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TURBULENT MIXING DUE TO RAYLEIGH-TAYLOR INSTABILITY

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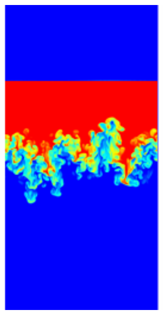
OUTLINE OF TALK

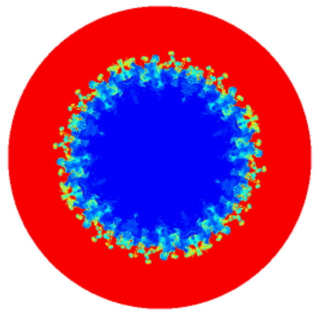
- What is Rayleigh-Taylor instability and where does it occur?
- The fundamental case – growth of a single sinusoidal mode
- Self-similar turbulent mixing

Constant acceleration, influence of initial conditions, internal structure, variable acceleration

- Examples of more complex flows

Break-up of a dense layer, a simple (ICF) implosion





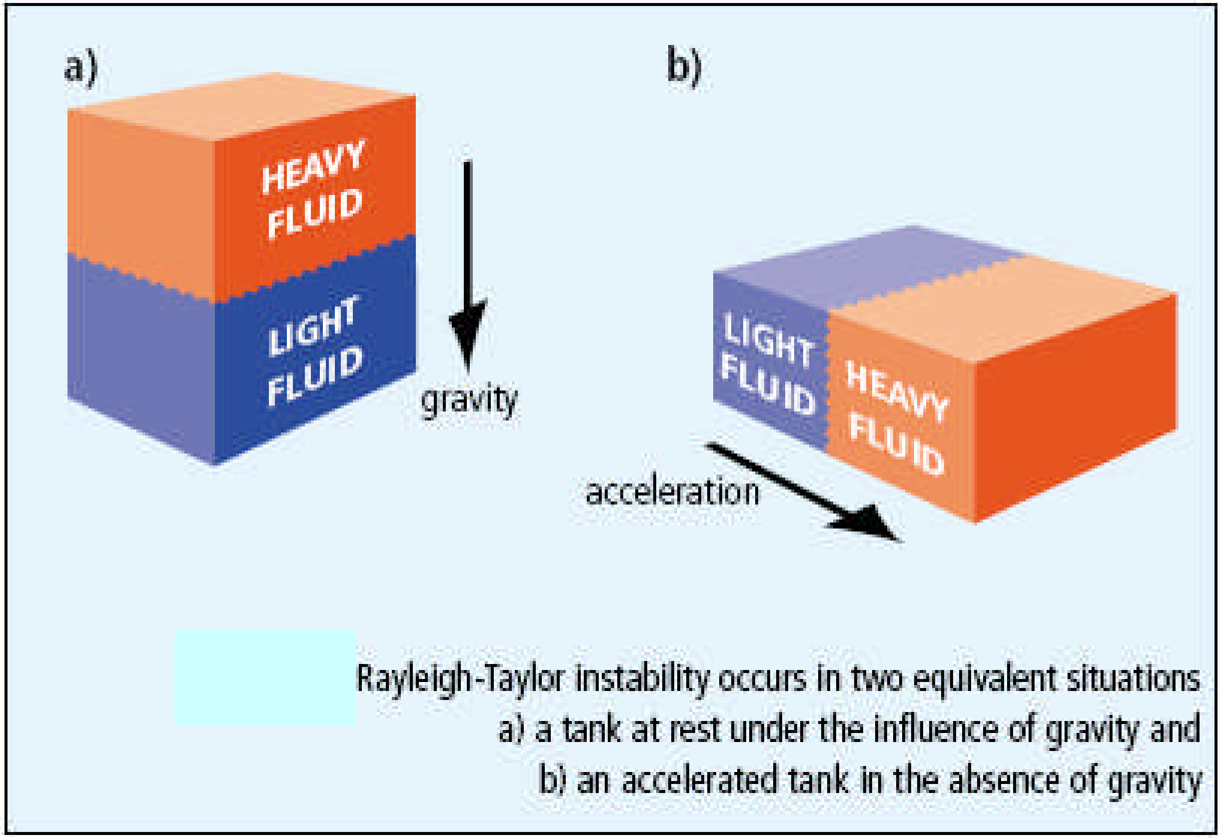
- Concluding remarks

Will give some historical background and show some experimental results
Will discuss major advances recently made using 3D simulation (and scope for further work)

similarities to turbulent shear flow

implications for engineering modelling (RANS models)

What is Rayleigh-Taylor (RT) instability?



Note: pressure gradient from heavy to light in both cases

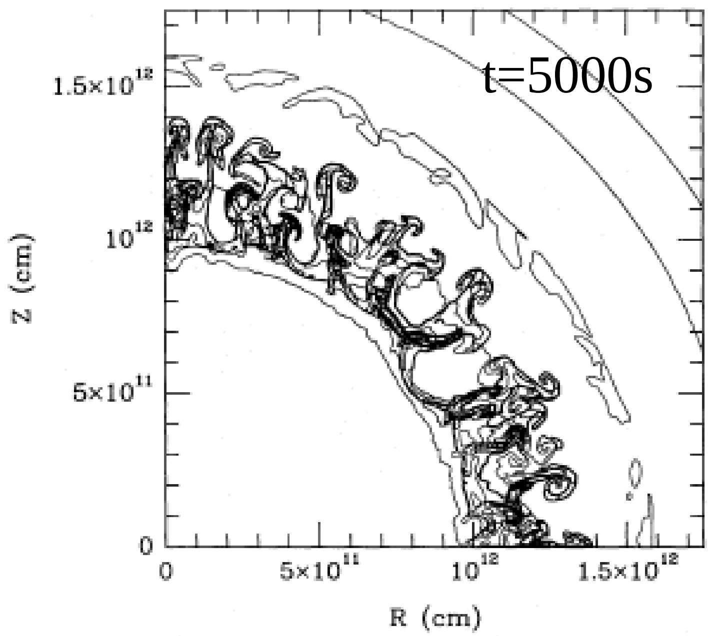
First publication, Rayleigh (1883)

Became an important research topic after the paper of G.I. Talyor (1950)

(related process – Richtmyer-Meshkov instability occurs when shock waves pass through perturbed interfaces)

A broad set of vital applications

Astrophysics

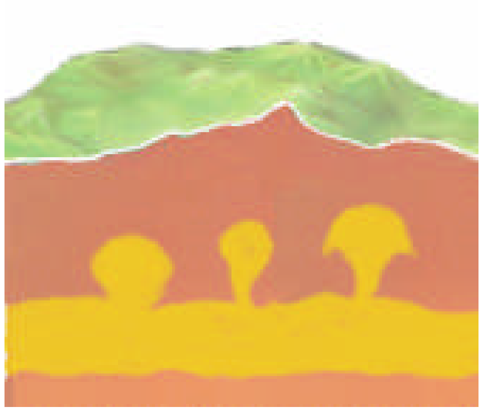


Instability and clumping in SN1987A. Müller et al. Astronomy and Astrophysics (1991)

Atmosphere / Ocean

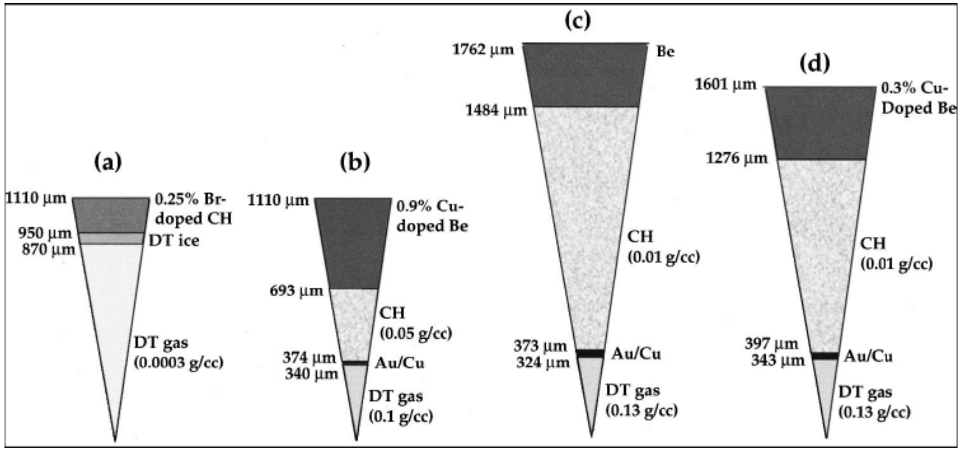
Rayleigh-Taylor instability traced by cirrus clouds (Picasa Web)

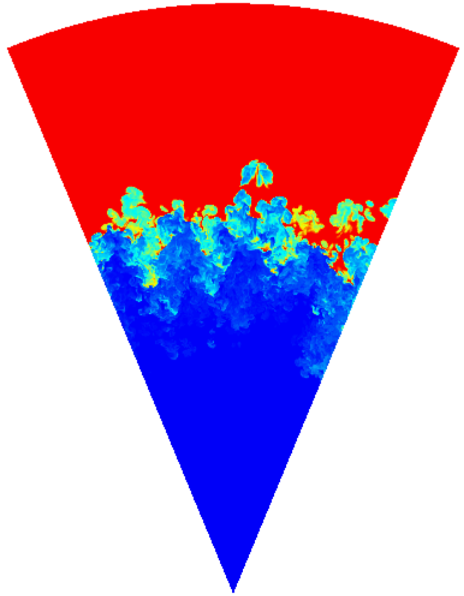
Geological flows (e.g. oil trapping in salt-domes)



wide range of distance and time scales, wide range of density ratios

Inertial Confinement Fusion (ICF) – nanosecond time-scale



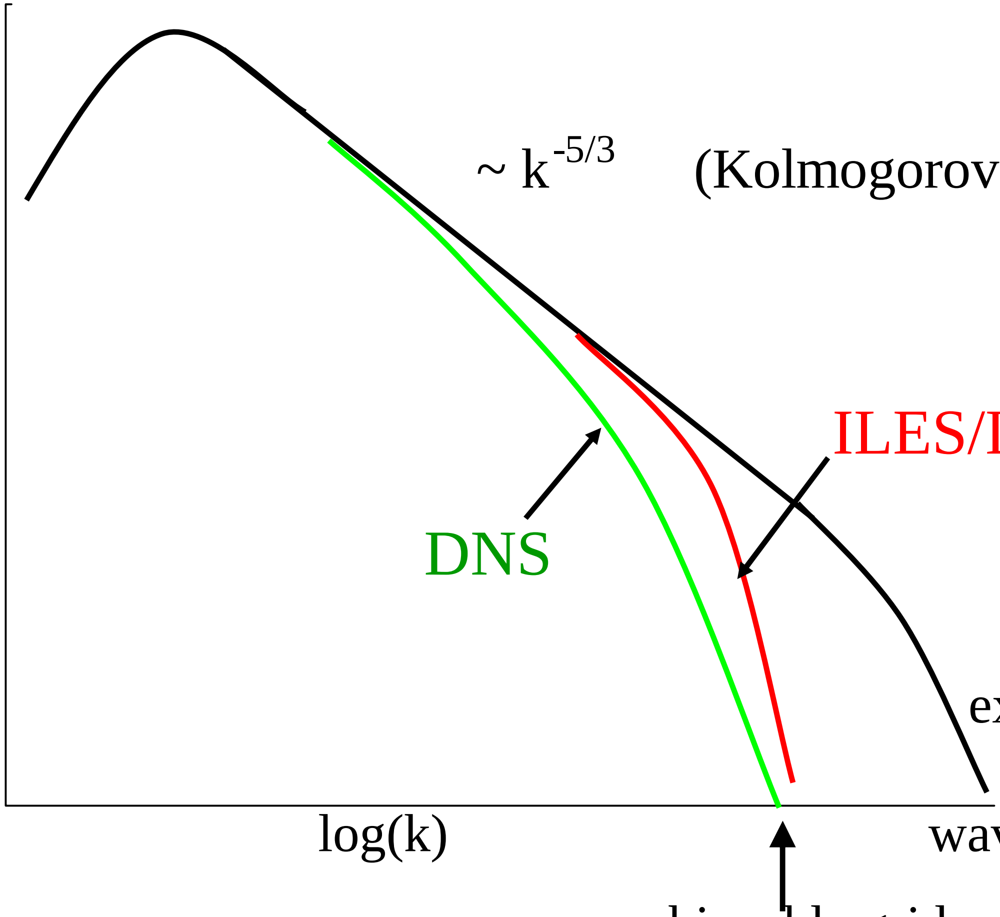


Amendt et al. Physics of Plasmas (2002) – degradation of capsule performance.

