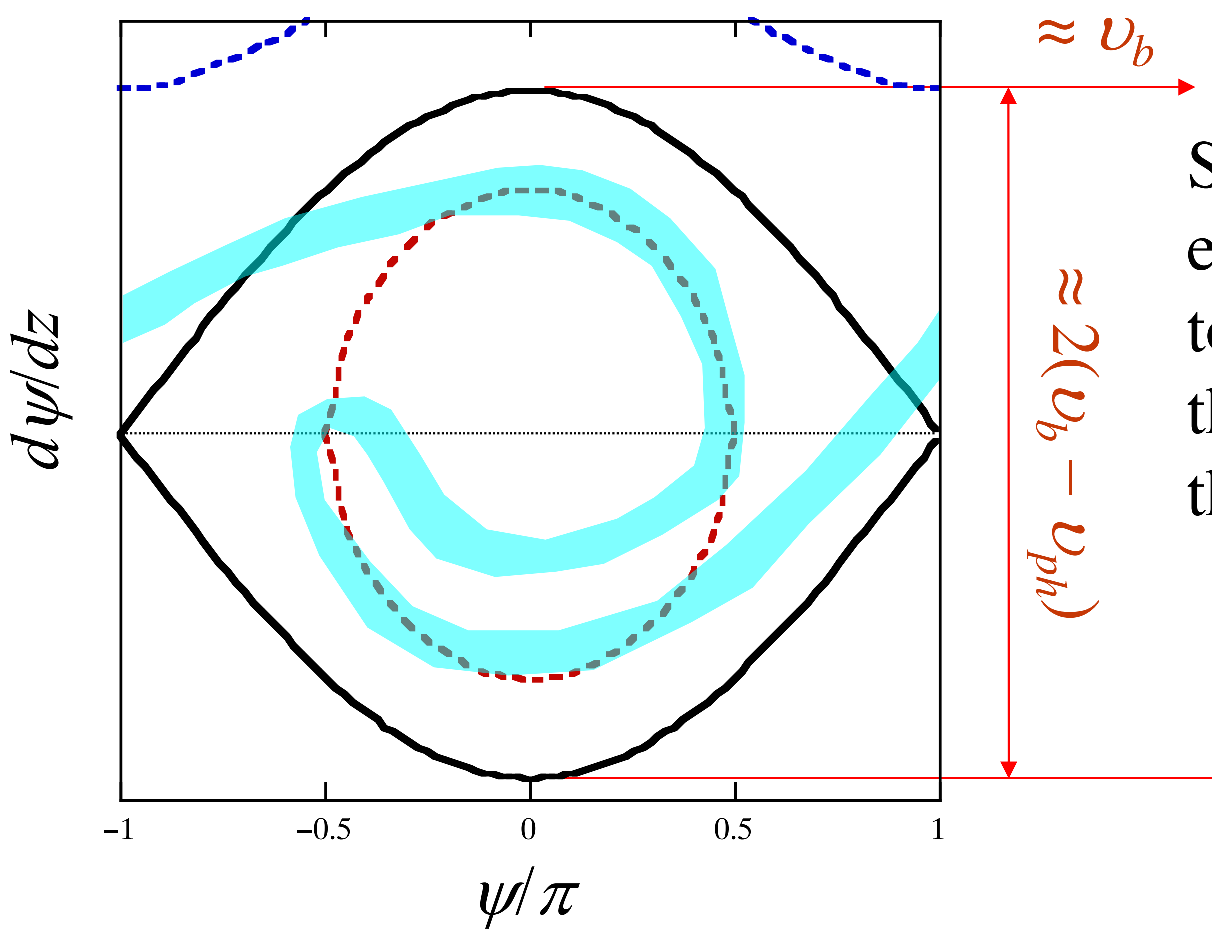


As the electron beam loses energy, it drops in the phase plane and develops a velocity modulation.

As the wave grows in power, the depth of the separatrix increases.

PHASE SPACE AT SATURATION

The nonlinear regime begins as the electrons begin to cross the separatrix onto trapped trajectories. Not all electrons, however, cross the separatrix, and this is a limiting factor in the ultimate extraction efficiency.



Saturation occurs for an energy loss corresponding to electrons dropping from the top to the bottom of the well, *i.e.*

$$\eta \approx 2\gamma_{\|}^2 \upsilon_b (\upsilon_b - \upsilon_{ph})/c^2$$

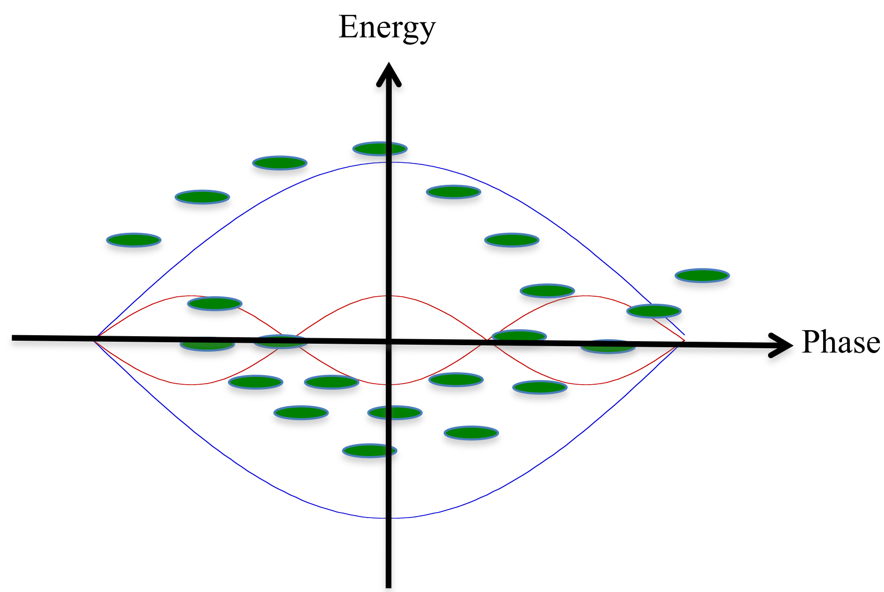
QUANTUM MECHANICAL EFFECTS

- Quantum mechanical effects become important when the spreading of the electron wave packet over the length of the wiggler is comparable to the wavelength, which correspond s to

$$\Delta z = \frac{N_w \lambda_c}{\gamma} << \lambda \qquad \lambda_c = Compton\ Wavelength \approx 0.024\ \text{Å}$$

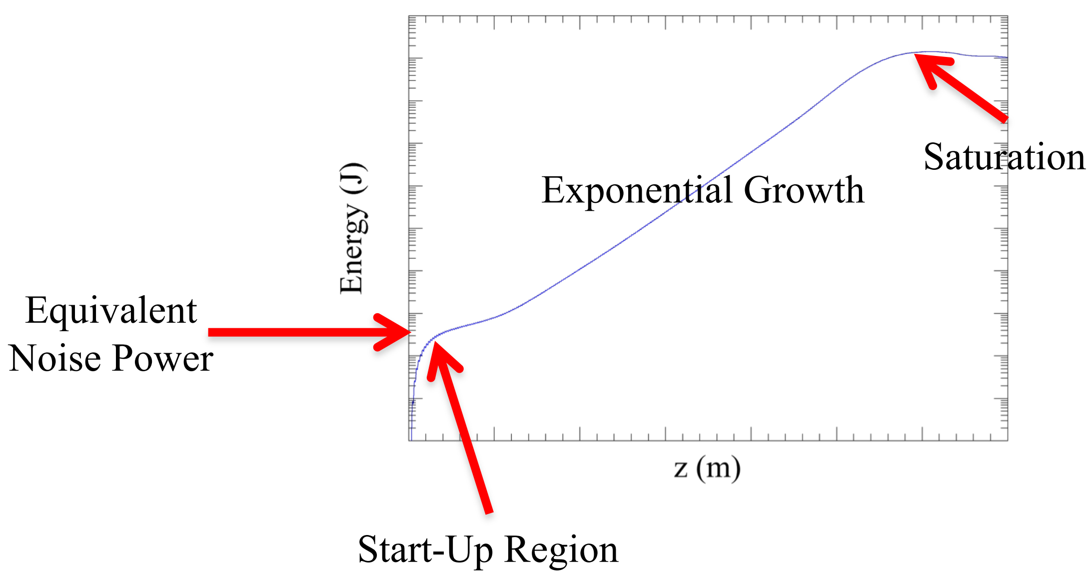
- When this happens, the electrons cannot be well-located within the ponderomotive bucket, and we cannot say for certain even whether all the electrons are on trapped or untrapped orbits
  - Becoming important for XFELs
  - This is even more restrictive for harmonic generation

