Slip, Swim, Mix, Pack:
Fluid Mechanics at the Micron Scale

Eric Lauga

Division of Engineering and Applied Sciences
Harvard University

Thesis advisors
Michael P. Brenner
Howard A. Stone

Current address: Department of Mathematics, MIT

Copyright 2006 Eric Lauga

Outline

SLIP

The no-slip boundary conditions in hydrodynamics

SWIM

Locomotion of swimming microorganisms near surfaces

MIX

3D flows in microchannels

PACK

Capillary-driven assembly of microparticles

Copyright 2006 Eric Lauga

Outline

SLIP

The no-slip boundary conditions in hydrodynamics

SWIM

Locomotion of swimming microorganisms near surfaces

MIX

3D flows in microchannels

PACK

Capillary-driven assembly of microparticles

Copyright 2006 Eric Lauga

Flow boundary conditions

What is the appropriate boundary condition for Newtonian liquid flow past a solid surface?

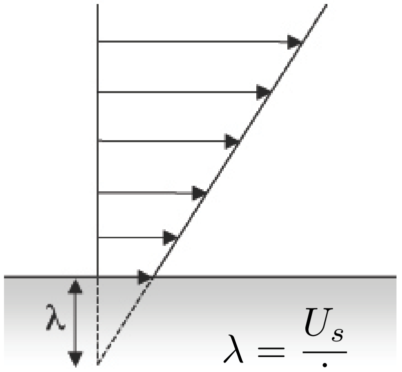
$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{u}\,, \quad \nabla \cdot \mathbf{u} = 0$$

$$\mathbf{u} \cdot \mathbf{n} = 0 \qquad \qquad \mathbf{u} \cdot \mathbf{t} = ?$$

D. Bernoulli, Euler, Coulomb, Girard,
Navier, Poisson, Poiseuille, Stokes, Hagen,
Darcy, Helmoltz, Maxwell, Couette...

Today, the no-slip boundary condition is in all textbooks.

Recent series of experiments: apparent breakdown of no-slip.



$$\lambda = \frac{U_s}{\dot{\gamma}}$$

(Navier) Slip length

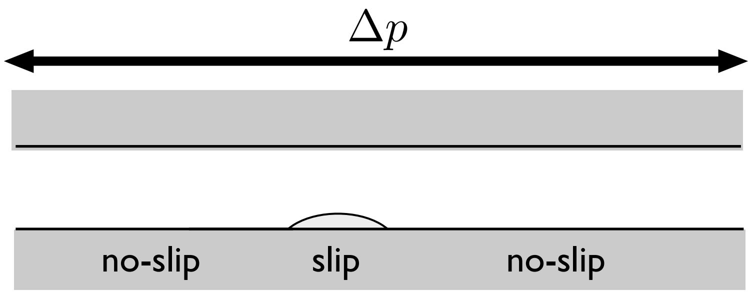
Copyright 2006 Eric Lauga

Apparent slip

Pressure drop vs. flow rate

Simple models for heterogeneous boundary conditions: distributed regions of (perfect) slip such as bubbles.
Two parameters: length scale, surface coverage.

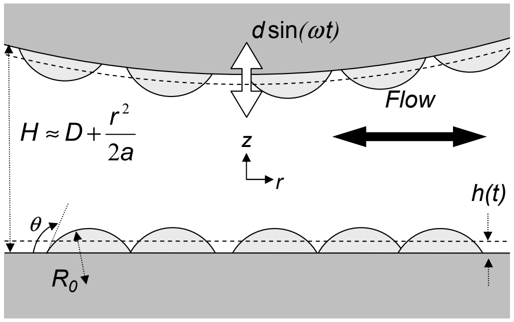
Lauga & Stone (2003) J. Fluid Mech. **489**, 55

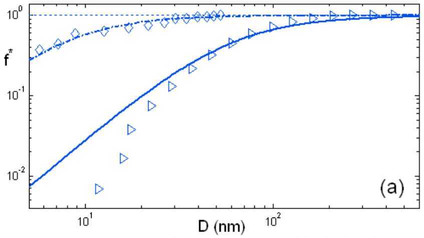


Drainage (squeeze-flow)

Surface-attached bubbles in lead to shear-dependent apparent slip due to bubble diffusion and compression.

Lauga & Brenner (2004) Phys. Rev. E. **93**, 026311



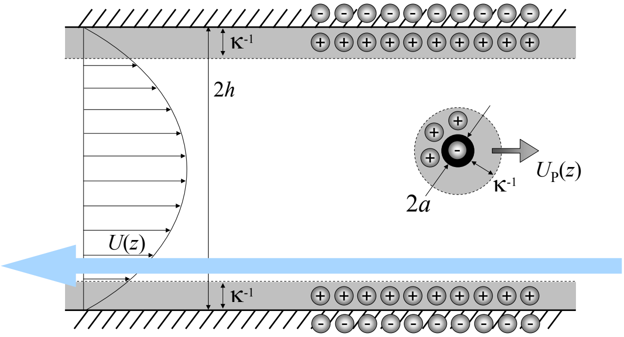


Zhu & Granick (2002)
Phys. Rev. Lett. **88**, 106102

PIV

Flow of an electrolyte: If the tracer particles are charged, their velocities can include a fake (electrokinetic) slip component.

Lauga (2004) Langmuir **20**, 8924



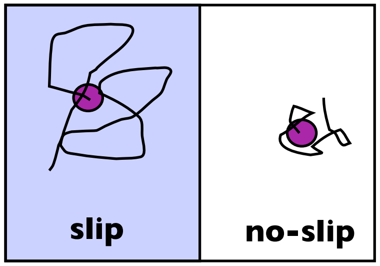
Copyright 2006 Eric Lauga

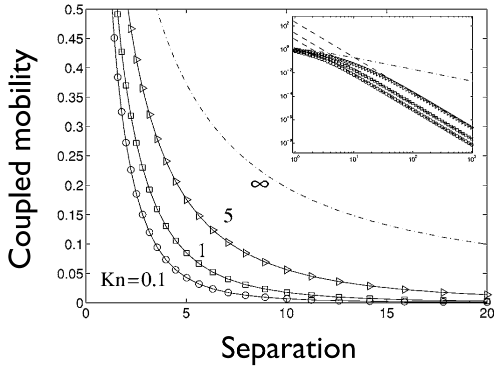
Other studies

A new method to measure slip

Diffusion of a colloidal probe near a slip surface can be used to infer slip lengths.