Focus of the present talk will be high-Reynolds no. mixing at initially sharp interfaces.

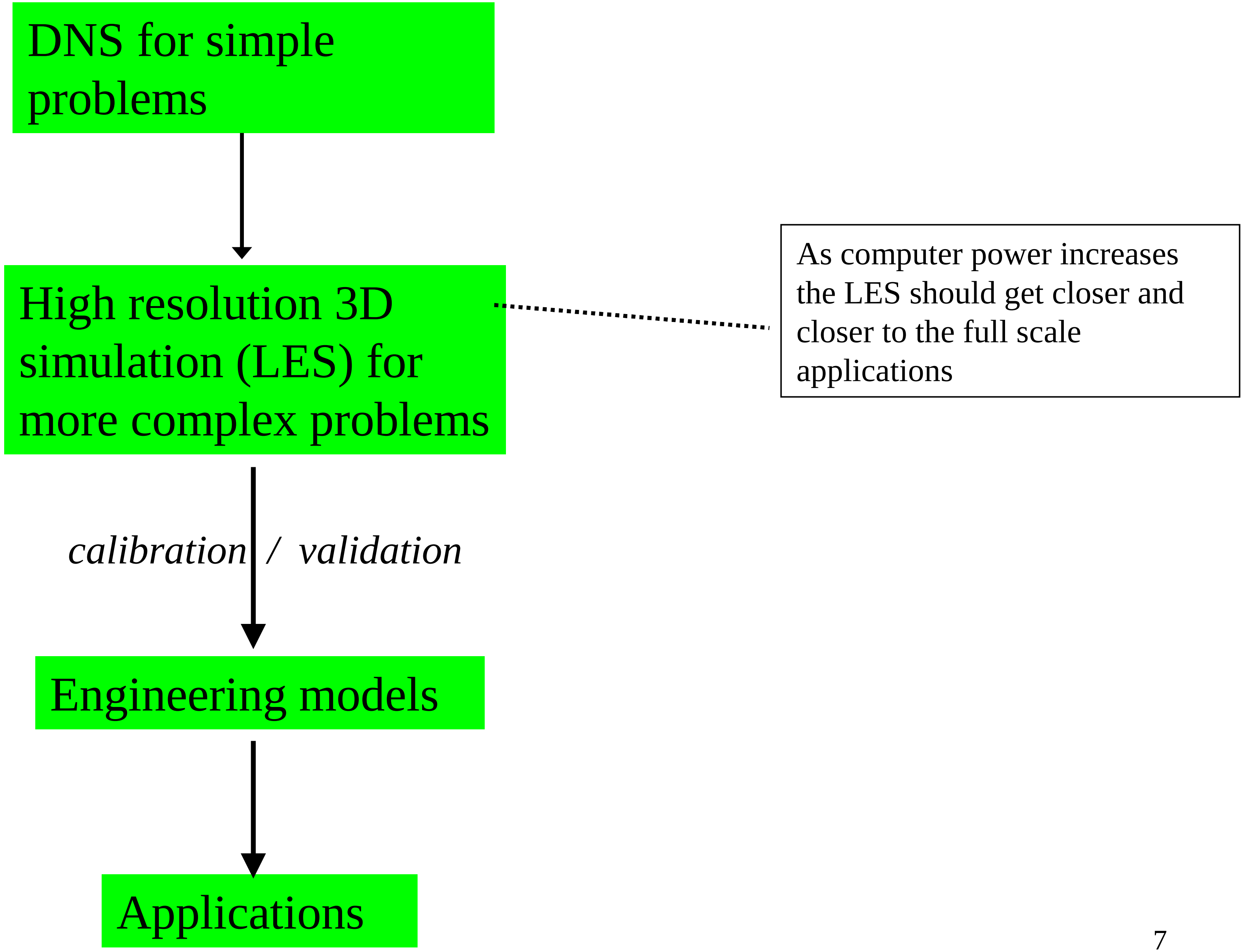
Flows may be compressible, but turbulence Mach no. (u′/c) is low ⇒ most key aspects of the RT process can be understood via incompressible experiments and simulations.

3D simulation has greatly enhanced our understanding of RT turbulence during the past decade



DNS: Direct Numerical Simulation – needed to understand the effect of Reynolds No.

LES: Large Eddy Simulation – best approximation to high-Reynolds No. mixing in more complex flows (explicit “sub-grid-scale” dissipation model or high-wavenumber dissipation Implicit in the numerical scheme: ILES)


DNS for simple problems
High resolution 3D simulation (LES) for more complex problems
As computer power increases the LES should get closer and closer to the full scale applications
calibration / validation
Engineering models
Applications

Historical Development (in the West)

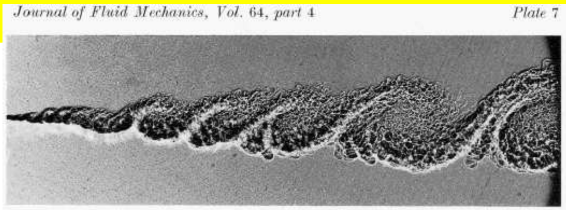
1950 – 1980s

Main focus on evolution of a single sinusoudal mode (small amplitude linear growth → large amplitude non-linear growth)
(see review paper by David Sharp, PhysicaD ,1984 – full references given at end of PDF file)

1980s – Present day

Emphasis changed to understanding turbulent mixing (evolution of mixing from random multimode perturbations)

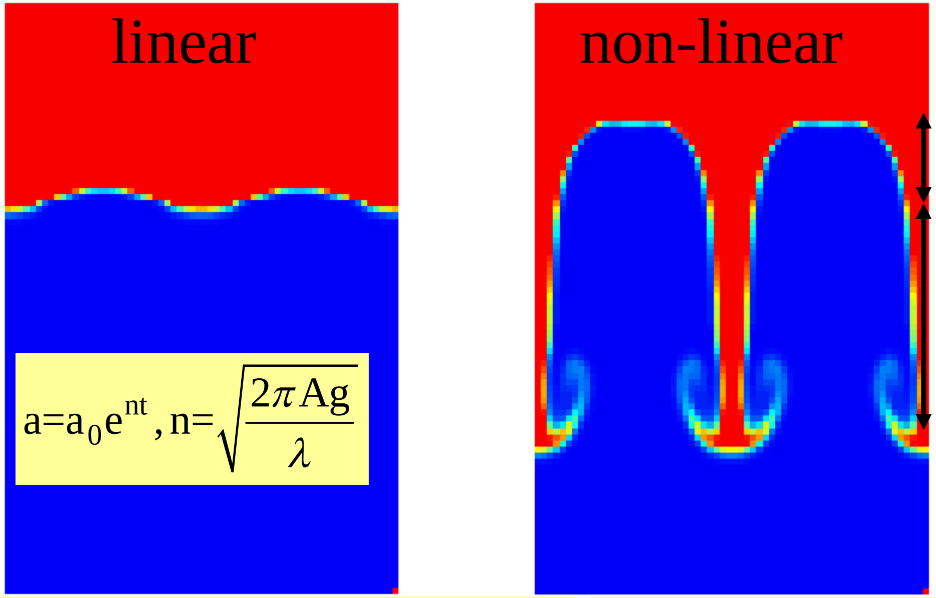
(note contrast with turbulent shear flow e.g. mixing layer - Liepmann & Laufer (1947) : self-similar turbulent mixing – Brown & Rosko (1974) : coherent structures)



Single mode Rayleigh-Taylor instability

2D simulation at density ratio $\rho_1 / \rho_2 = 20$

Atwood number: $A = \dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$



$a = a_0 e^{nt}, \; n = \sqrt{\dfrac{2\pi A g}{\lambda}}$

early time: exponential growth (19th C theory for interfacial waves with sign of ρ_1 - ρ_2 reversed)