XFEL	Energy	λ	$\Delta z_f/\lambda$
LCLS	14 GeV	1.5 Å	2%
SACLA	8.5 GeV	0.6 Å	10%
MaRIE	12 GeV	0.3 Å	12%



TYPICAL OPERATION

- There are typically three stages of the interaction
  - The start-up regime
    - Start-up can be from noise or from a seed laser
    - There is gain in this regime that scales as z^3 - the so-called low gain regime
  - The exponential growth regime
    - If the wiggler is sufficiently long, then there is a transition to exponential growth
    - This usually occurs after about 2 exponentiation lengths
  - Saturation
    - If start-up is from noise, this usually occurs after about 5 – 7 orders of magnitude growth



FEL CONFIGURATIONS

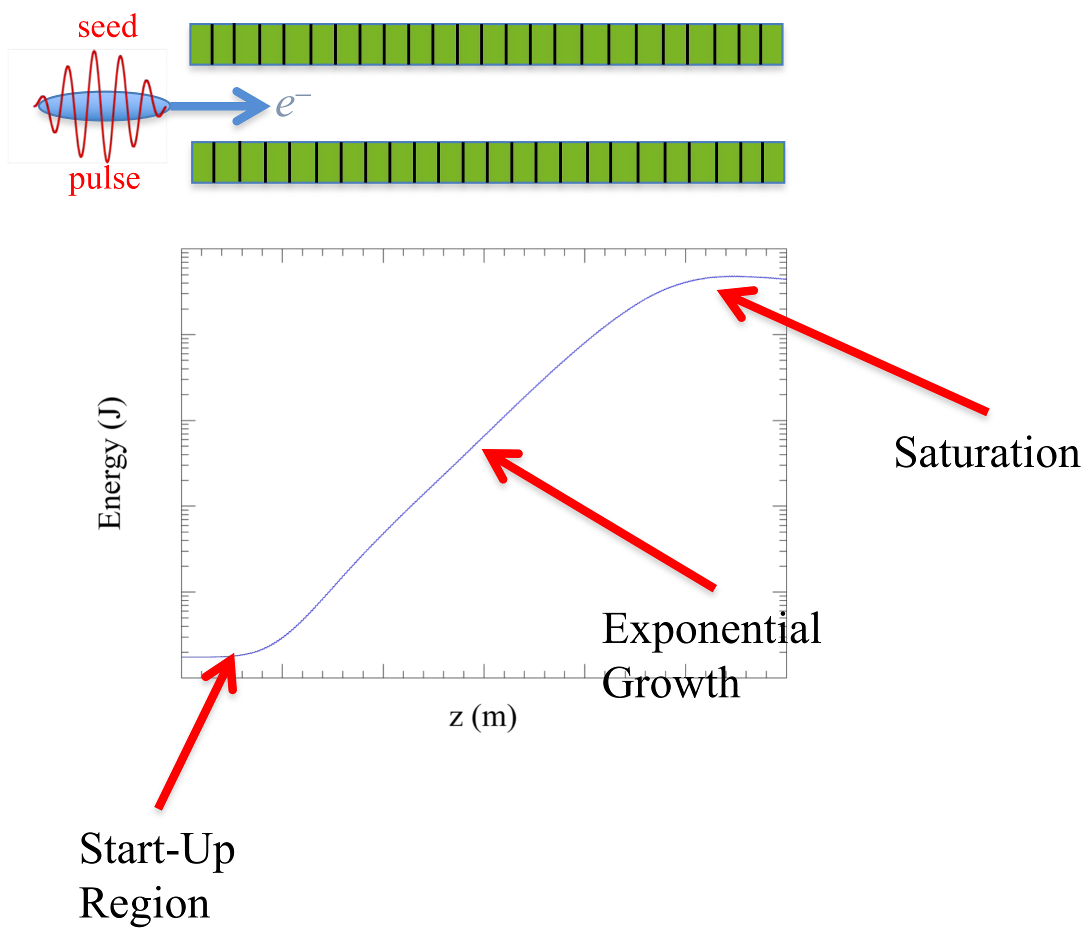
- Amplifier/MOPA (Master Oscillator Power Amplifier):
  - Seed power from an external source is amplified as it co-propagates through the wiggler with the electron beam
  - Advantage: Stable operation
  - Disadvantage: Needs high charge and long wiggler

- SASE (Self-Amplified Spontaneous Emission):
  - Shot noise on the electron beam is amplified over a single pass through a long wiggler
  - Advantage: No external seed is required (*i.e.*, x-ray FELs)
  - Disadvantage: Shot-to-shot fluctuations in wavelength, power, *etc*.

- Oscillator:
  - Short wiggler/feedback via resonator cavity
  - Shot noise is amplified cumulatively over many passes through the wiggler
  - Typical oscillator has low gain/high-Q
  - "Regenerative Amplifier" has high gain/low-Q
  - Advantages: High charge not required, short wiggler, no external seed
  - Disadvantages: Limit cycle oscillations, limitations imposed by mirror technology (*i.e.*, x-rays), mirror degradation at high power

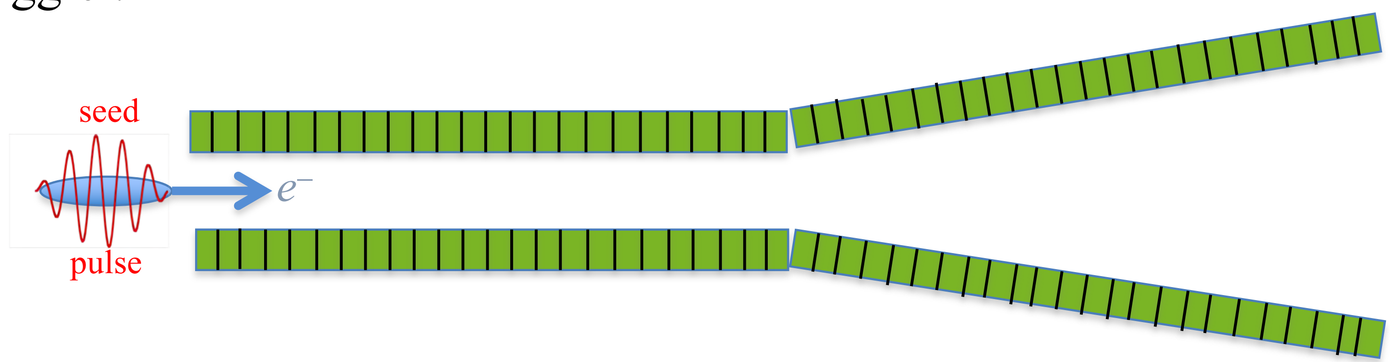
FEL AMPLIFIERS – UNIFORM WIGGLER

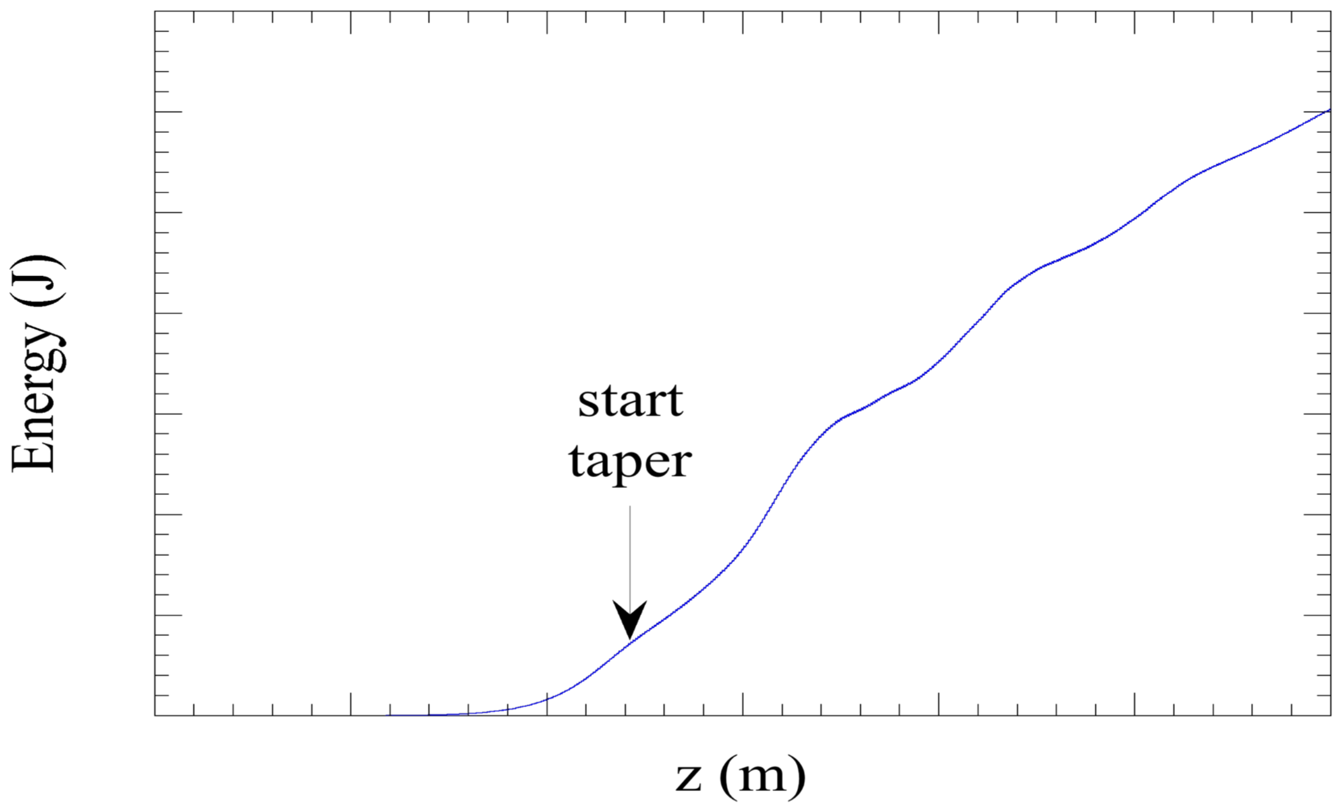
- An amplifier (sometimes called a MOPA or Master Oscillator Power Amplifier) consists of a long wiggler into which an electron bunch and a seed laser pulse is injected in synchronism