Lauga & Squires (2005) Phys. Fluids **17**, 103102

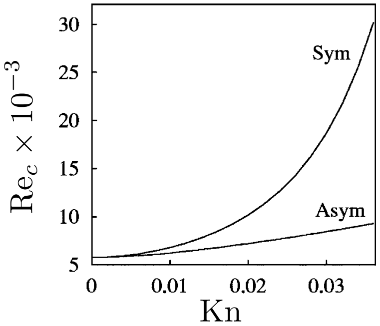




Influence of slip on flow stability

Slip has large stabilizing effect on normal modes but negligible effect on transient growth.

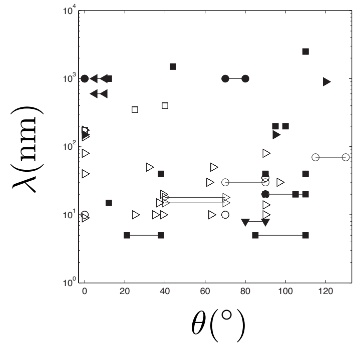
Lauga & Cossu (2005) Phys. Fluids **17**, 088106



Review

Discussion of theory, simulations, experiments.

Lauga, Brenner & Stone (2006) *Handbook of Experimental Fluid Dynamics* - In press



Copyright 2006 Eric Lauga

Outline

SLIP

The no-slip boundary conditions in hydrodynamics

SWIM

Locomotion of swimming microorganisms near surfaces

MIX

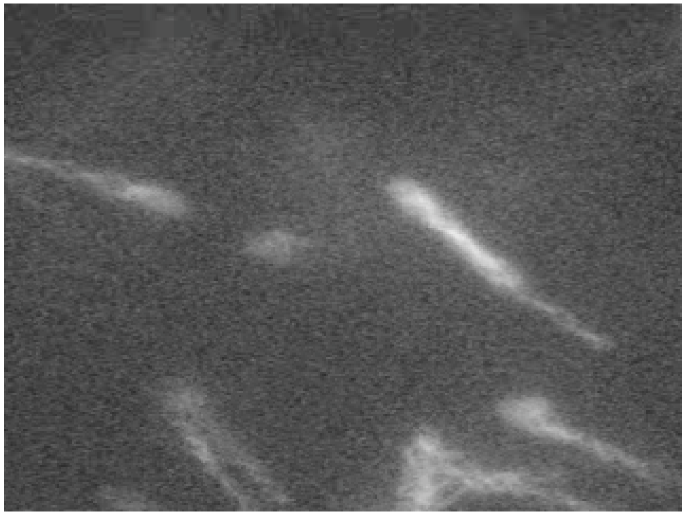
3D flows in microchannels

PACK

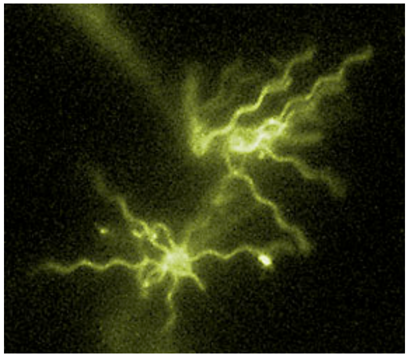
Capillary-driven assembly of microparticles

Copyright 2006 Eric Lauga

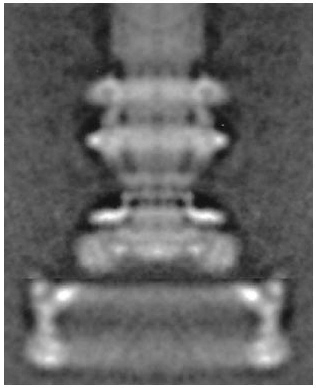
Swimming bacteria - *Escherichia coli*



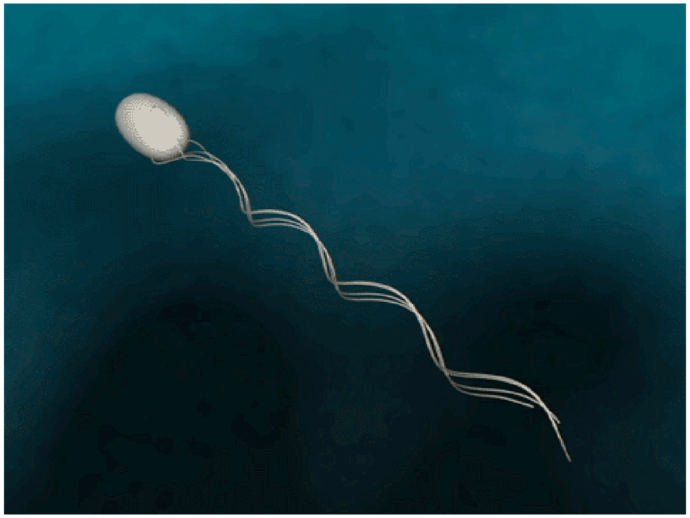
Berg Lab - Rowland Institute at Harvard



Berg Lab

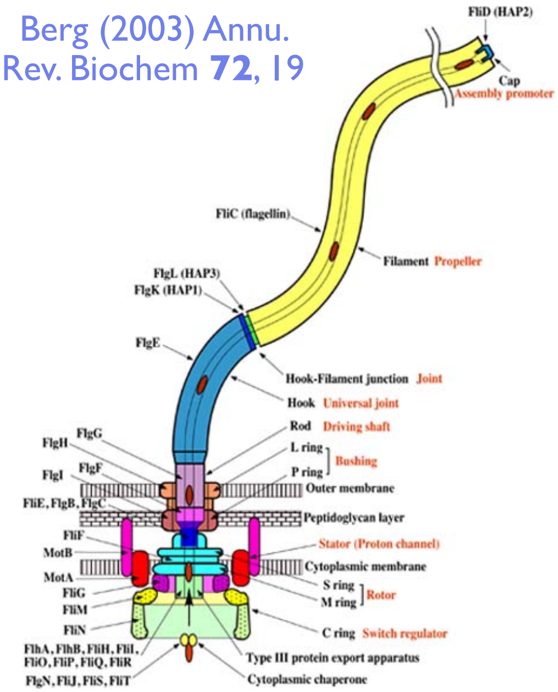


Berg (2003) Annu. Rev. Biochem **72**, 19



Protonic Nanomachine Project
http://www.npn.jst.go.jp/

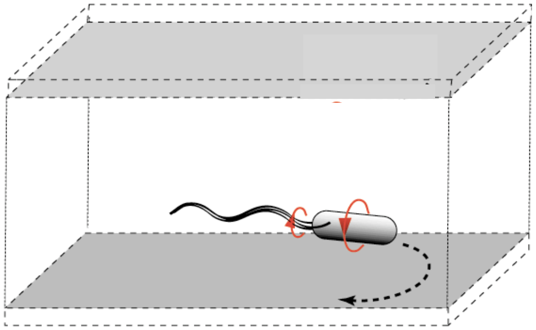
D. Kunkel Microscopy Inc.