late-time: bubbles rise with velocity: $V \sim \sqrt{A g \lambda}$

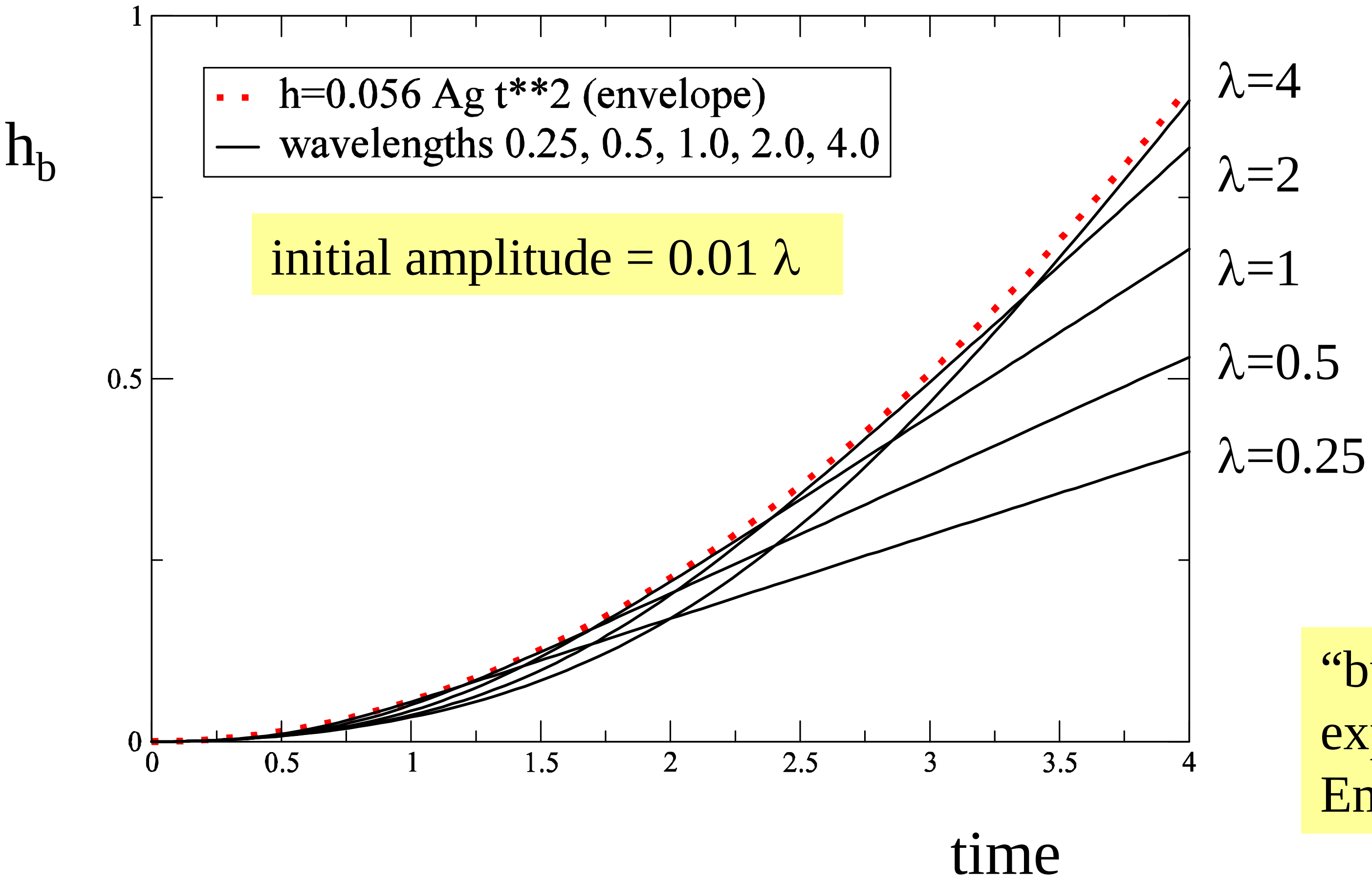
Atwood No. close to 1 : Equation due to Layzer (1955) gives a good approximation to the

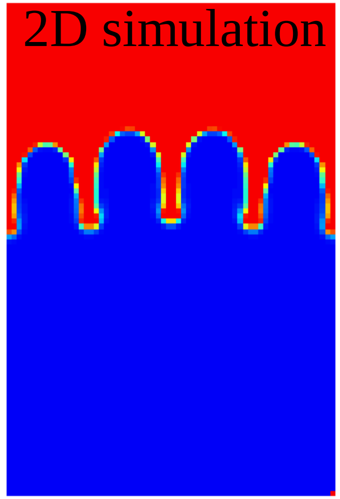
bubble velocity $V = \frac{dh_b}{dt}$: $\quad (2+E)\dfrac{dV}{dt} = (1-E)Ag - \dfrac{6\pi V^2}{\lambda}$ $\quad$ where $E = \exp\left(-\dfrac{6\pi h_b}{\lambda}\right)$

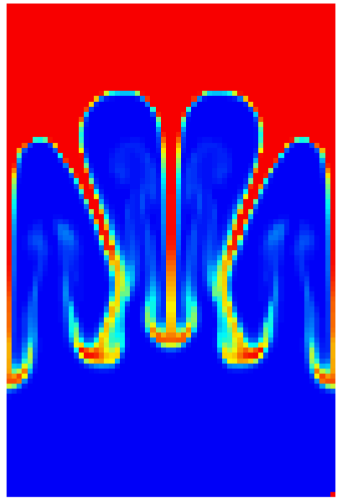
Low Atwood No. : More complex behaviour, Ramaprahbu et al. (2006) - 3D simulations

Multimode initial perturbations

Apply Layzer equation to a range of wavelengths (similar technique used by G. Birkhoff, Los Alamos report, 1954)



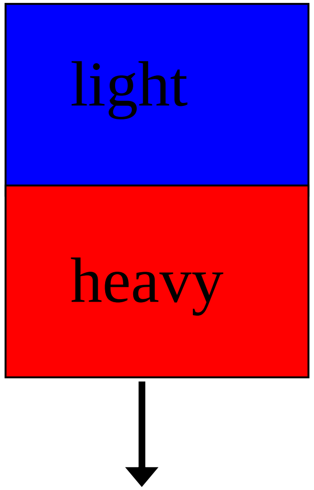




"bubble competition" experiments of Emmons et al. 1960.

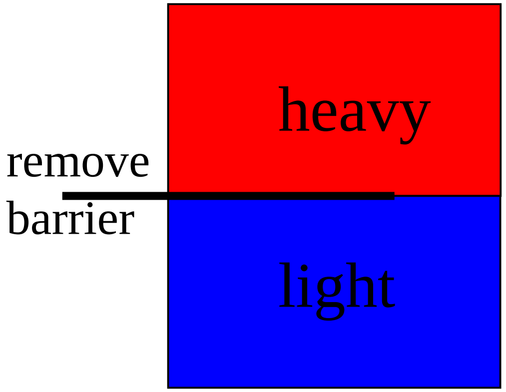
Experiments used for Rayleigh-Taylor turbulence (1980 onwards)

accelerated tank



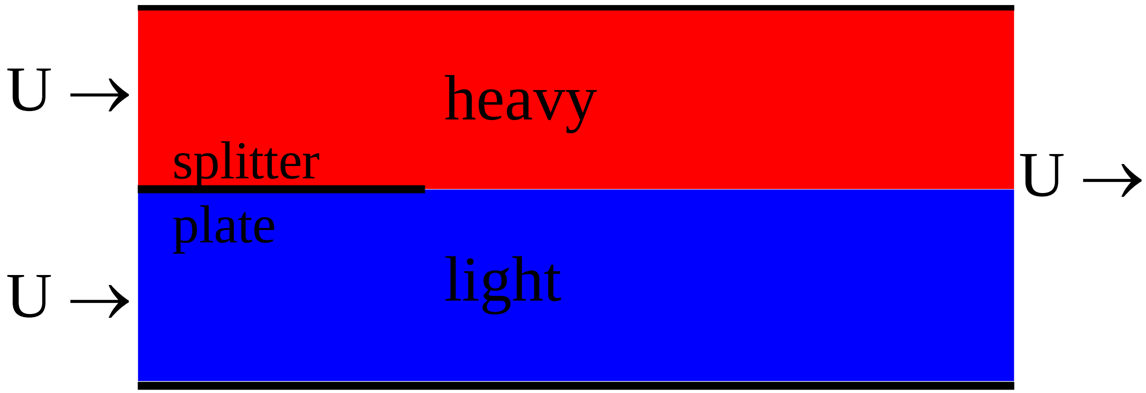
AWE , LLNL
Chelyabinsk-70,
U Arizona

static tank



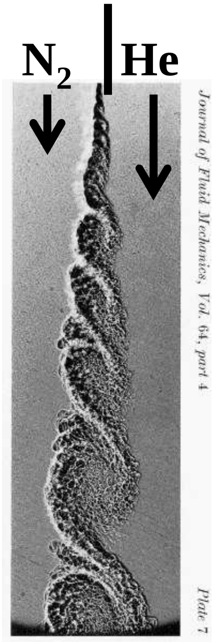
Cambridge U
Chelyabinsk-70

water/gas channel, h_b =f(t) where t=x/U



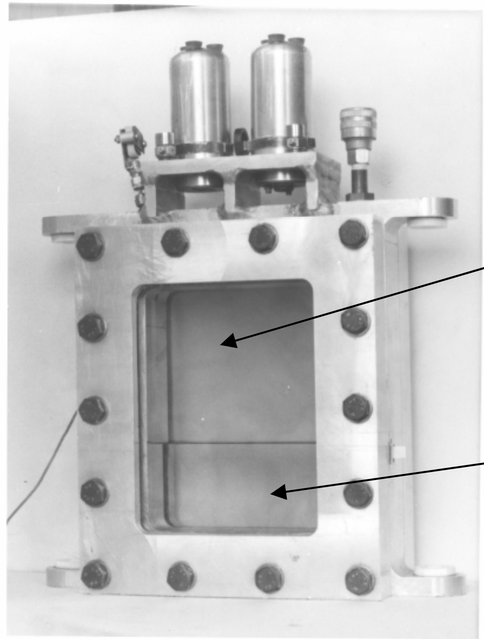
Texas A&M U

Rocket-Rig RT experiment - AWRE Foulness ,1980s, (see, Read ,1984)



mixing layer:
Brown and Rosko,
JFM, (1974)

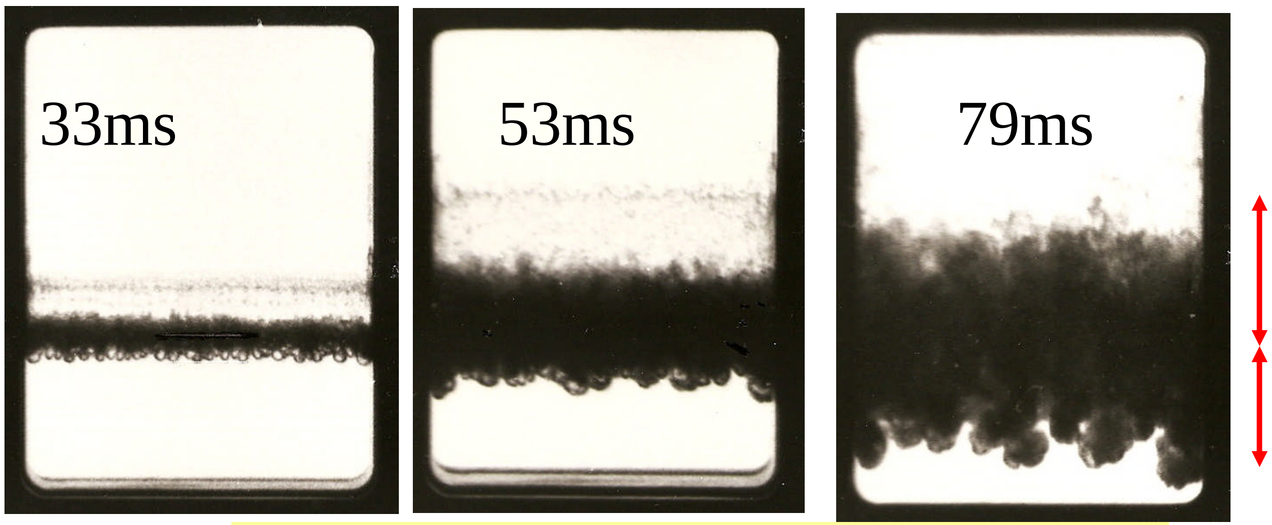
length scale increases by vortex pairing