FEL AMPLIFIERS – TAPERED WIGGLER

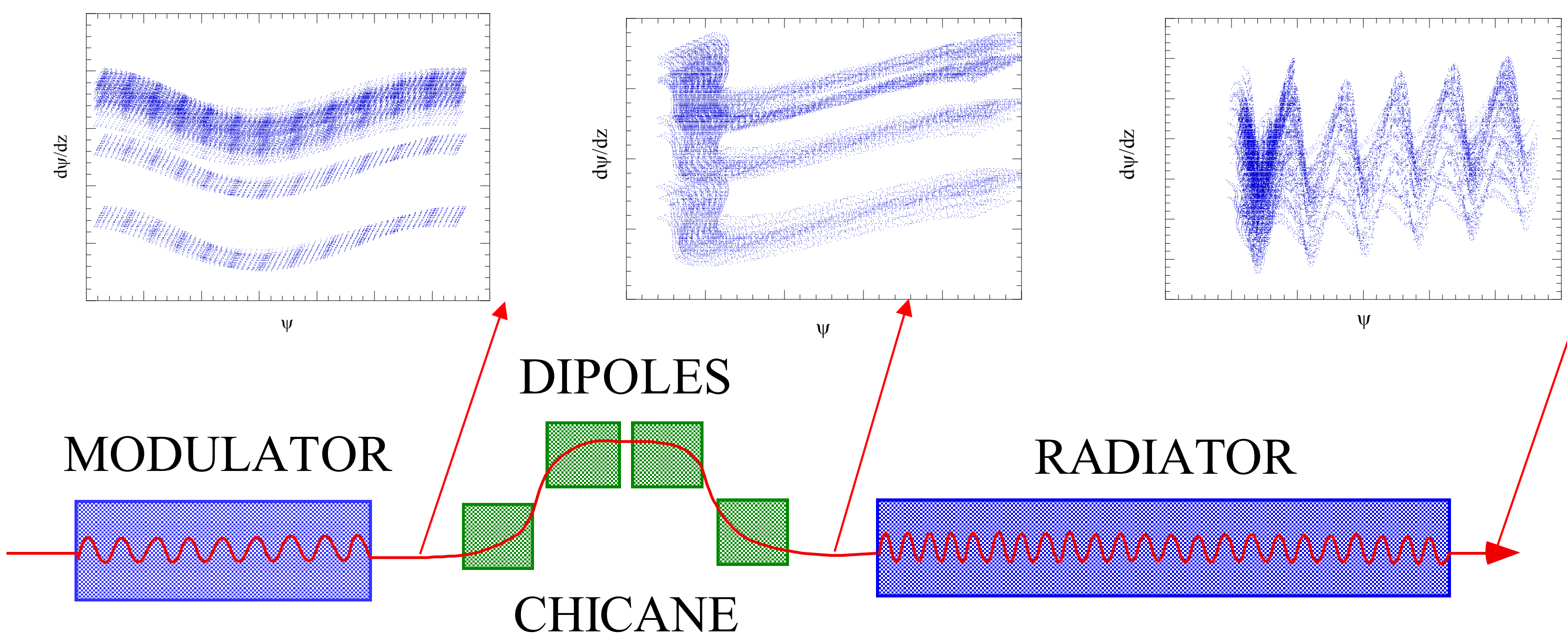
- At saturation the electron beam energy drops out of resonance and the interaction terminates. Higher efficiency and an extended interaction length can be achieved using a tapered wiggler.





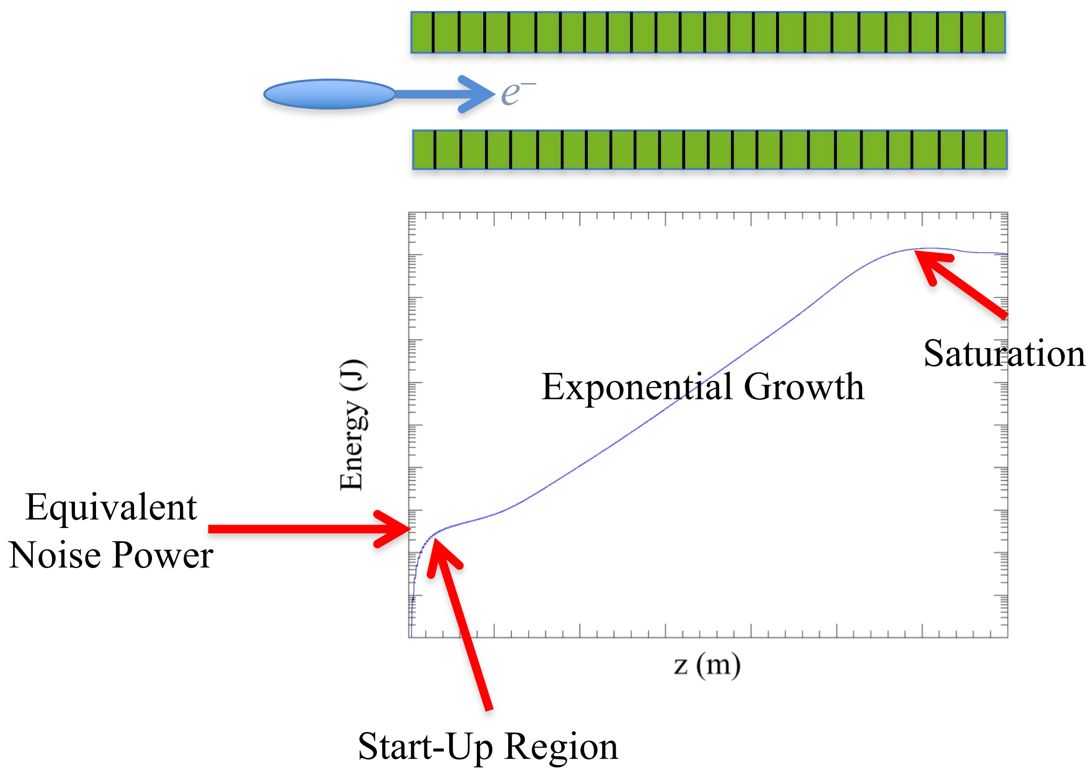
OPTICAL KLYSTRON AMPLIFIERS

- The Optical Klystron (OK) and High Gain Harmonic Generation (HGHG) concepts are similar.
    - Two wigglers are separated by a magnetic chicane.
    - The beam is injected into the Modulator together with a high power resonant seed pulse that induces a modulation on the beam.
    - The chicane enhances the bunching induced in the modulator.
    - The bunch radiates strongly in the Radiator - tuned to either the fundamental (OK) or a harmonic (HGHG) of the Modulator seed.