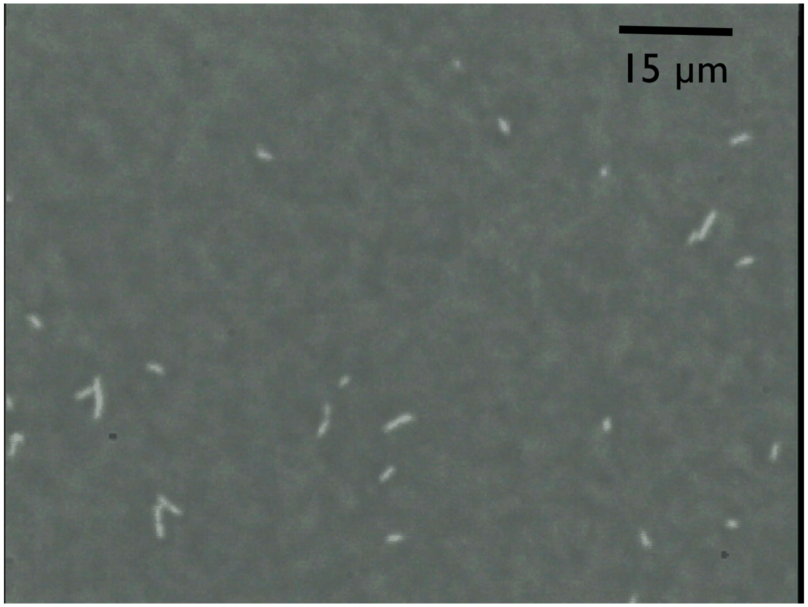
Protonic Nanomachine Project
http://www.npn.jst.go.jp/

Copyright 2006 Eric Lauga

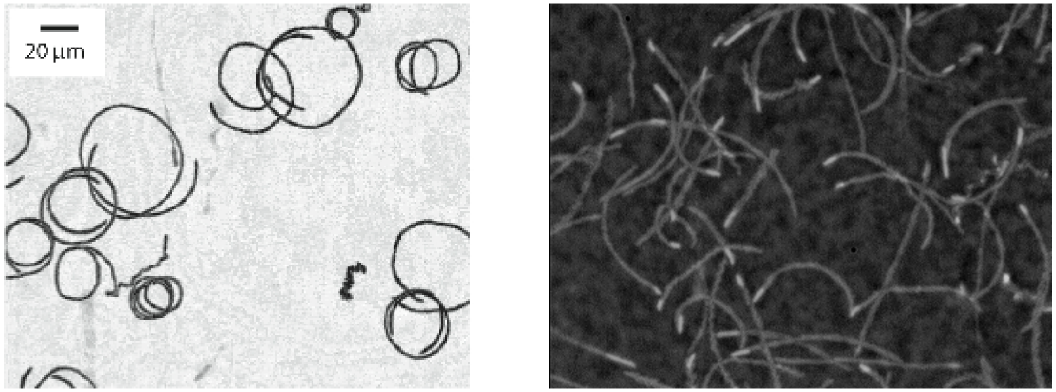
E. coli swimming near a surface



- Smooth-swimming *E. coli* in growth media
- Real time
- Velocity ~30 µm/s
- Microchannel height = 105 µm
- Solid surface: PDMS



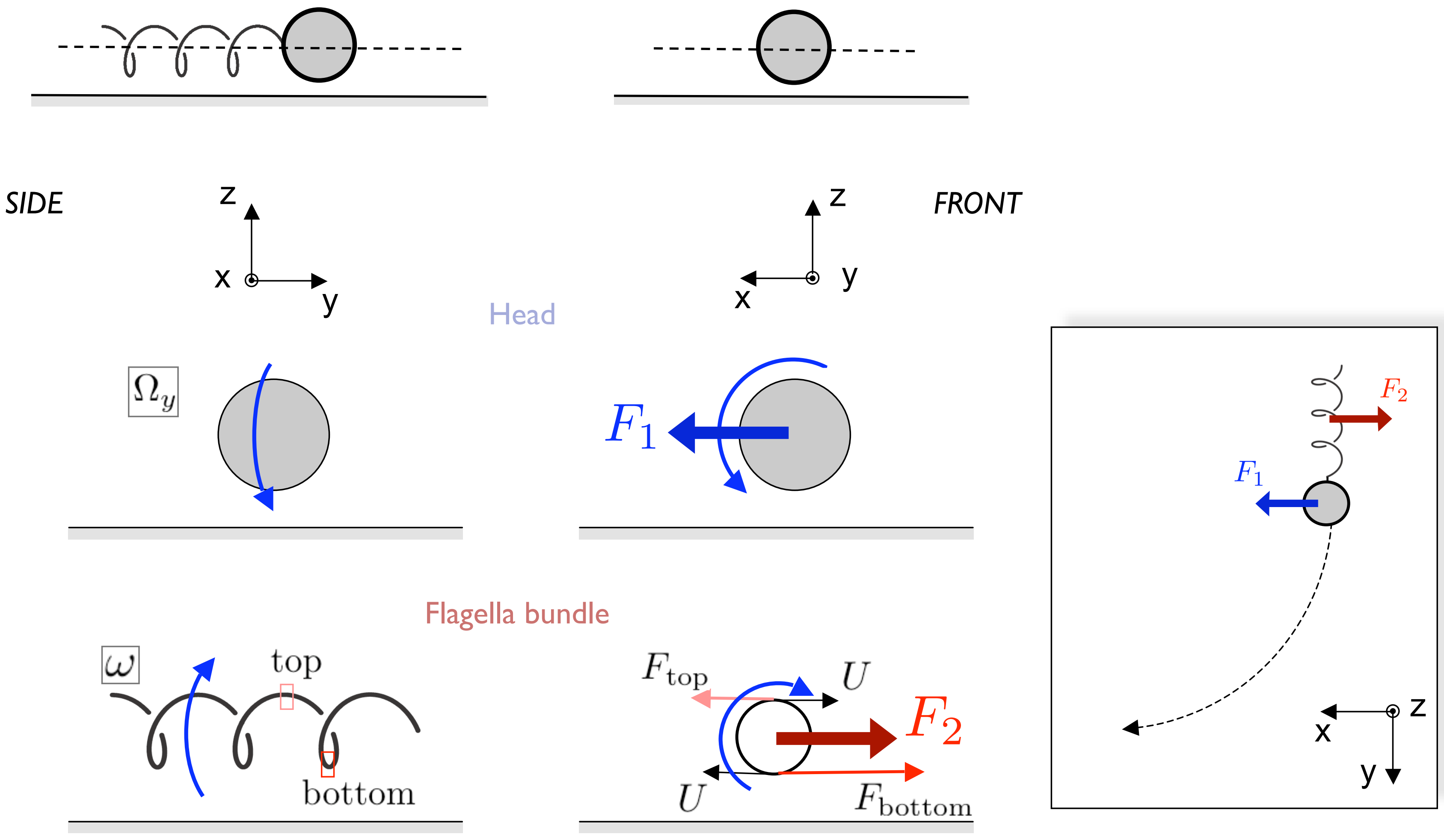
Experiment by Willow DiLuzio, DEAS, Harvard University