Compressed SF_6

Pentane

$$\frac{\rho_1}{\rho_2} = 8.5$$



h_s (spike)

h_b (bubble)

length scale increases by bubble competition

Experiments show increase in length scale as mixing evolves – if mixing is self-similar, dimensional analysis suggests

mixing zone width $$W = f\left(\frac{\rho_1}{\rho_2}\right) gt^2$$

The Rocket-rig experiments showed

$$h_b = \alpha \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2} gt^2$$ where $\alpha \simeq 0.06$

and h_s/h_b a slowly increasing function of ρ_1 / ρ_2

very simple pattern for the amount of mixing

h_b = penetration of dense fluid (bubble distance)

h_s = penetration of light fluid (spike distance)

More recent Linear Electric Motor experiments at LLNL, (Dimonte & Schnieder,1996) gave $\alpha \sim 0.05$, Texas A&M α=0.07

Also similar results from experiments performed by Kucherenko's group at Chelyabinsk-70 (Kucherenko et al., 1991)

LOSS OF MEMORY OF INITIAL CONDITIONS (Youngs 1984)

If the initial surface consists of small random short wavelength perturbations then, after a short time:
dominant length scale » viscous scale
dominant length scale increases by mode coupling
$\Rightarrow$ expect loss of memory of the initial conditions to occur (as assumed in turbulent shear flow, Townsend, 1976*)
$\Rightarrow$ unique value of α

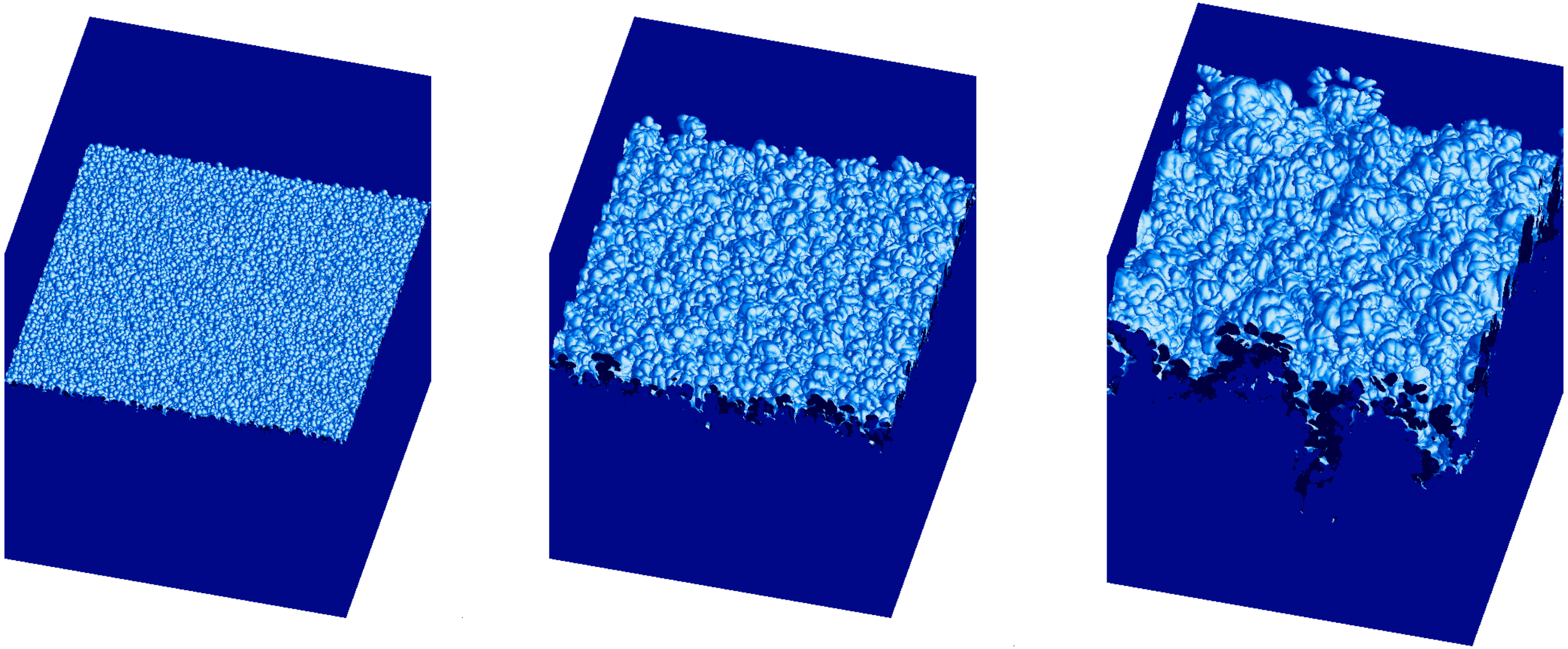
It was noted that mixing would be enhanced if long-wavelength initial perturbations with sufficiently high amplitudes were present.

However, before high-resolution 3D simulation was feasible it was thought that loss of memory of initial conditions would apply to low end of the observed range of α values

* was known at the time that shear layer growth varied from experiment to experiment

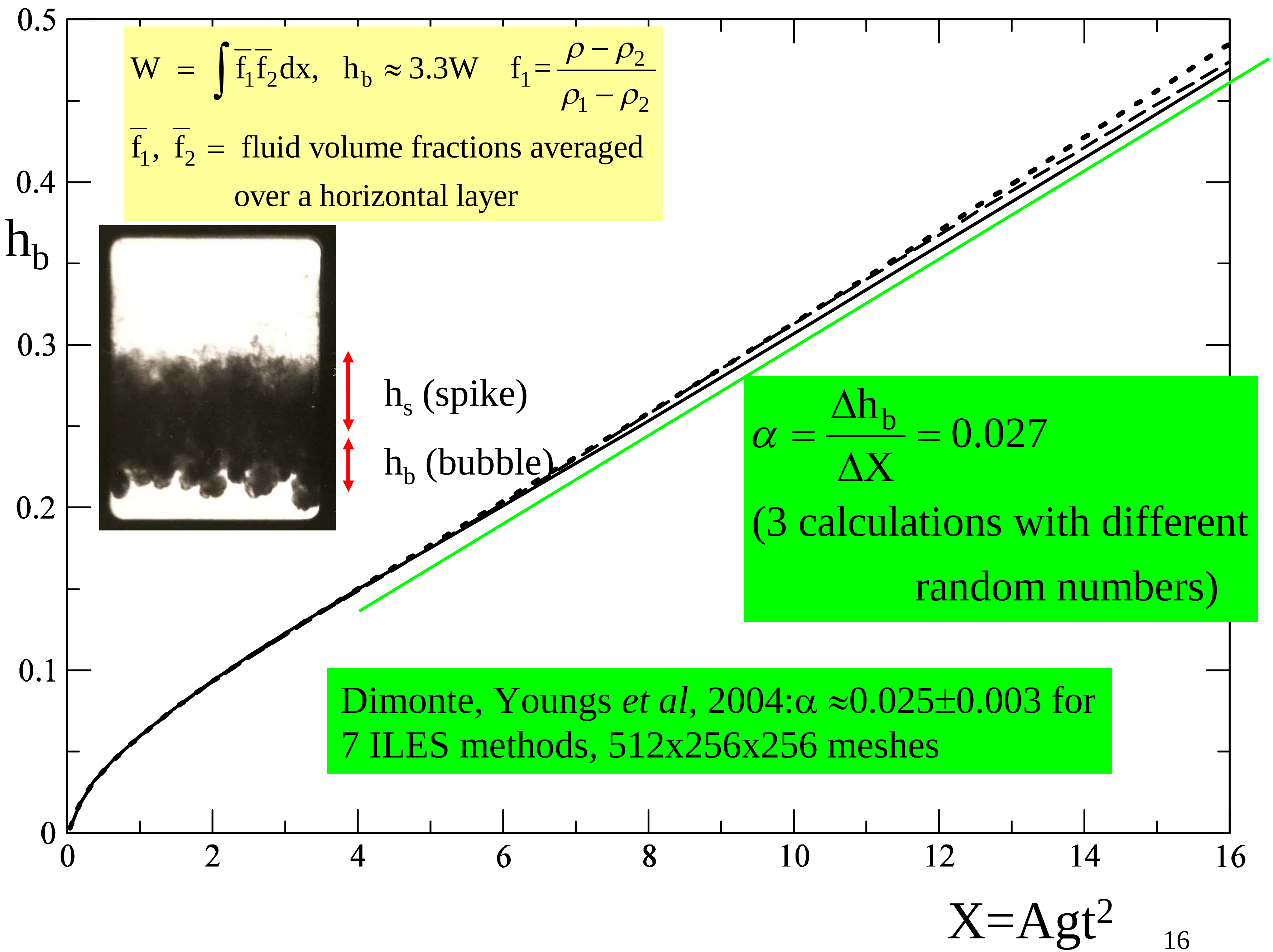
TURMOIL MILES ($\rho_1 / \rho_3 = 3$) 720 x 600 x 600 meshes (simple explicit compressible code run at low Mach no)

Short wavelength initial perturbations : random combination of Fourier modes: wavelengths:- 4 Δx to 8 Δx, amplitude s.d.:- 0.04 Δx