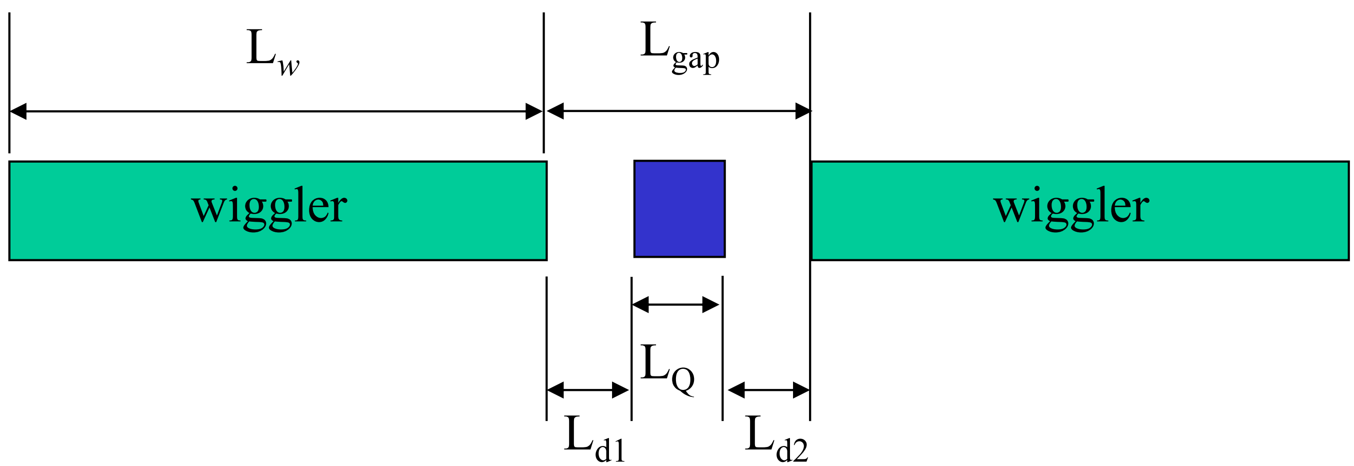
SASE FREE-ELECTRON LASERS

- Self-Amplified Spontaneous Emission (SASE) can be used when no seed laser is available
  - Radiation grows from shot noise in a single pass through the wiggler(s)
  - Because the growth is from noise, relatively large fluctuations are observed in the output light
    - Power, Wavelength, Linewidth
  - Used for XFELs
    - Long gain lengths require extremely long wigglers
      - Segmented wiggler with quadrupole focusing



SEGMENTED WIGGLER/FODO LATTICE

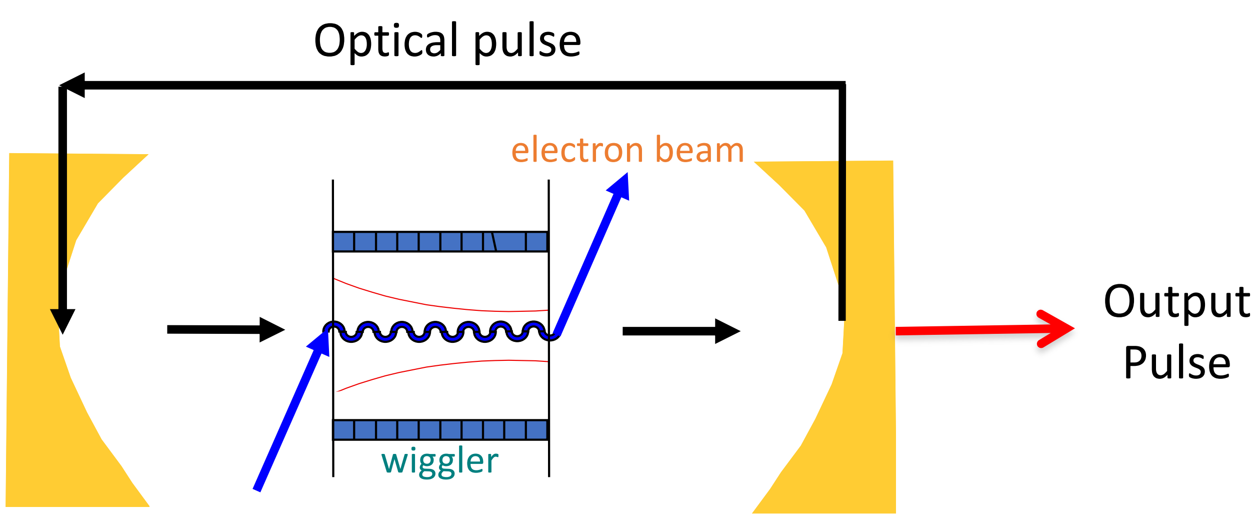
- X-ray FELs require extremely long wiggler lines (≈ 100m)
  - Impractical to build a single wiggler → segmented wigglers
  - Additional focusing required → FODO lattice
    - Shorter β-functions possible than for weak focusing wigglers
      - This results in shorter gain lengths



- Phase matching between the wiggler segments is important because the gain length is often comparable to the wiggler length
  - Destructive interference between wiggler segments is possible if the phase advance through the gap is not properly matched

FEL OSCILLATORS

- FEL oscillators consist of a (usually) short wiggler enclosed in an optical resonator
  - Radiation grows from noise in multiple passes through the resonator
  - Round trip path of the light through the resonator must be adjusted so that the returning optical pulse is in synchronism with the electron bunch frequency
    - $L_{cav} = c/2f_{rep}$
  - Single-pass gain must exceed resonator losses
    - This sets a minimum wiggler length
    - In the nonlinear regime, saturation occurs when the gain drops to balance the losses
      - $G = L/(1 - L)$
  - The electron beam is focused to a waist near the center of the wiggler to *match* the optical mode waist



OPTICAL GUIDING

- The electron beam acts like a optical fiber that can confine the bulk of the radiation to within the electron beam
  - Gain Guiding: Rays stop growing when they exit the e-beam
  - Refractive Guiding: Wavenumber shift → increases index of refraction in the e-beam → e-beam acts like an optical fiber

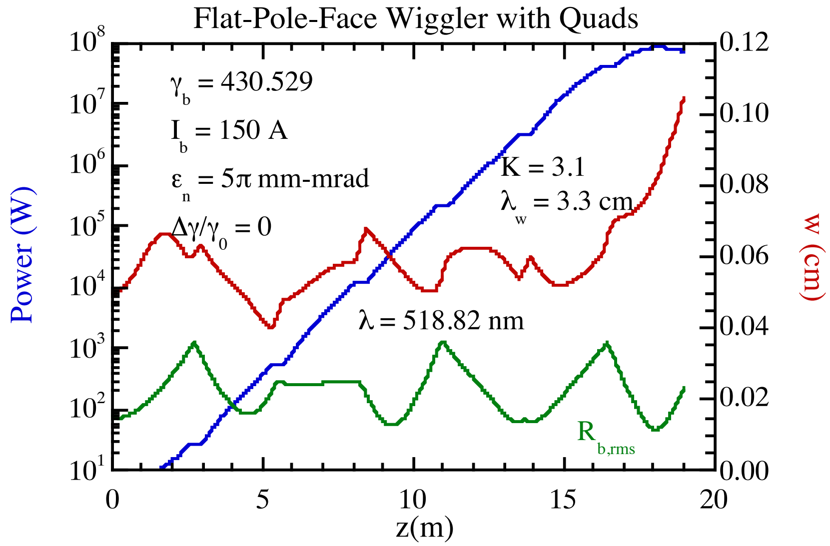
- **High-Gain FEL:** When $L_G \leq z_R$ the light is optically guided by two related mechanisms

Leads to an extended interaction length where the coupling is high.

LEUTL at Argonne Nat'l Lab →

$L_G \approx 0.5$ m

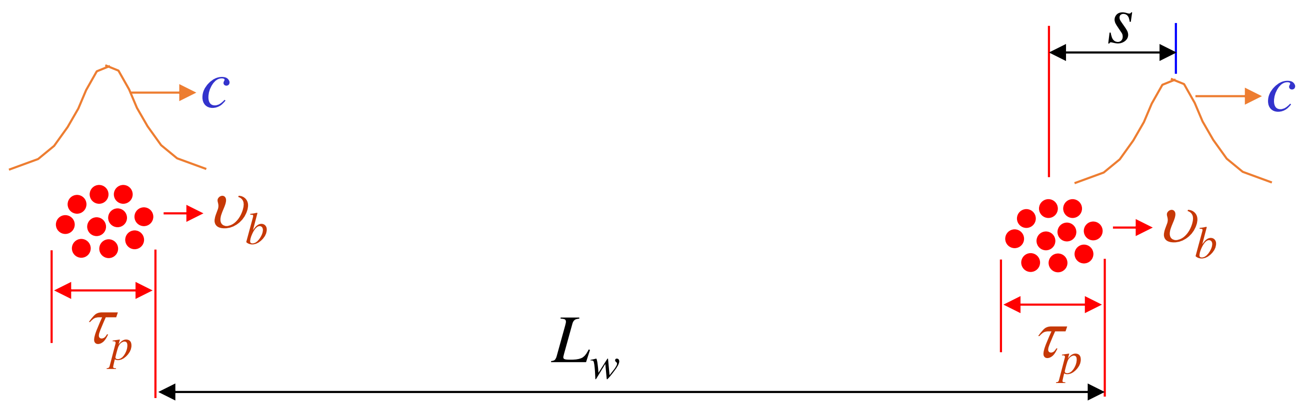
$z_R \approx 1.5$ m



- **Low-Gain FEL:** When the gain/pass in an oscillator reaches about 100%, then the mode is focused during propagation through the wiggler and the resonator mode also is affected