Superimposed images showing clockwise, circular paths

Copyright 2006 Eric Lauga

Why does the bacterium rotate?



Hydrodynamic interactions between the free-swimming bacterium and the surface lead to an out-of-plane torque, and since the cell has to be torque-free, it will rotate.

Copyright 2006 Eric Lauga

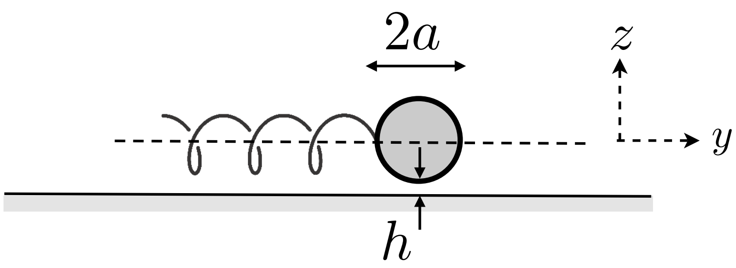
Computing the circle radius

Axial propulsive force balances translational drag

$$\mathcal{M}^{\mathrm{F}\Omega}_{\mathrm{yy}}(\omega-\Omega_{\mathrm{y}}) \approx (\mathcal{M}^{\mathrm{FU}}_{\mathrm{yy}}+\mathcal{N}^{\mathrm{FU}}_{\mathrm{yy}})U_{\mathrm{y}}$$

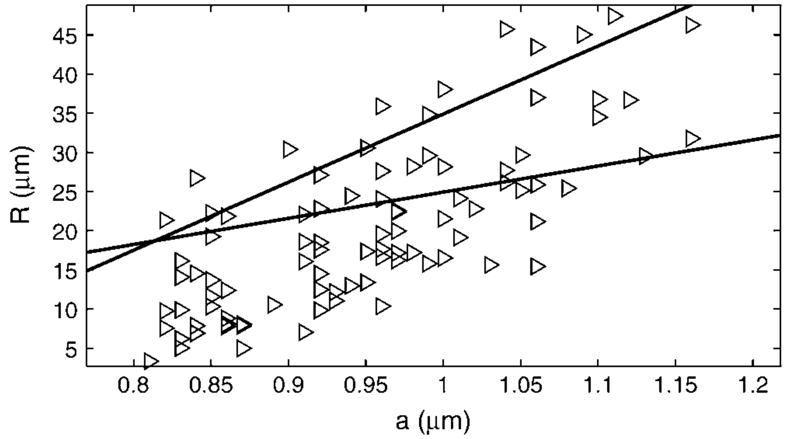
Wall-induced torque balances rotational drag

$$\mathcal{W}^{\mathrm{L}\Omega}_{\mathrm{zy}}(\omega-\Omega_{\mathrm{y}}) \approx -\mathcal{M}^{\mathrm{L}\Omega}_{\mathrm{zz}}\Omega_{\mathrm{z}}$$



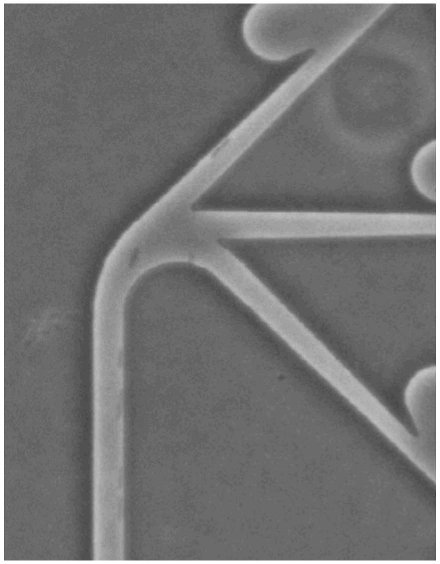
$$\boxed{\mathcal{R} \approx \frac{U_{\mathrm{y}}}{|\Omega_{\mathrm{z}}|} \approx \frac{\mathcal{M}^{\mathrm{L}\Omega}_{\mathrm{zz}}\mathcal{M}^{\mathrm{F}\Omega}_{\mathrm{yy}}}{\mathcal{W}^{\mathrm{L}\Omega}_{\mathrm{zy}}(\mathcal{M}^{\mathrm{FU}}_{\mathrm{yy}}+\mathcal{N}^{\mathrm{FU}}_{\mathrm{yy}})}}$$

Radius of the circle



Exploiting the results

$$\boxed{\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}a} > 0}$$



Lauga, DiLuzio, Whitesides & Stone (2006) *Biophys. J.* **90**, 400

Copyright 2006 Eric Lauga

Outline

SLIP

The no-slip boundary conditions in hydrodynamics

SWIM

Locomotion of swimming microorganisms near surfaces

MIX

3D flows in microchannels

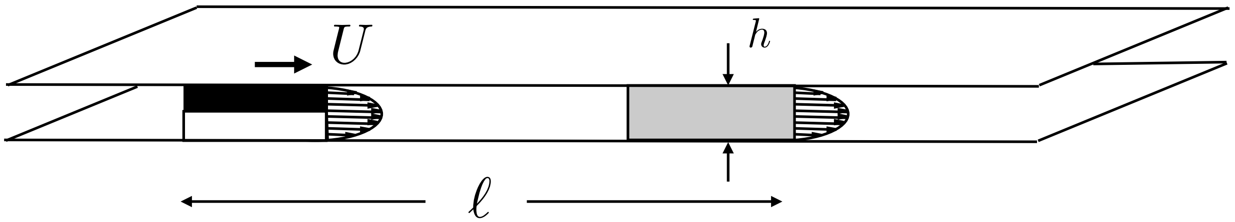
PACK

Capillary-driven assembly of microparticles

Copyright 2006 Eric Lauga

Mixing on (not too) small scales

Typically, low Reynolds number but high Peclet number.
How long downstream will molecular diffusion mix?