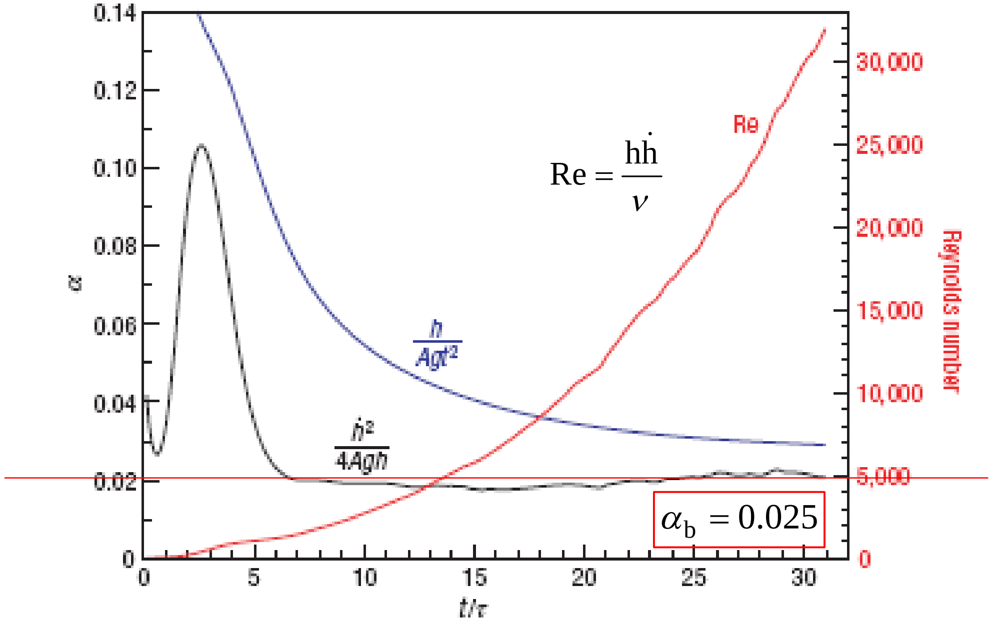


t = 0.8 t = 2.0 t = 3.8

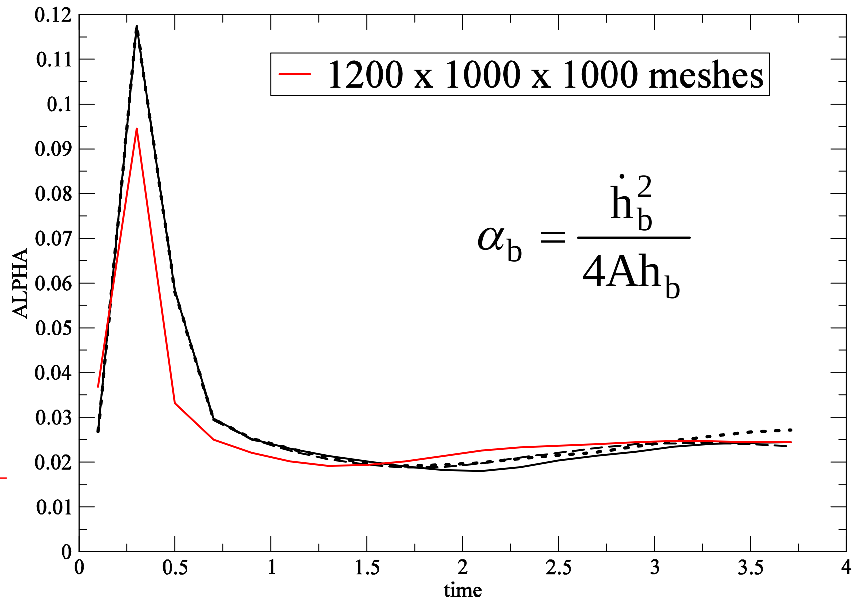
Calculations performed on the AWE Cray XT3 (8000 processing elements). For these calculations: 360 processors for 24 hours


$W = \int \bar{f}_1 \bar{f}_2 dx, \quad h_b \approx 3.3W \quad f_1 = \frac{\rho - \rho_2}{\rho_1 - \rho_2}$
$\bar{f}_1, \bar{f}_2 =$ fluid volume fractions averaged over a horizontal layer
h_s (spike)
h_b (bubble)
$\alpha = \frac{\Delta h_b}{\Delta X} = 0.027$
(3 calculations with different random numbers)
Dimonte, Youngs *et al*, 2004: $\alpha \approx 0.025 \pm 0.003$ for 7 ILES methods, 512x256x256 meshes
h_b
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
$X = Agt^2$

Cabot & Cook,LLNL, 2006: 3072^3 DNS



TURMOIL (ILES) 720x600x600



DNS suggests that α may increase slowly with Re ⇒ some uncertainty in high Re limit.

TURMOIL shows similar limiting behaviour to the much higher resolution DNS.

For the ideal situation of "small random perturbations" a range of both LES and DNS results have all given values of α much less than observed $\Rightarrow$ need to assume that in experiments low levels of initial long wavelength perturbations have enhanced mixing.

A model for enhanced self-similar growth was proposed by Inogamov(1999):
Long wavelength initial perturbations with amplitude $\propto$ wavelength, up to size of experiment.

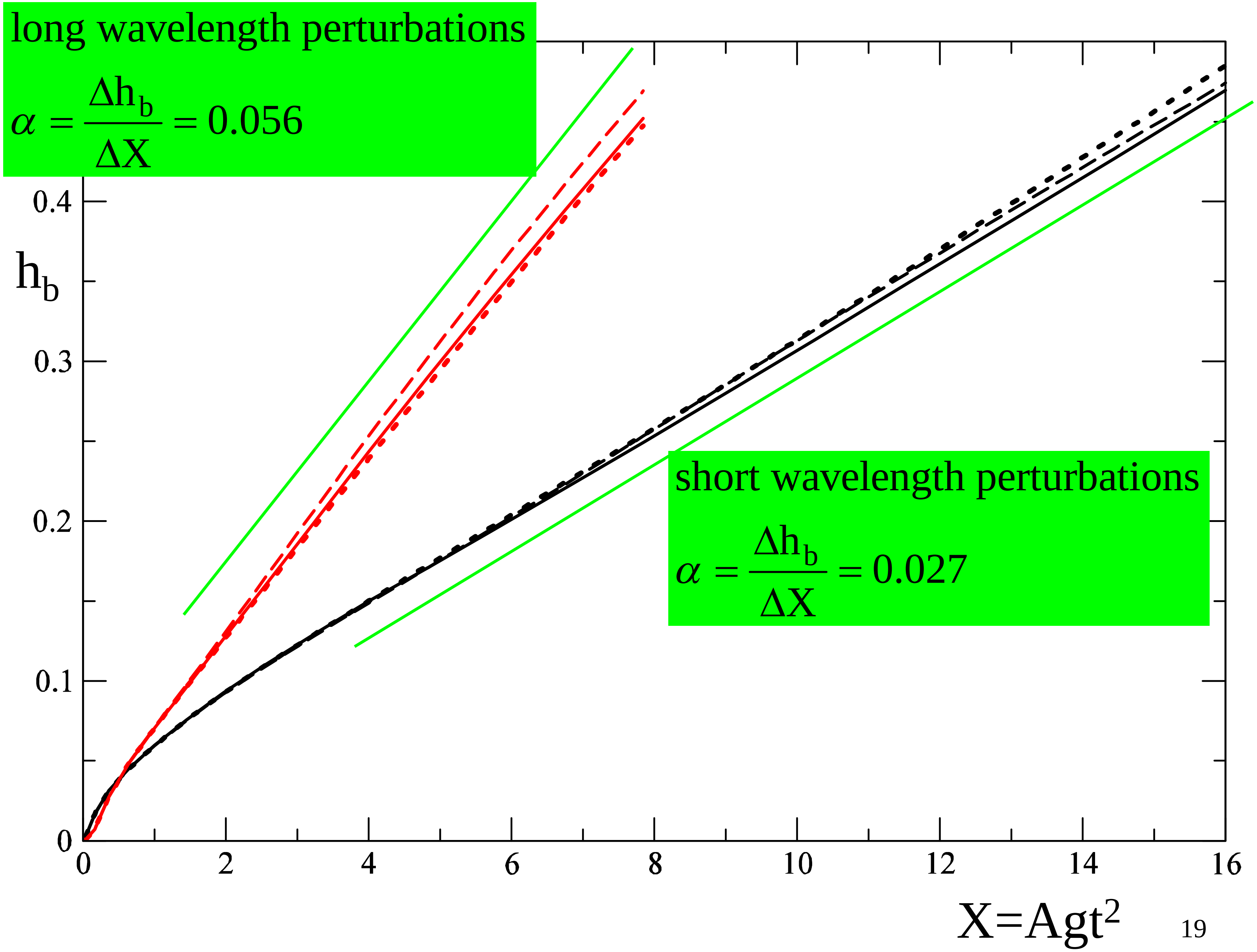
Note similarity to Birkhoff's arguement

In mathematical terms

s.d of surface = σ, where $\sigma^2 = \int P(k)dk$ with $P(k) \sim 1/k^3$

have used here: wavelengths up to $\frac{1}{2}$ box width

and $\sigma = 0.00025 \times$ box width (a very small value)



long wavelength perturbations
$\alpha = \frac{\Delta h_b}{\Delta X} = 0.056$
short wavelength perturbations
$\alpha = \frac{\Delta h_b}{\Delta X} = 0.027$
h_b
0
0.1
0.2
0.3
0.4
0
2
4
6
8
10
12
14
16
$X=Agt^2$

The results suggest that low levels of long-wavelength initial perturbations are the explanation of the higher observed growth rates. (note recent experiments, Kucherenko et.al, 2003, have indicated a lower α~0.04)

Experiments are finite:

The basic problem is not mixing at an infinite plane boundary with finite s.d. (this should asymptote to α~0.025)
but mixing in a finite domain of size L, with low levels of perturbations with wavelengths up to size L. Then expect influence of initial conditions to persist throughout the duration of the experiment.

Similar conclusion apply to turbulent shear flows (influence of upstream conditions) – W.K.George, Freeman Scholar Lecture , ASME Fluids Engineering Meeting, 2008.

A simple quatitative model for the dependence on initial conditions

Initial perturbation at wavelength λ: $a = \varepsilon\lambda = \sqrt{\int_{k-\Delta k}^{k+\Delta k} P(k')dk'}$

Perturbation due to mode-coupling = $\varepsilon_0 \lambda$ (invariant)

$\lambda_d =$ dominant scale at time t, then (approximately)

$$a_{\lambda_d} e^{nt} = \tfrac{1}{2}\lambda_d \qquad \text{where } n = \sqrt{\frac{2\pi Ag}{\lambda}}$$

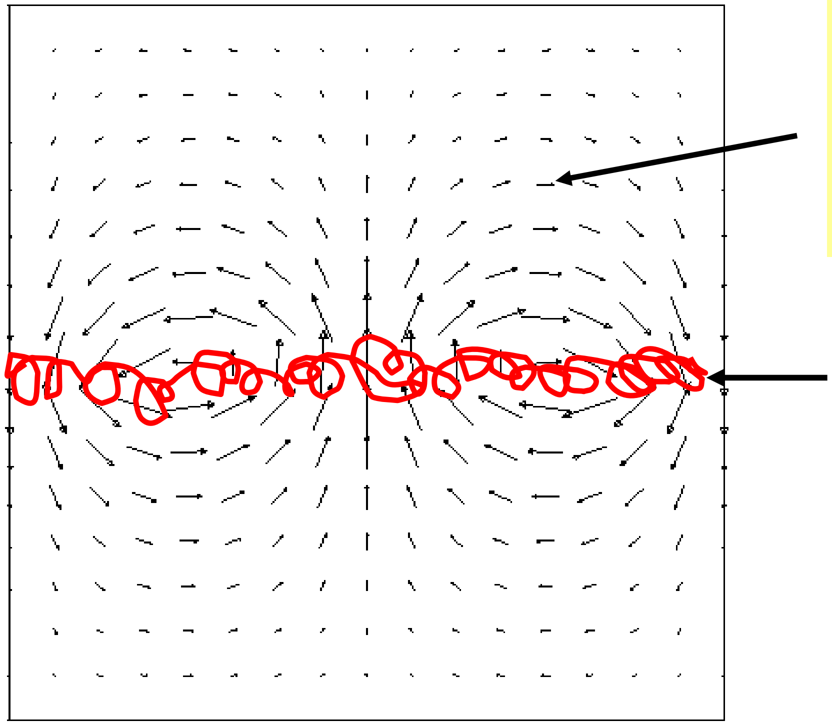
Then $h_b \sim \lambda_d = 2\pi \left\{ \ln\left(2\sqrt{\varepsilon^2 + \varepsilon_0^2} \right) \right\}^2 Agt^2$

Suggests weak logarithmic dependence in initial conditions

(more quantitative argument given by Dimonte, 2004)

Important Implications for Engineering Modelling
{RANS models, one-point closure models e.g. (k,ε) model}