SHORT PULSE EFFECTS – SLIPPAGE

SLIPPAGE: Describes the tendency of the e-beam to lag behind the light pulse. Unimportant when $s < \upsilon_{\parallel} \tau_p$.



$$t_{light} = \frac{L_w}{\upsilon_{gr}} \approx \frac{L_w}{c} \qquad t_{electrons} = \frac{L_w}{\upsilon_{\parallel}}$$

- The distance the light (when on resonance) slips ahead of the electrons is given by

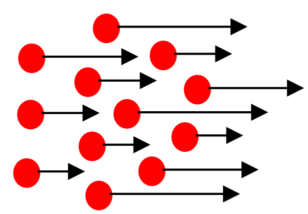
$$s = c\left(t_{electrons} - t_{light}\right) = L_w\left(1 - \frac{\upsilon_{\parallel}}{c}\right) \approx N_w \frac{\lambda_w}{2\gamma_{\parallel}^2} \approx N_w \lambda$$

3D EFFECTS - BEAM QUALITY

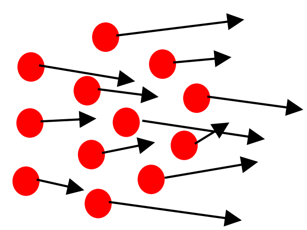
• The FEL interaction depends on the axial bunching of the beam, and is sensitive to the axial energy spread. We require that

$$\left(\frac{\delta\gamma}{\gamma}\right)_{rms} << \frac{1}{2}\eta$$

• **Energy Spread:** may be correlated or uncorrelated



• **Emittance:** phase space area ($R_b\Delta\theta$) → angular spread



• **Wiggler Gradients:** transverse shear in wiggler → velocity shear

$$\frac{\Delta\gamma_z}{\gamma_b} = \frac{\gamma_z}{\gamma_b}\frac{\Delta\gamma_{th}}{\gamma_b} + \frac{1}{2}\left(\frac{\varepsilon_n}{R_b}\right)^2\left(\frac{\gamma_z}{\gamma_b}\right)^3 + \left(\frac{k_w R_b}{2}\frac{K}{\gamma_b}\right)^2$$

THE LOW GAIN REGIME

- The low gain regime is principally relevant to short wigglers in low gain oscillators where there is not enough wiggler to reach the exponential gain regime. The peak gain and the linewidth scales as

$$G(dB) \approx 0.59\,\frac{\omega_b^2}{\gamma c^2 k_w^2}\,\frac{K^2}{\gamma^2}\,(k_w L_w)^3 \qquad \frac{\Delta\omega}{\omega_{res}} \approx \frac{1}{N_w}$$

- The efficiency scales as

$$\eta \approx \frac{1}{2N_w}$$

- Designs of oscillators balance gain and loss. Saturation occurs when the gain drops to $G = L/(1 - L)$, so the small-signal gain must be higher than this. This, in turn, yields a minimum wiggler length, which specifies the efficiency

THE HIGH-GAIN REGIME

- Vlasov-Maxwell analysis yields a quartic dispersion equation

Positive- and Negative-energy space-charge waves

Forward- and Backward-propagating EM waves

$$\left(\left[\omega-(k+k_w)\upsilon_z\right]^2-\frac{\omega_b^2}{\gamma_0\gamma_z^2}\right)\left(\omega^2-c^2k^2-\frac{\omega_b^2}{\gamma_0}\right)=\frac{\upsilon_w^2}{2\upsilon_z^2}\frac{\omega^2\omega_b^2}{\gamma_0\gamma_z^2}$$

Neglect backward wave

$$\approx -2\omega\left(ck-\sqrt{\omega^2-\frac{\omega_b^2}{\gamma_0}}\right)$$

- This gives us the well-known cubic dispersion equation

Negative-Energy SC wave

$$\left[\omega-(k+k_w)\upsilon_z-\frac{\omega_b}{\gamma_0^{1/2}\gamma_z}\right]\left[\omega-(k+k_w)\upsilon_z+\frac{\omega_b}{\gamma_0^{1/2}\gamma_z}\right]\left(ck-\sqrt{\omega^2-\frac{\omega_b^2}{\gamma_0}}\right)\approx-\frac{\upsilon_w^2}{4\upsilon_z^2}\frac{\omega\omega_b^2}{\gamma_0\gamma_z^2}$$

Positive-Energy SC wave

Forward EM wave

THE RAMAN & COMPTON REGIMES

- The cubic dispersion equation can be written as

$$\delta k\left(\delta k+\frac{2\omega_b}{\gamma_0^{1/2}\gamma_z\upsilon_z}\right)(\delta k-\Delta k)=-\frac{\upsilon_w^2}{2\upsilon_z^2}\frac{\omega_b^2}{\gamma_0 c^2}k_w$$

$$\delta k=k-\frac{\omega}{\upsilon_z}-\frac{\omega_b}{\gamma_0^{1/2}\gamma_z\upsilon_z} \qquad \Delta k=k_w\left(1-\frac{\omega}{\omega_{res}}\right)-\frac{\omega_b}{\gamma_0^{1/2}\gamma_z\upsilon_z}$$

- **High-Gain Compton Regime:** Ponderomotive wave dominates

$$\frac{\omega_b}{ck_w}<<\frac{\gamma_0^{3/2}}{16}\frac{\gamma_0^2\upsilon_w^2/c^2}{\left(1+\gamma_0^2\upsilon_w^2/c^2\right)^{3/2}}$$

$$\delta k^2(\delta k-\Delta k)=-\frac{\upsilon_w^2}{2\upsilon_z^2}\frac{\omega_b^2}{\gamma_0 c^2 k_w^2}k_w^3$$

→ Cubic equation with maximum growth when $\Delta k\approx k_w(1-\omega/\omega_{res})=0$

- **High-Gain Raman Regime:** Space-Charge wave dominates

$$\delta k(\delta k-\Delta k)=-\frac{\upsilon_w^2}{4\upsilon_z^2}\frac{\omega_b}{\gamma_0^{1/2}c}k_w$$

→ Quadratic equation with maximum growth when $\Delta k=0$ (neg-energy sc-wave)

THE PIERCE PARAMETER

- The cubic dispersion equation can be written in terms of the Pierce parameter as

$$\delta k^2(\delta k - \Delta k) = -(2\rho k_w)^3$$

$$\longrightarrow \Gamma = \mathrm{Im}(\delta k) = \sqrt{3}\rho k_w \longrightarrow L_G = \frac{1}{2\Gamma} = \frac{\lambda_w}{4\sqrt{3}\pi\rho}$$

- The Pierce parameter for a helical wiggler is, therefore, given by

$$(2\rho)^3 = \frac{\upsilon_w^2}{2\upsilon_z^2}\frac{\omega_b^2}{\gamma_0 c^2 k_w^2}$$

$$\frac{\omega_b^2}{\gamma_0 c^2 k_w^2} = \frac{4\pi e}{\gamma_0 m_e c^3}\frac{e n_b c}{k_w^2} = \frac{4\pi e}{\gamma_0 m_e c^3}\frac{I_b}{k_w^2 \pi R_b^2} = \frac{4}{\gamma_0 k_w^2 R_b^2}\frac{I_b}{I_A}$$

$$\longrightarrow (2\rho)^3 = \frac{2\upsilon_w^2}{\upsilon_z^2}\frac{1}{\gamma_0 k_w^2 R_b^2}\frac{I_b}{I_A}$$

$$\longrightarrow \boxed{\rho^3 = \frac{\upsilon_w^2}{4\upsilon_z^2}\frac{1}{\gamma_0 k_w^2 R_b^2}\frac{I_b}{I_A}}$$

THE COMPTON REGIME EFFICIENCY

- We know from the phase trapping argument that the efficiency is

$$\eta = 2\gamma_z^2 \frac{\upsilon_z}{c}\left(\frac{\upsilon_z - \upsilon_{ph}}{c}\right)$$

- The maximum growing solution for the wavenumber is $k = k_{res} + \delta k$, where

$$\frac{\delta k}{k_w} = \frac{1 + i\sqrt{3}}{2}\left(\frac{\upsilon_w^2}{2c^2}\frac{\omega_b^2}{\gamma_0 c^2 k_w^2}\right)^{1/3}$$