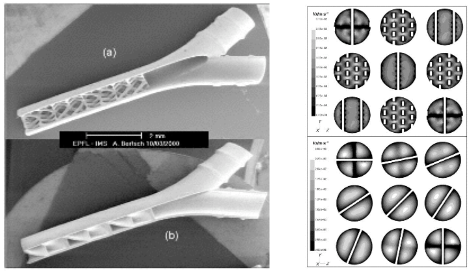


Diffusion time across the channel $\tau \sim h^2/D$

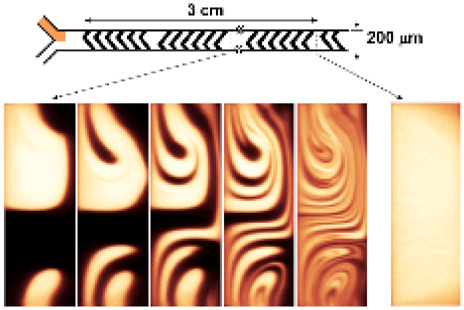
Distance traveled $\ell \sim U\tau$

$$\boxed{\frac{\ell}{h} \sim \frac{Uh}{D} = \mathrm{Pe} \gg 1}$$

Solution: generate transverse flows to replace diffusive transport by convective transport



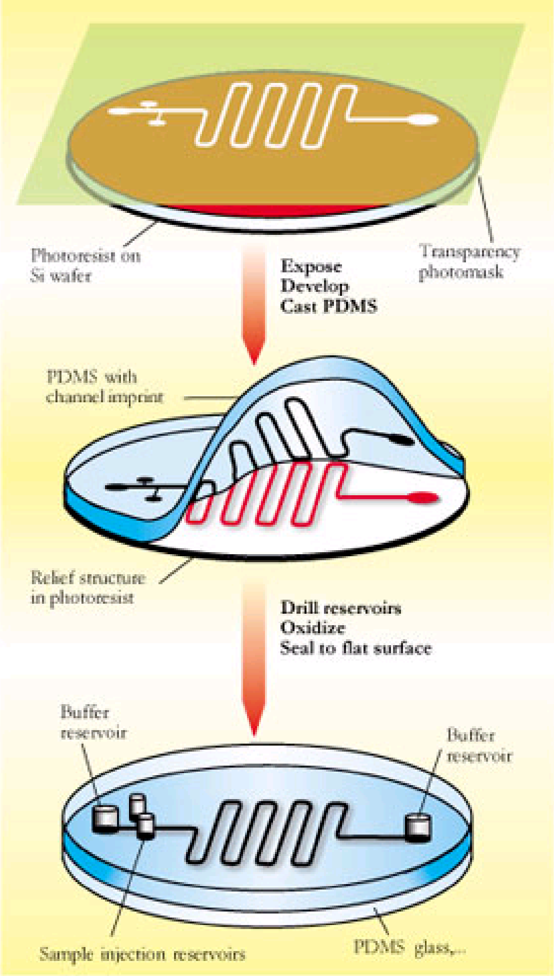
Bertsch et al. (2001) Lab Chip **1**, 56



Stroock et al. (2000) Science **295**, 5555

Copyright 2006 Eric Lauga

Fabrication constraints



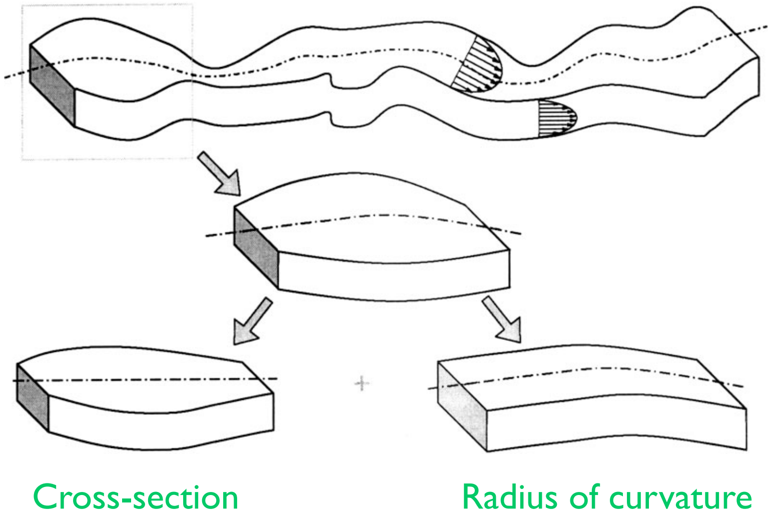
Whitesides & Stroock (2001) Physics Today **54**, 42

Soft Lithography
Simplest: one step of microfabrication

↓

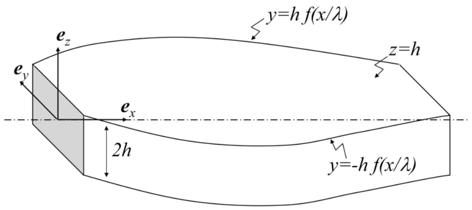
The channels will have a fixed height
Will the resulting channel be a good mixer?