Given set of model coefficients ⇒ a given value of α - does not capture the dependence on initial conditions (also applies to shear flow modelling – W. K. George)



velocity field due to long wavelength initial perturbation – will enhance mixing at late-time (not captured in one-point closure models)

current region of turbulent mixing

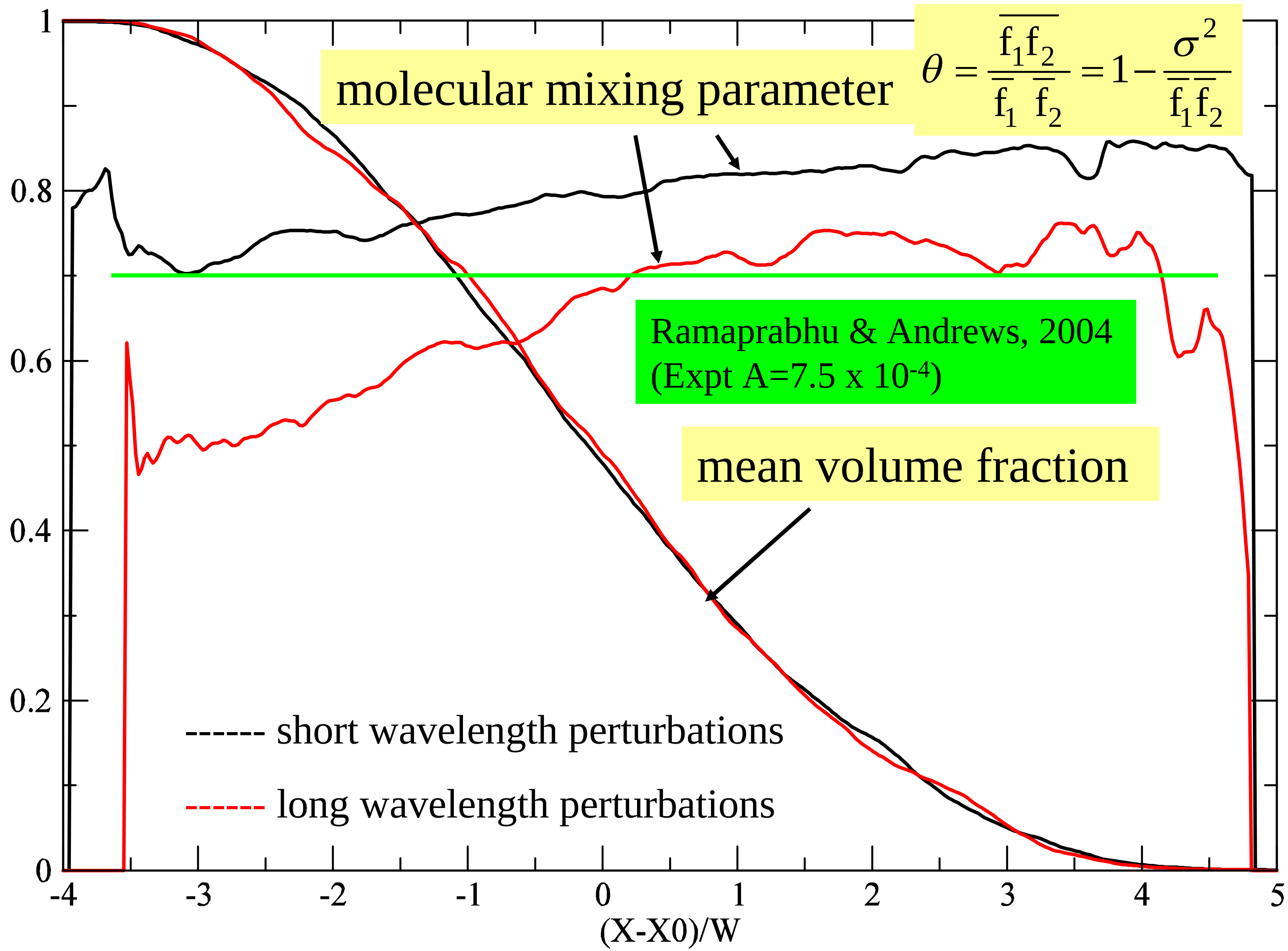
Solution adopted here

- Note that , for a given experimental series, assuming α= a const., works quite well (weak dependence on initial conditions)

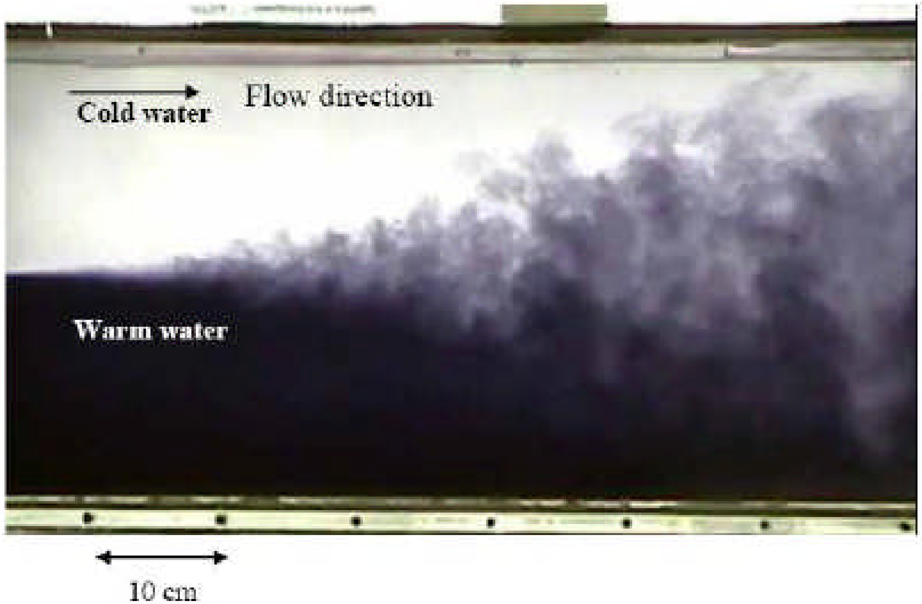
- Derive model coefficient sets for a range of values of α, using 3D LES (or DNS) results for enhanced self-similar mixing (Inogamov)

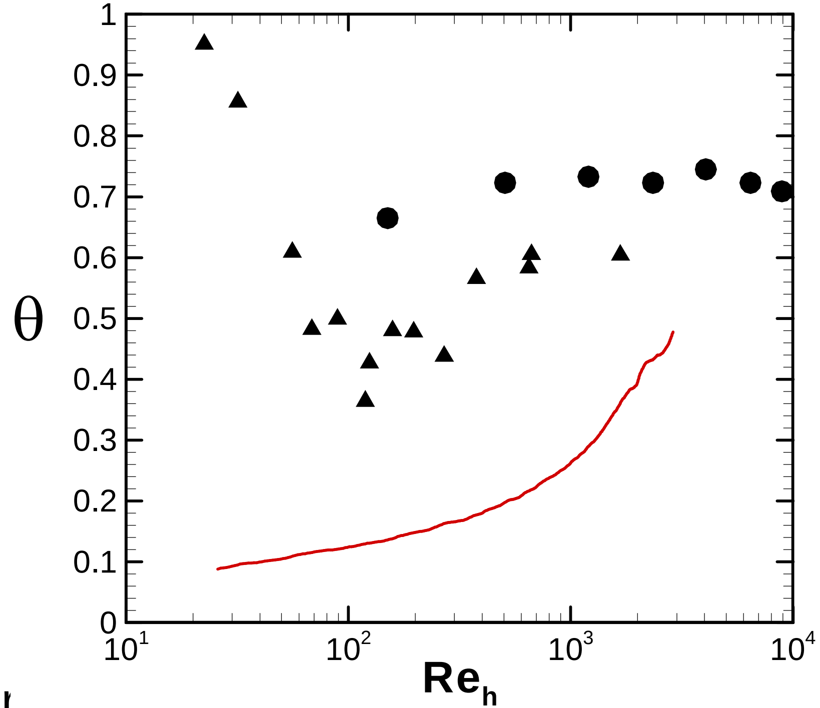
- Use LES for simplified versions of the real problems, with estimates of realistic initial conditions, to estimate the appropriate $\alpha_{effective}$ for a given application.

The 3D simulations also give much detailed information on the internal structure – for ICF important to know the amount of molecular mixing



Measurements of molecular mixing parameter: Malcolm Andrews group at Texas A&M





$$Sc = \frac{\nu}{D} = \left[\frac{L_{vis}}{L_{diff}}\right]^2$$

— Sc=700 (brine/water)

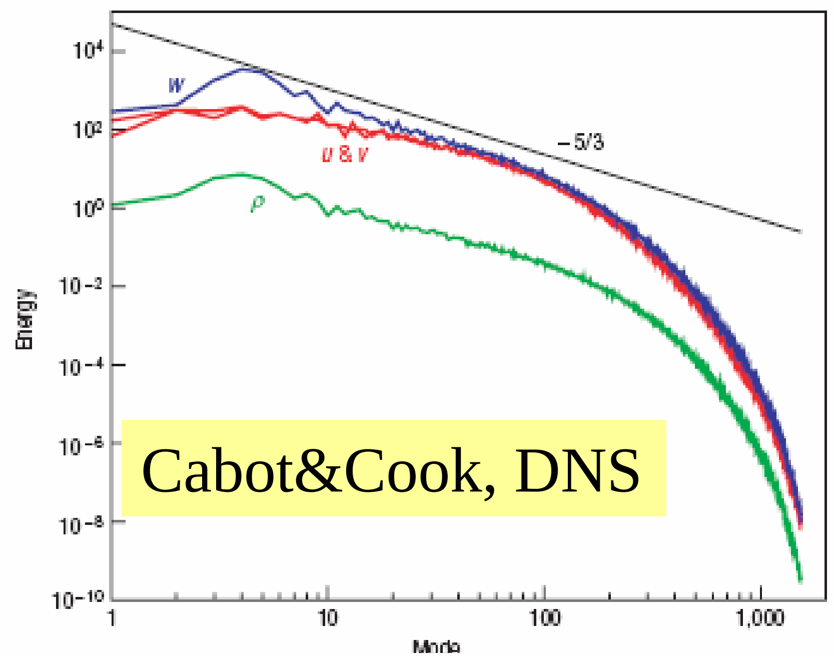
▲ Pr=Sc=7 (cold/hot water)