- This results in a shift in the phase velocity

$$\upsilon_{ph} = \frac{\omega}{k_{res} + k_w + \mathrm{Re}(\delta k)}$$

$$\longrightarrow \quad \frac{\upsilon_z - \upsilon_{ph}}{c} = \frac{(k_{res} + k_w)\upsilon_z - \omega + \mathrm{Re}(\delta k)\upsilon_z}{k_{res} + k_w + \mathrm{Re}(\delta k)} \approx \frac{\mathrm{Re}(\delta k)\upsilon_z}{k_{res} + k_w}$$

- So the efficiency becomes

$$\eta = \frac{1}{2}\left(\frac{\upsilon_w^2}{2c^2}\frac{\omega_b^2}{\gamma_0 c^2 k_w^2}\right)^{1/3} = \rho$$

SIMULATION/VALIDATION

- Numerical simulation of FELs has reached a mature level that provides confidence in their application to FEL design

- Simulation techniques can be classed in two general classes
  - Slowly-Varying Envelope Approximation (SVEA)
    - Fast – can be used for rapid design/optimization
    - First developed to simulate microwave tubes such as TWTs
    - Limited to pulse durations of about 5 – 10 wave periods
  - Particle-in-Cell (PIC)
    - Slow: too slow for general design activity and not in common use
    - Can treat ultra-short pulses

- I will discuss the validation of the SVEA simulation codes

SIMULATION – E&M FIELDS

- **3D E&M FIELDS**
  - Polychromatic SVEA approximation
  - Time-dependent and/or polychromatic physics
  - Gaussian Modal decomposition of the fields → extremely low memory requirement
    - Planar, Helical, or Elliptic polarizations
  - Adaptive eigenmode expansion (SDE) for field propagation minimizes the number of modes
  - Slippage can be applied at arbitrary intervals

SIMULATION - PARTICLES

- **PARTICLE DYNAMICS** are treated from first principles
  - Internal field models for Planar (FPF & PPF), Helical, and APPLE-II Undulators
    - The *JJ*-factor is implicitly included for planar & elliptic undulators
  - The undulator field can be imported from a 3D field map
  - Internal models for quadrupoles and dipoles (chicanes)
  - Harmonics & sidebands implicitly included
  - Start-up from noise (Fawley algorithm)/Prebunched beam

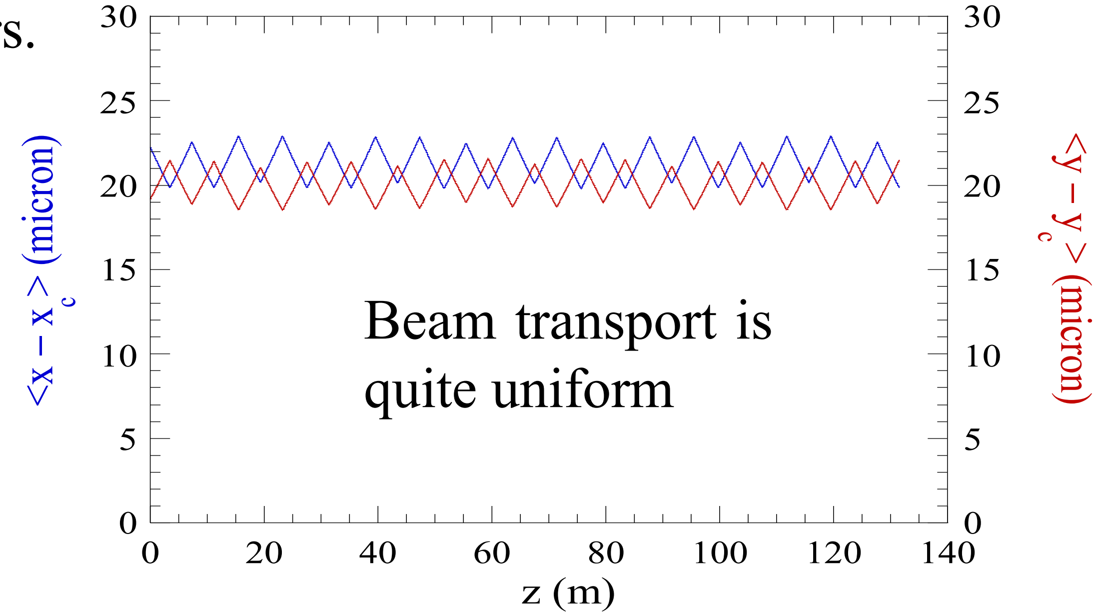
SIMULATION ALGORITHM

- Runge-Kutta Integration of particle **and** field equations simultaneously
  - $6N_{particles} + N_{harmonics}(2N_{modes} + 2)$ ODE (equations) per slice
  - Must use 20 – 30 steps/undulator period to resolve wiggle-motion
  - Can use either 2nd or 4th order Runge-Kutta

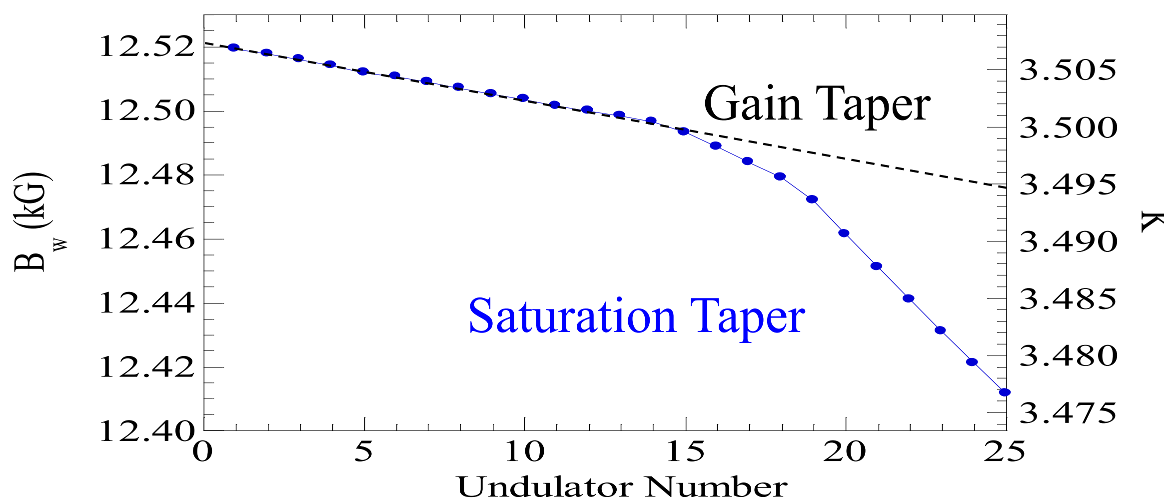
LCLS SIMULATION – GAIN TAPER

Electron Beam	
Energy	13.64 GeV
Bunch Charge	250 pC
Bunch Duration	83 fsec
Peak Current	3000 A (flat-top)
x-Emittance	0.4 mm-mrad
y-Emittance	0.4 mm-mrad
rms Energy Spread	0.01%
rms Size (*x*)	215 microns
α_x	1.1
β_x	30.85 m
rms Size (*y*)	195 microns
α_y	-0.82
β_y	25.38 m
Undulators	33 segments
Period	3.0 cm
Length	113 Periods
Amplitude (1st segment)	12.4947 kG
K_{rms} (1st segment)	2.4748
Taper Slope	-0.0016 kG
Gap Length	0.48 m
Quadrupoles	
Length	7.4 cm
Field Gradient	4.054 kG/cm

- The actual magnet strengths and positions are used in MINERVA including the undulators and FODO lattice for both the gain & saturation tapers.