Design: two degrees of freedom



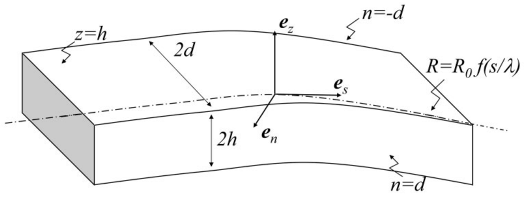
Copyright 2006 Eric Lauga

The flow is always 3D



Straight channel of
varying width

Flow is 3D unless constant width



Curved channel of constant width
and varying curvature

Flow is 3D unless constant curvature

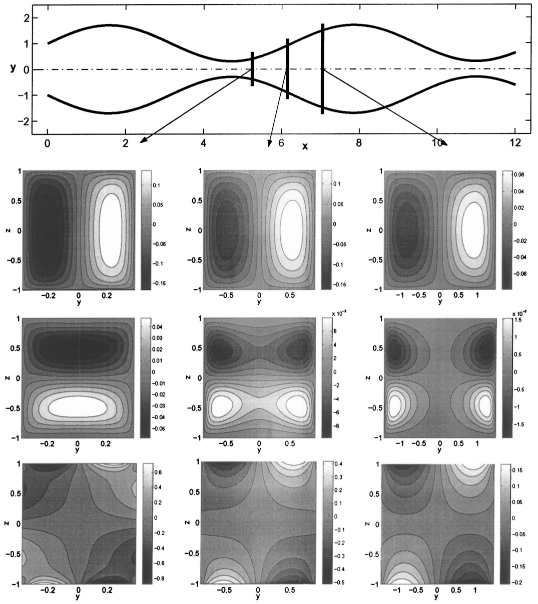


Illustration on periodic channel

Lauga, Stroock & Stone (2004) Phys. Fluids **16**, 3051

Copyright 2006 Eric Lauga

Outline

SLIP

The no-slip boundary conditions in hydrodynamics

SWIM

Locomotion of swimming microorganisms near surfaces

MIX

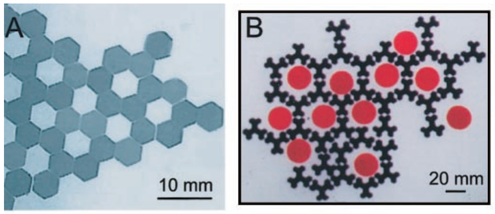
3D flows in microchannels

PACK

Capillary-driven assembly of microparticles

Copyright 2006 Eric Lauga

Self-assembly on small scales



Whitesides et al. (2002) PNAS **99**, 4769
Kralchevsky and Denkov (2001) Curr. Opin. Colloid Int. Sci. **6**, 383

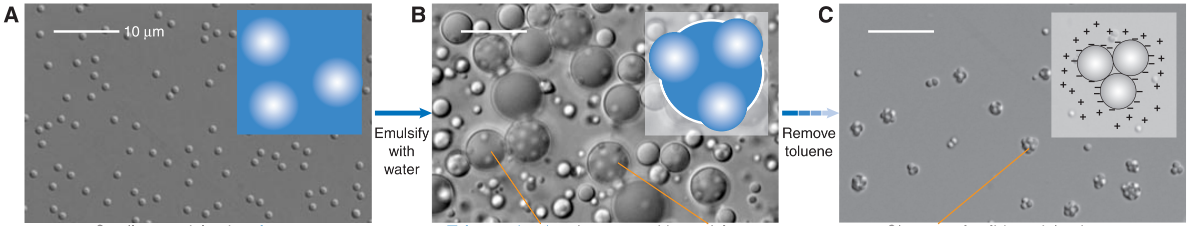
Reducing length scales:
Design an energy landscape

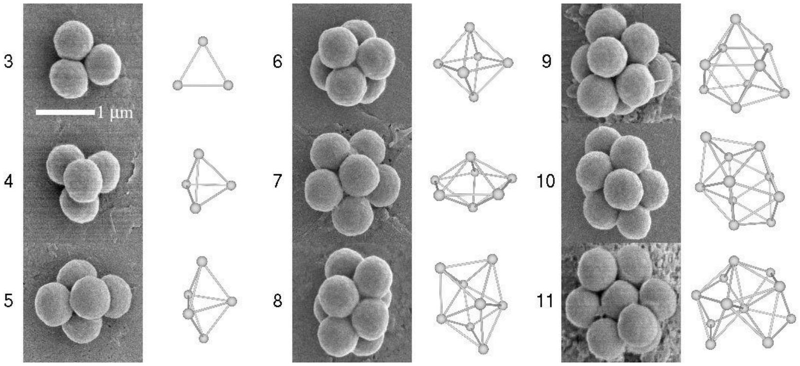
Usually many local minima

Ex: Packing of N spheres using vdW forces

Number of Spheres	6	7	8	9	10	11	12	13
Number of Local minima	2	4	8	18	57	145	366	988

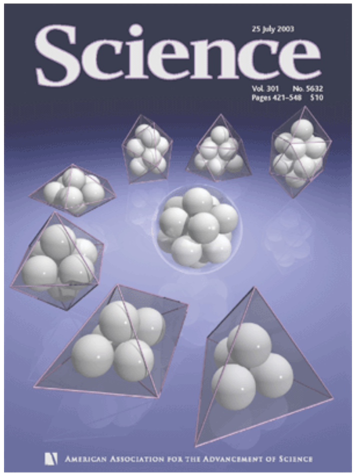
Hoare & McInnes (1976) Faraday Discuss. Chem. Soc. **61**, 12





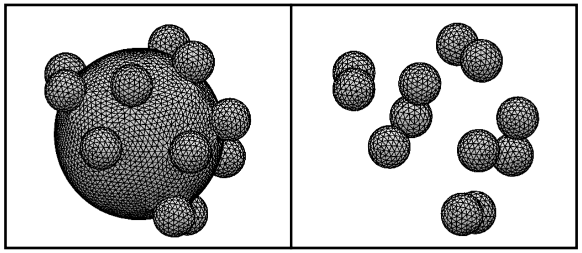
Final packings are unique
(no local minima)

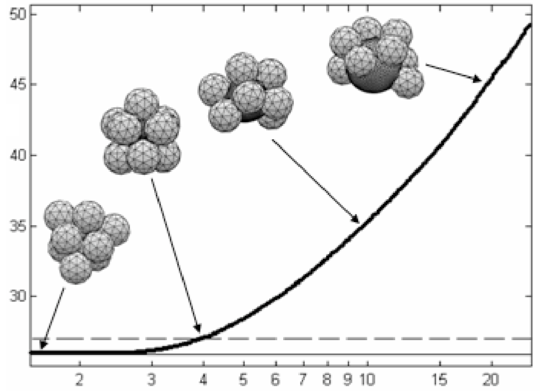
Manoharan et al. (2004)
Science **301**, 483