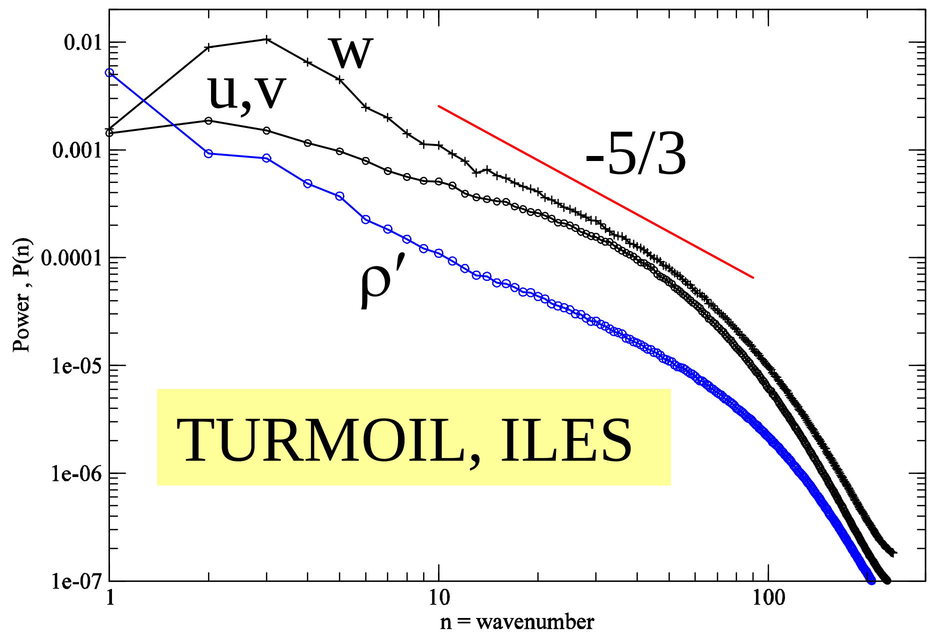
● Sc=0.7 (gases)

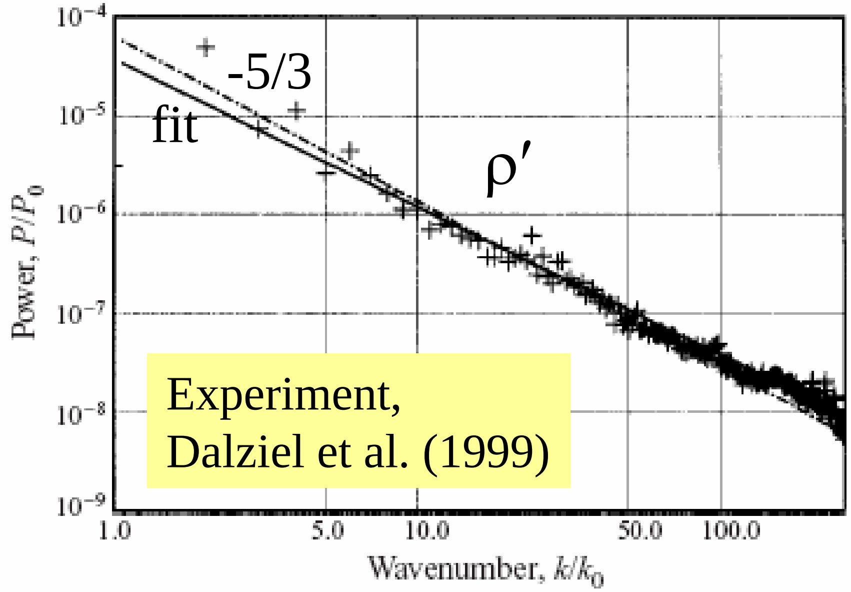
- **Combining passive scalar and reacting scalar experiments at multiple equivalence ratios provide a measure of the volume fraction variance, Meuschke et al (2008)**
- **Measurement of molecular mixing demonstrates a large influence of Schmidt number at small Re, but tending toward saturation at high Re ~10^4 *a challenge for DNS*.**
- **Note similarity to jet mixing, Dimotakis (2000) –**

"mixing transition" at Re =Uδ/ν = 1-2 x 10^4

Power spectra: w (vertical velovity), u&v (horizontal velocity) and ρ (density)







DNS and ILES show similar behaviour for velocity spectra at low wavenumber.

DNS/ILES/Experiment all show spectra for ρ′ slightly flatter than $k^{-5/3}$

Departures from Kolmogorov's $k^{-5/3}$ law
(see also O. Poujade, 2006)

• Both DNS and LES velocity spectra show some asymmetry at high-wavenumbers – suggests buoyancy effects persist at high-wavenembers (some evidence for this from TAMU experiments)

• Corrections to the $k^{-5/3}$ law have been proposed, for slowly varying turbulence (Yoshizawa,1994: Woodruff &Rubinstein, 2006)

$$E(k)=C_K\varepsilon^{2/3}k^{-5/3}+C_N\dot{\varepsilon}\varepsilon^{-2/3}k^{-7/3}+\ldots\ldots\, , \quad \dot{\varepsilon}=\frac{D\varepsilon}{Dt}$$

Now for RT mix with constant g

$$\varepsilon=\frac{u'^3}{\ell}\sim t$$

But expect to get more extreme variations than this ……

Have so far assumed, acceleration, g = a constant – not very realistic approximation in many applications (e.g supernova explosion, ICF implosion).

$g \sim t$ $g \sim 1/t$ highly likely to occur (the LLNL Linear Electric Motor experiments did consider variable g)

Antoine Llor, CEA (2004) has advocated studying self-similar RT mixing problems, also O. Poujade APS-DFD 2008:

$$g = Ct^n \quad \Rightarrow \quad h_b = \alpha_n A C t^{n+2}$$

Very useful for extended validation of the one-point closure models.

Also interesting to look at power spectra

$g \sim t \Rightarrow \varepsilon \sim t^3$: $g \sim 1/t \Rightarrow \varepsilon \sim 1/t$

expect more extreme variations in time than many other types of turbulent flow

Finally some more complex RT flows will be considered:

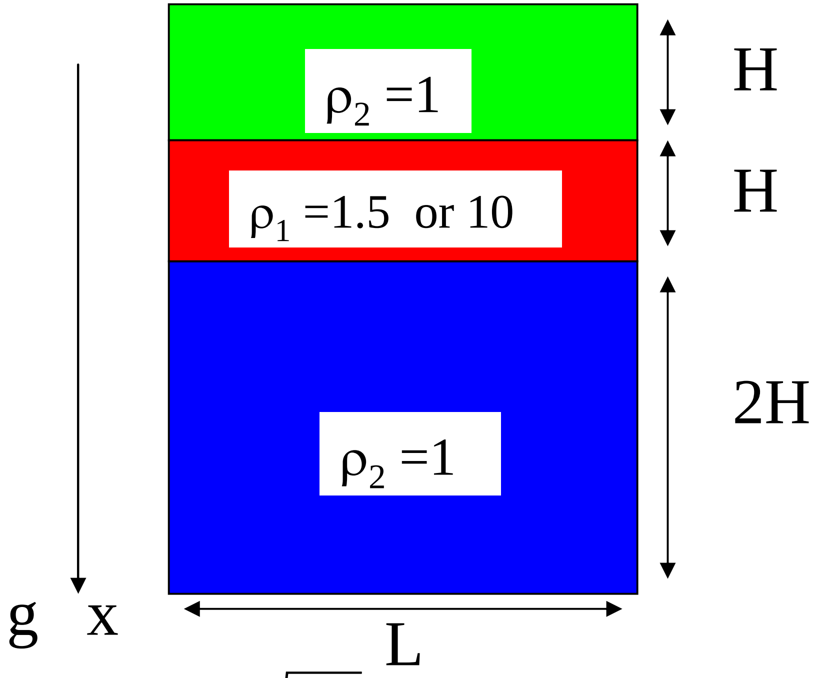
(a) Break-up of a dense fluid layer due to Rayleigh-Taylor instability.

(b) A simplified spherical implosion

Will show that ILES currently makes an essential contribution to engineering model validation

model used here : multiphase flow equations +turbulent diffusion terms + decay of concentration fluctuations (a type of RANS model)

BREAK-UP OF A DENSE FLUID LAYER DUE TO RAYLEIGH-TAYLOR INTABILITY

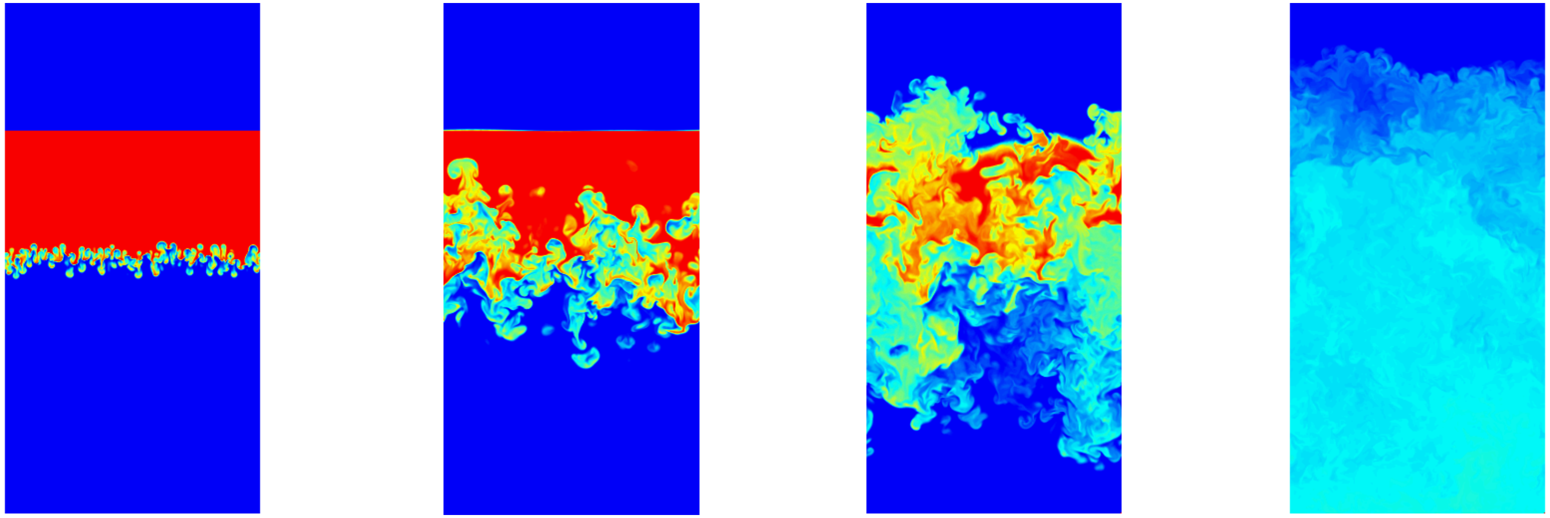


dimensionless time $\tau = \sqrt{\frac{Ag}{H}}t$ (Atwood no. or Boussinesq scaling) $A = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$