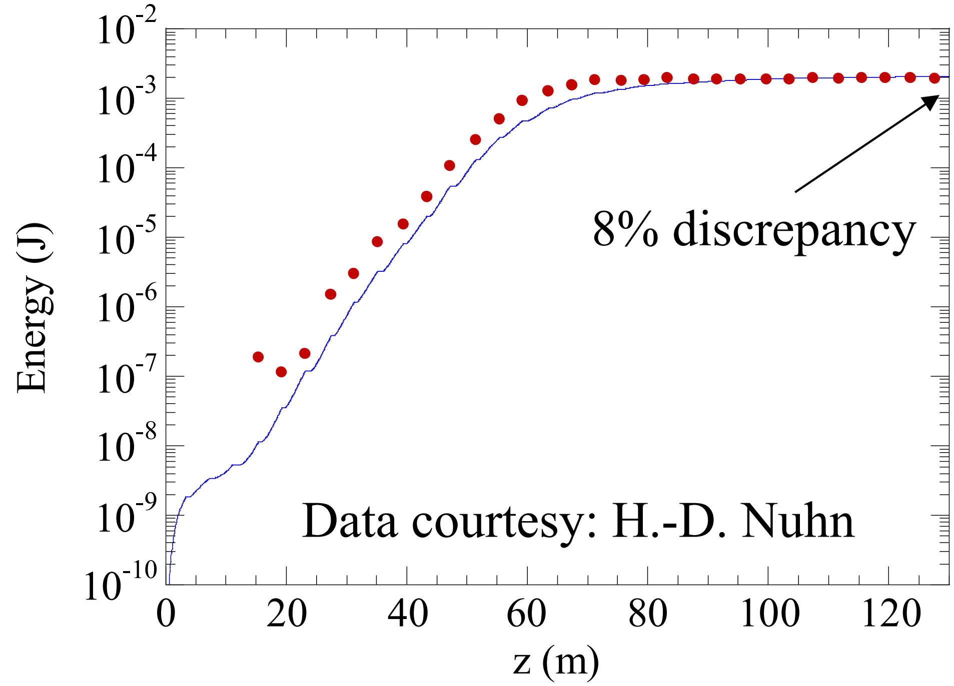


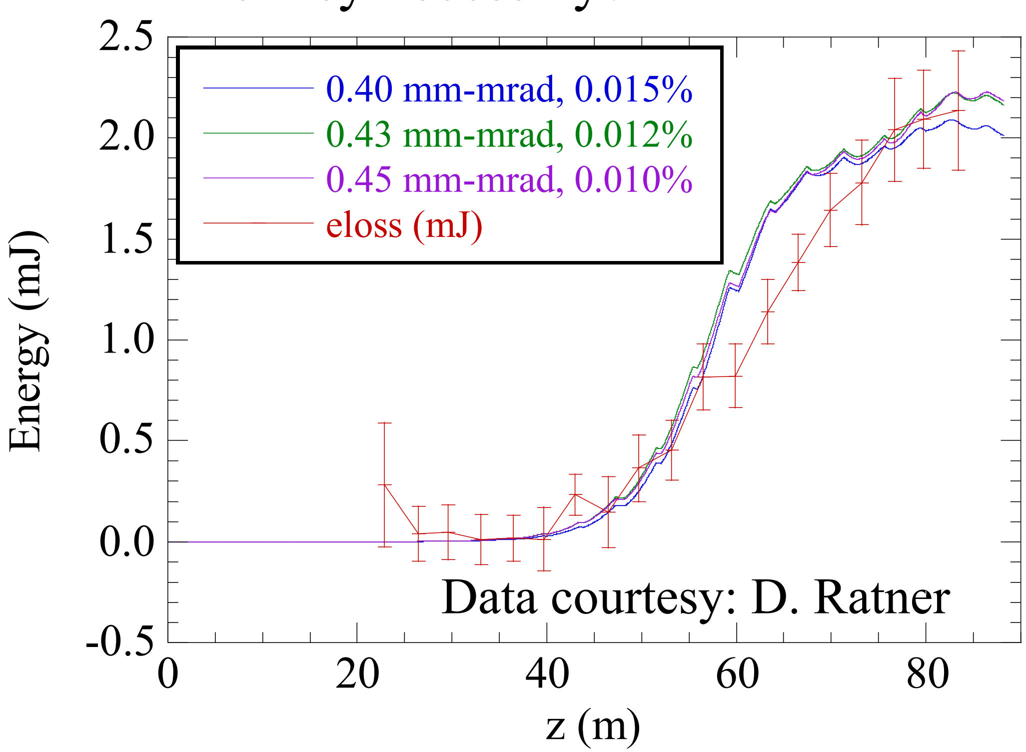
- The undulator field strengths are taken from the actual values for the gain & saturation tapers.



LCLS – GAIN & SATURATION TAPER

- MINERVA simulations (25 noise seeds) of the original LCLS with the gain taper have shown good agreement with the data.
  - The gain taper is still a taper and we see a small positive slope.
  - Question: Neither ICR nor wakefields have been included – are they necessary?



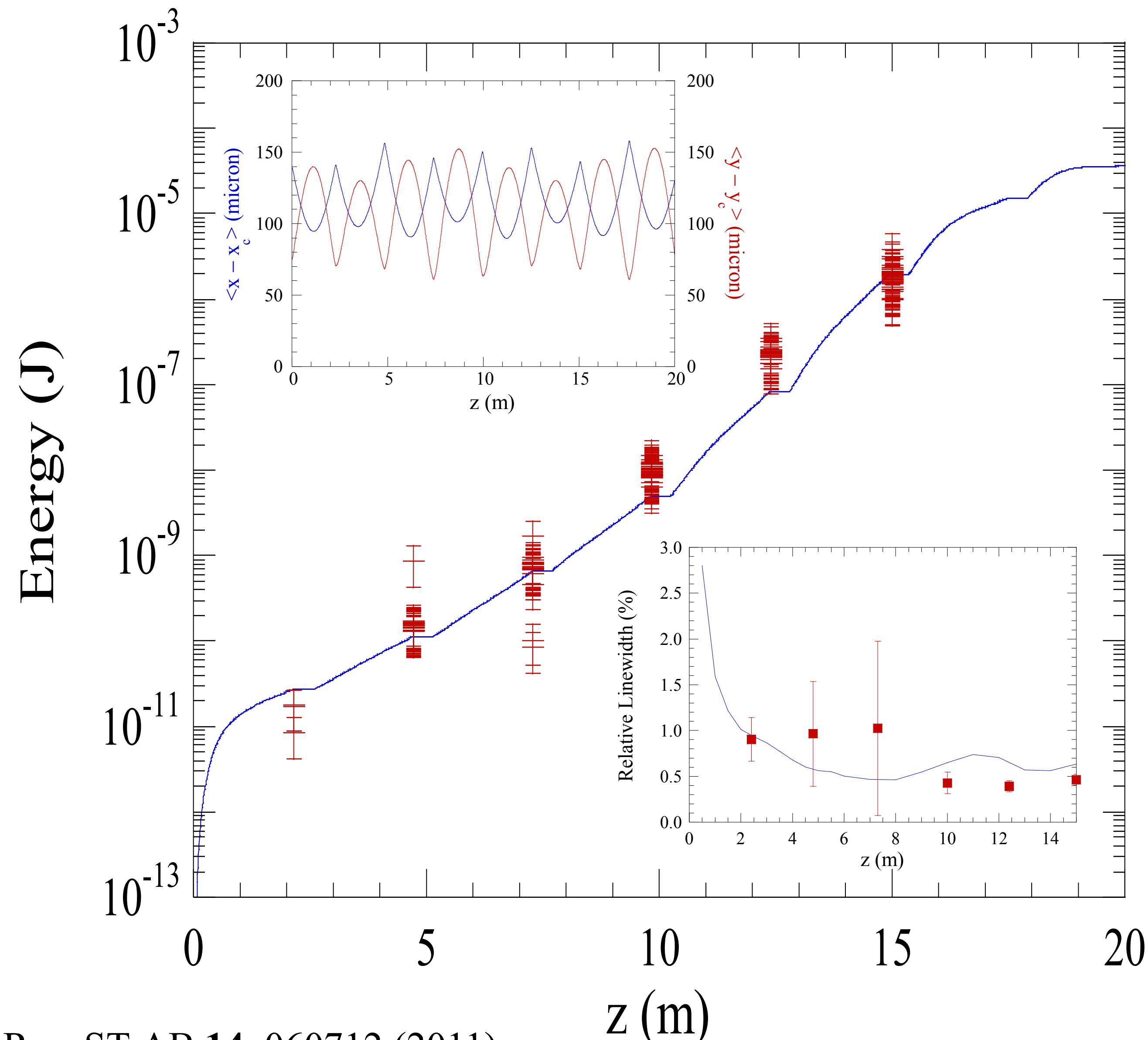


- MINERVA is in reasonable agreement with the saturation taper data for a range of beam parameters.

SPARC SIMULATION

MINERVA is in good agreement with the SPARC experimental data.