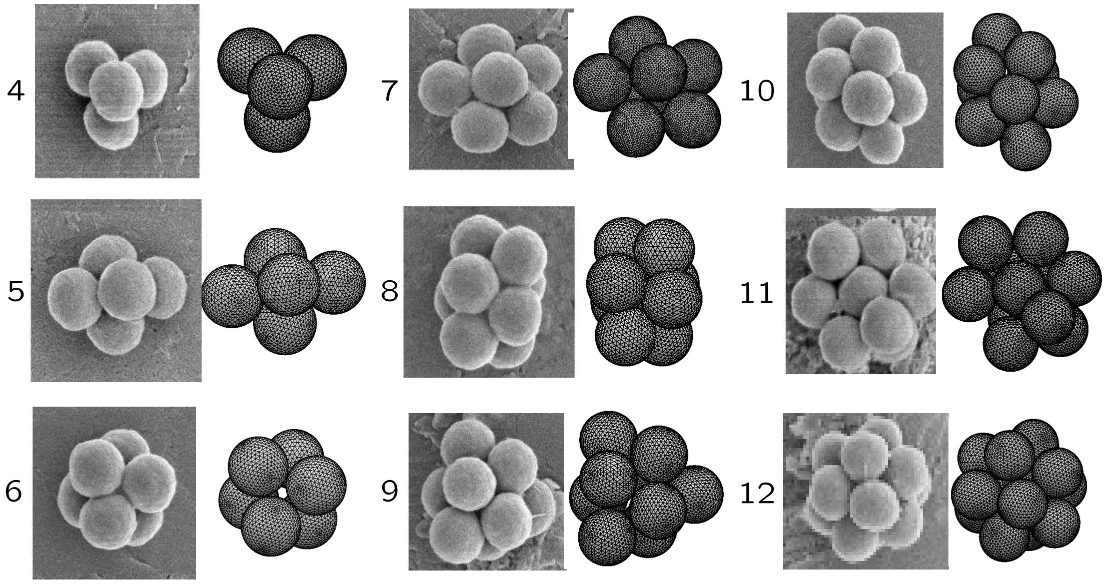


Copyright 2006 Eric Lauga

Simulations





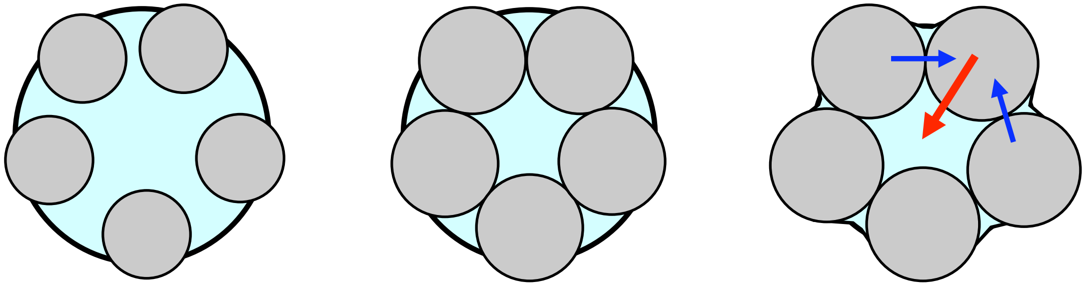


Numerical simulations of hard spheres on droplet reproduce the packings obtained by Manoharan et al.
Lauga & Brenner (2004) Phys. Rev. Lett. **93**, 238301

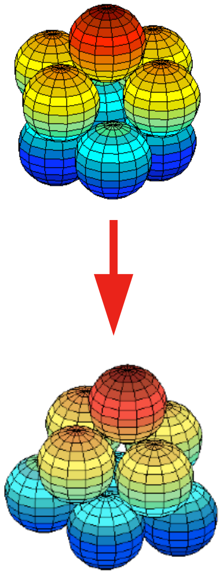
Experimental pictures from Manoharan et al. (2004) Science **301**, 483

Copyright 2006 Eric Lauga

Theory

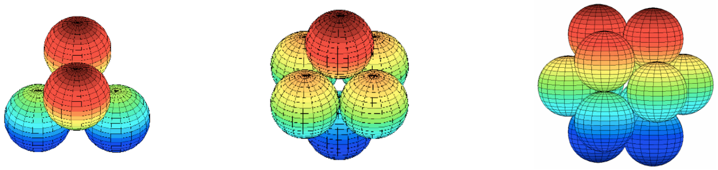


Rearrangement of particles is uniquely determined by geometry.

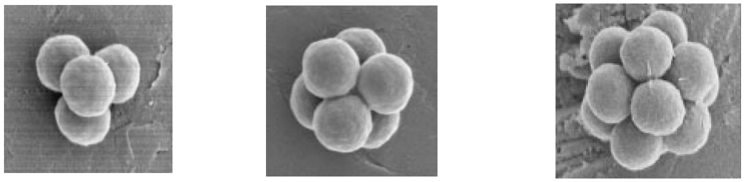


Simple model
$N = 9$

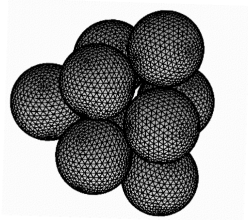
Tetrahedron Octahedron Icosahedron



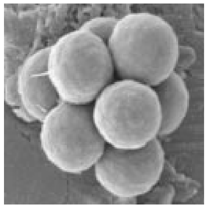
$N = 4$ $N = 6$ $N = 12$



Some final packings correspond to surface jamming



Full simulations

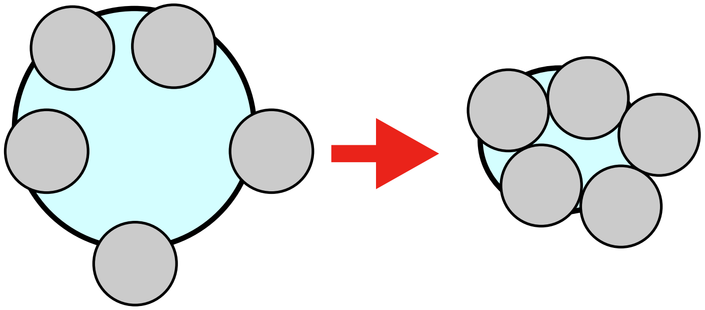


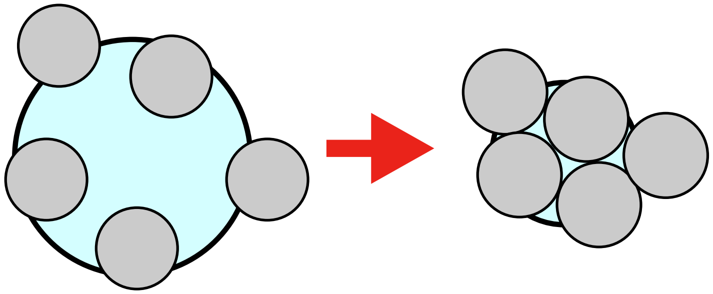
Experiments

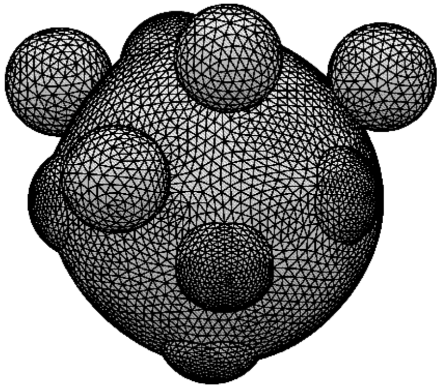
Experimental pictures from Manoharan et al. (2004) Science **301**, 483