Electron Beam	
Energy	151.9 MeV
Bunch Charge	450 pC
Bunch Duration	12.67 psec
Peak Current	53 A (parabolic)
x-Emittance	2.5 mm-mrad
y-Emittance	2.9 mm-mrad
rms Energy Spread	0.02%
rms Size (*x*)	132 microns
α_x	0.938
rms Size (*y*)	75 microns
α_y	-0.705
Undulators	6 segments
Period	2.8 cm
Length	77 Periods
Amplitude	7.8796 kG
K_{rms}	1.457
Gap Length	0.40 m
Quadrupoles	Centered
Length	5.3 cm
Field Gradient	0.9 kG/cm



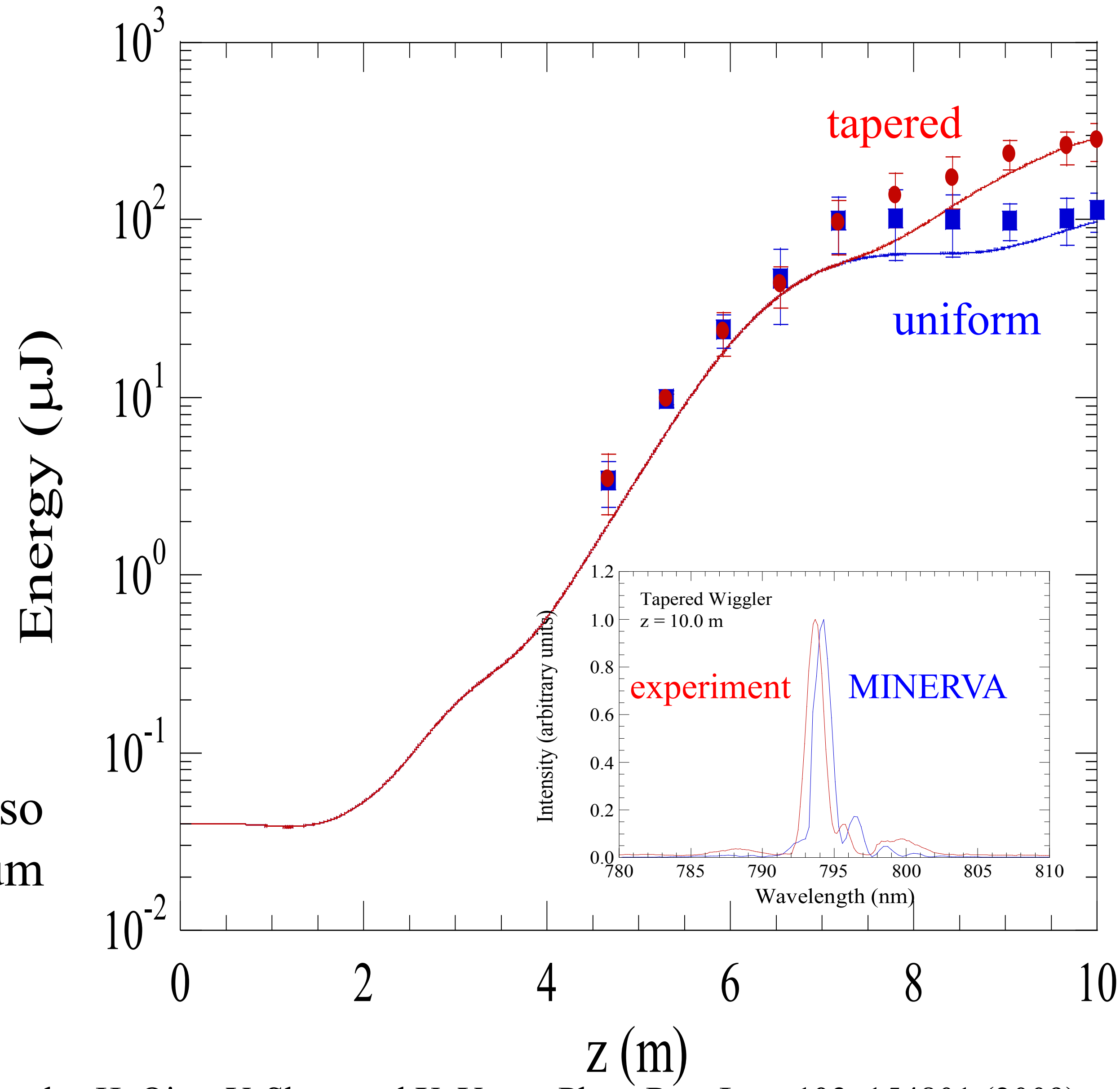
Reference: L. Giannessi *et al.*, Phys. Rev. ST-AB **14**, 060712 (2011)

BNL TAPERED AMPLIFIER

- MINERVA was validated for an IR, tapered amplifier exp't. at Brookhaven National Laboratory – good agreement found

Electron Beam	
Energy	100.86 MeV
Bunch Charge	360 pC
Bunch Duration	1.8 psec
Peak Current	300 A (parabolic)
Emittance	4 mm-mrad
rms Energy Spread	0.1%
NISUS Undulator	weak focusing
Period	3.89 cm
Amplitude (uniform)	3.03 kG
K_{rms}	0.848
Length	10 m
Start Taper Point	7.0 m
Optimal Taper	-4%
Optical Field	
Wavelength	793.5 nm
Seed Power	10 kW
Pulse Duration	6 psec

- Reasonable agreement also found for the output spectrum and 3rd harmonic power.



Ref.: X.J. Wang, H.P. Freund, W. Miner, J. Murphy, H. Qian, Y. Shen, and X. Yang,, Phys. Rev. Lett. **103**, 154801 (2009)

THE JLAB 10kW UPGRADE

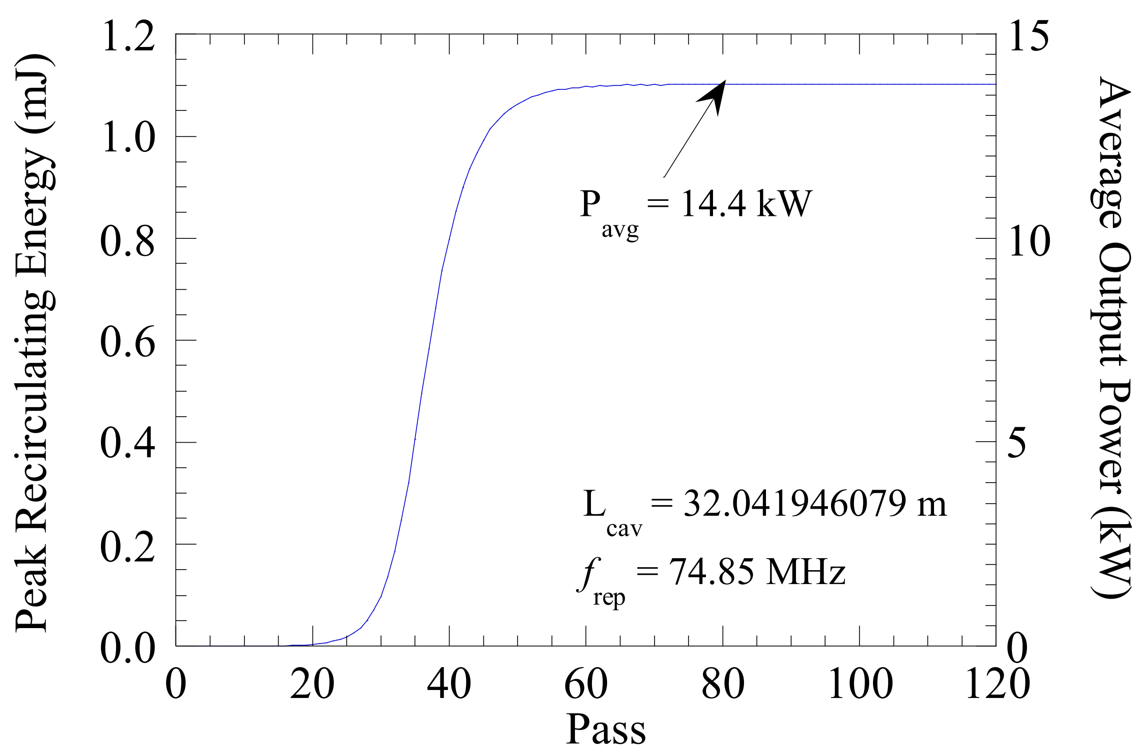
Electron Beam
Beam Energy: 115 MeV
Bunch Charge: 115 pC
Bunch Length: 390 fsec
Bunch Frequency: 74.85 MHz
Emittance: 9 mm-mrad (wiggle plane), 7 mm-mrad
Energy Spread: 0.3%

Wiggler
Period: 5.5 cm
Amplitude: 3.75 kG
Length: 30 periods

Radiation/Resonator
Wavelength: 1.6 microns
Resonator Length: 32 m
Rayleigh Range: 0.75 m
Out-Coupling: 21% (transmissive)

- The experiment recorded an average output power of 14.3 ± 0.72 kW.
- Earlier simulation with MEDUSA/OPC was in good agreement and found an average output power of 12.3 kW [reference: P.J.M. van der Slot *et al.*, Phys. Rev. Lett. **102**, 244802 (2009)].

- MINERVA/OPC is also in good agreement and finds an average output power of 14.4 kW.



SUMMARY

- The development of the theory and simulation of FELs has reached a point where we can be confident in the designs of future FELs

- The pace of design, construction, and operation of FELs is accelerating throughout the world over a spectrum ranging from THz through x-rays