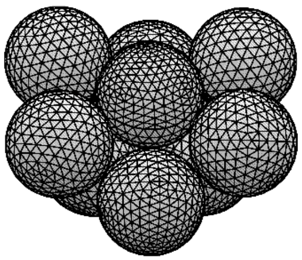
Copyright 2006 Eric Lauga

How to create different packings



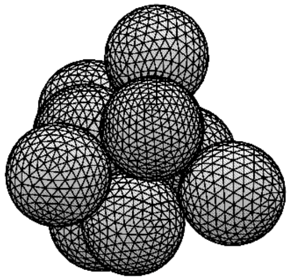






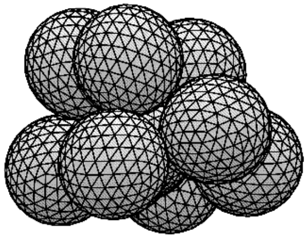
(a)

4 spheres with 160°
5 spheres with 20°



(b)

6 spheres with 160°
3 spheres with 20°



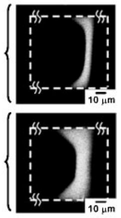
(c)

8 spheres with 160°
1 spheres with 20°

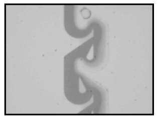
Lauga & Brenner (2004) Phys. Rev. Lett. **93**, 238301
Schnall-Levin, Lauga & Brenner (2006) Langmuir **22**, 4547

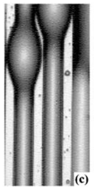
Copyright 2006 Eric Lauga

Fluid Mechanics at the micron scale...

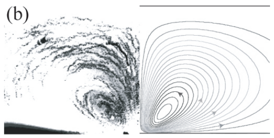


Ismagilov *et al.* (2000)


Hong *et al.* (2001)

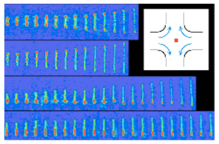


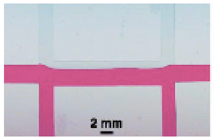
Gau *et al.* (1999)



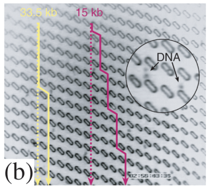
Green *et al.* (2002)

Delamarche *et al.* (2005)

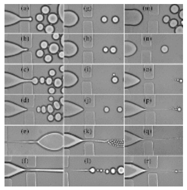

Perkins *et al.* (1997)

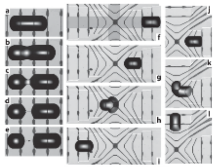


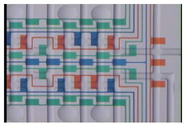
Zhao *et al.* (2001)

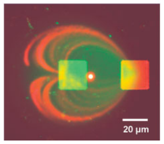


Chou *et al.* (2000)

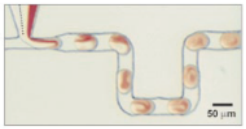

Anna *et al.* (2003)


Darhuber *et al.* (2003)

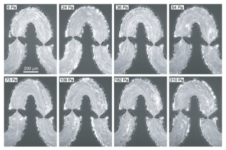

S. Maerkl

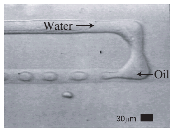


Juncker *et al.* (2005)

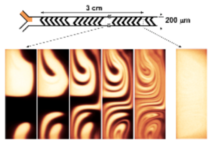


Song *et al.* (2003)

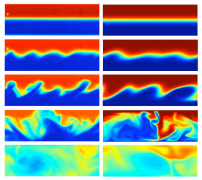

Groisman *et al.* (2003)

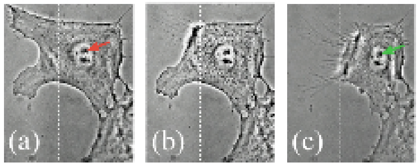


Thorsen *et al.* (2001)

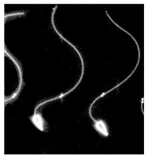


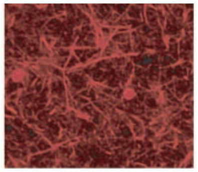
Stroock *et al.* (2002)


Lin *et al.* (2004)



Takayama *et al.* (2003)


C. Brokaw


Shin *et al.* (2004)

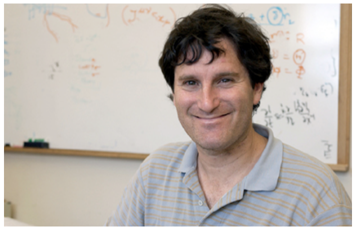
... is not just the study of viscous flows:
there is a lot of additional physics

Surface effects and wetting
Interface deformations and two-phase flow
Advective transport vs. diffusion
Slip and non-continuum effects
Elastic and non-Newtonian forces
Electrokinetics and electrical forces
Intermolecular forces
Heat transfer
Porous material
Acoustic streaming and sound waves
Suspensions
Living cells
Chemical reactions

Beebe et al. (2002) Annu. Rev. Biomed. Eng. **4**, 261
Stone, Stroock & Adjadri (2004) Ann. Rev. Fluid Mech. **36**, 381
Darhuber & Troian (2005) Ann. Rev. Fluid Mech. **37**, 425
Delamarche et al. (2005) Adv. Mat. **17**, 2911
Squires & Quake (2005) Rev. Mod. Phys. **77**, 977

Copyright 2006 Eric Lauga

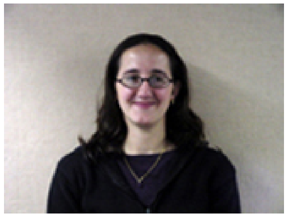
Thank you



Michael P. Brenner

Howard A. Stone

Carlo
Cossu



Willow
DiLuzio

Todd
Squires

Abraham
Stroock

Manouk Abkarian, Silas Alben, Shelley Anna, Jacquie Ashmore, Raymond Bergmann, Thomas Bewley, Nathalie Bontoux, Henry Chen, Marc Durand, Joel Frenzer, Jose Gordillo, Patrick Huerre, John Hutchinson, Stephan Koehler, Srinivas Paruchuri, Marcus Roper, Jim Rice, Michael Schnall-Levin, Steven Subotnick, Thomas Ward, George Whitesides, Kate Zirpolo, and many others...

Funding: NSF, Harvard MRSEC, ONR, Ecole des Mines de Paris

American Physical Society

Copyright 2006 Eric Lauga