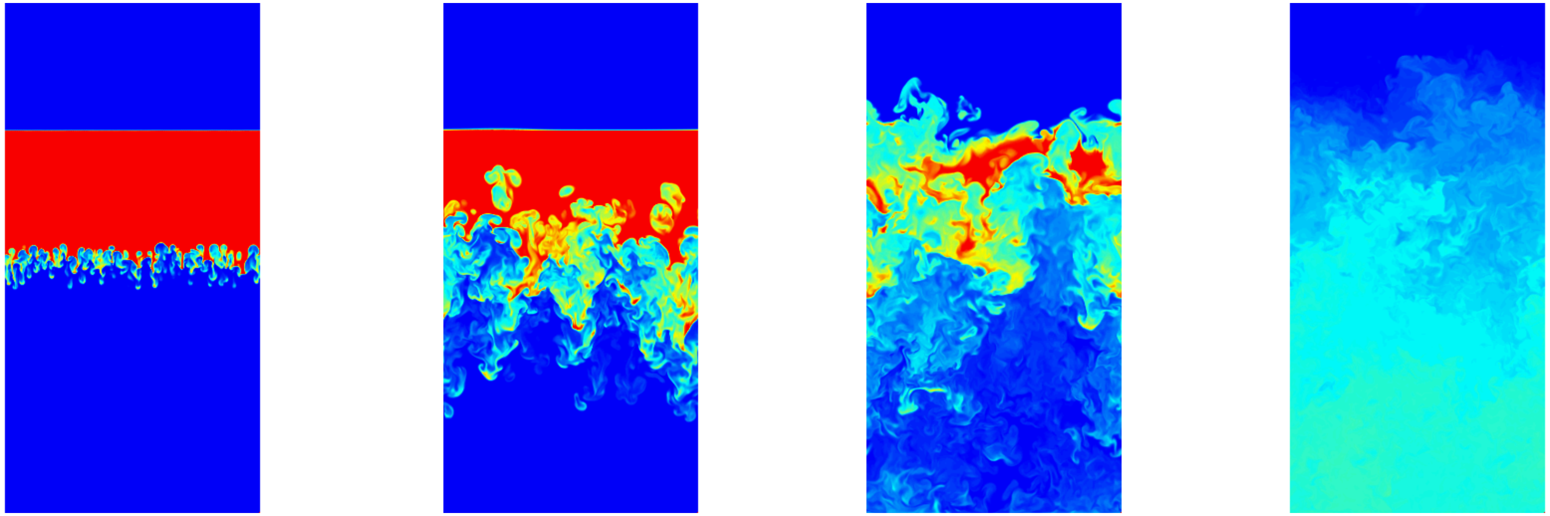
Initial perturbations at lower boundary P(k)~ 1/k^3 (as before)

Corresponding low Atwood no. experiments, Jacobs & Dalziel (2005)

$\rho_1 / \rho_2 = 1.5$, 3D simulation, sections for fluid 2 volume fraction



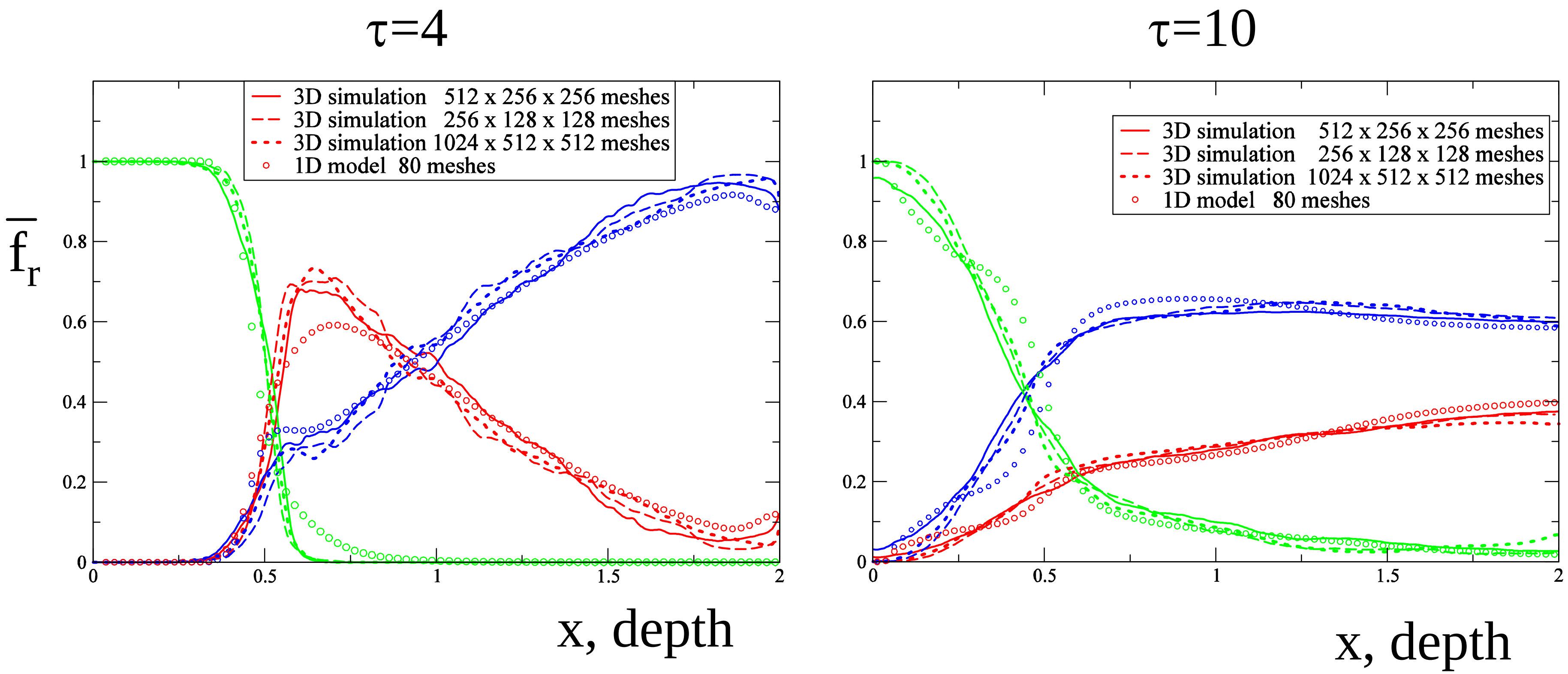
$\rho_1 / \rho_2 = 10$, 3D simulation, sections for fluid 2 volume fraction



$\tau = 1$ $\tau = 2.5$ $\tau = 4$ $\tau = 10$

mixing time scale is confirmed by experiments of Jacobs & Dalziel (2005)

Mean fluid volume fractions (averaged over horizontal layers) for $\rho_1 / \rho_2 = 10$



1D engineering model uses coefficient set for α=0.056

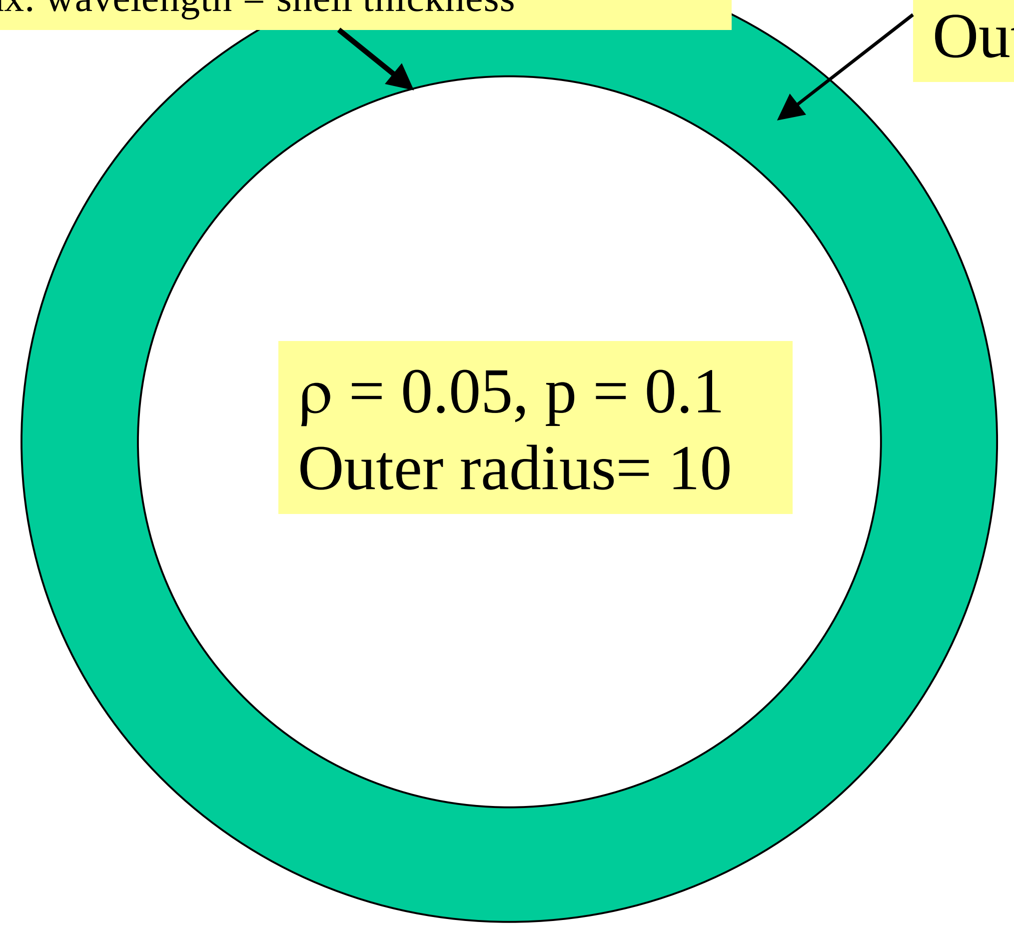
A simple spherical implosion (dimensionless units) – relevant to Inertial Confinement Fusion

Perturbation spectrum, $P(k) \sim \frac{1}{k^2}$, s.d.=0.0005

max. wavelength = shell thickness

$\rho = 1.0 \quad p = 0.1$

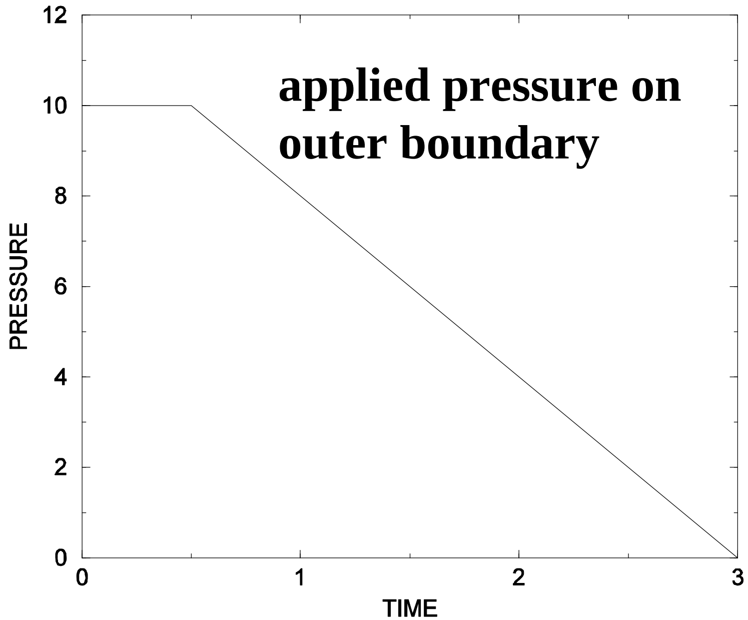
Outer radius = 12



$\rho = 0.05$, $p = 0.1$

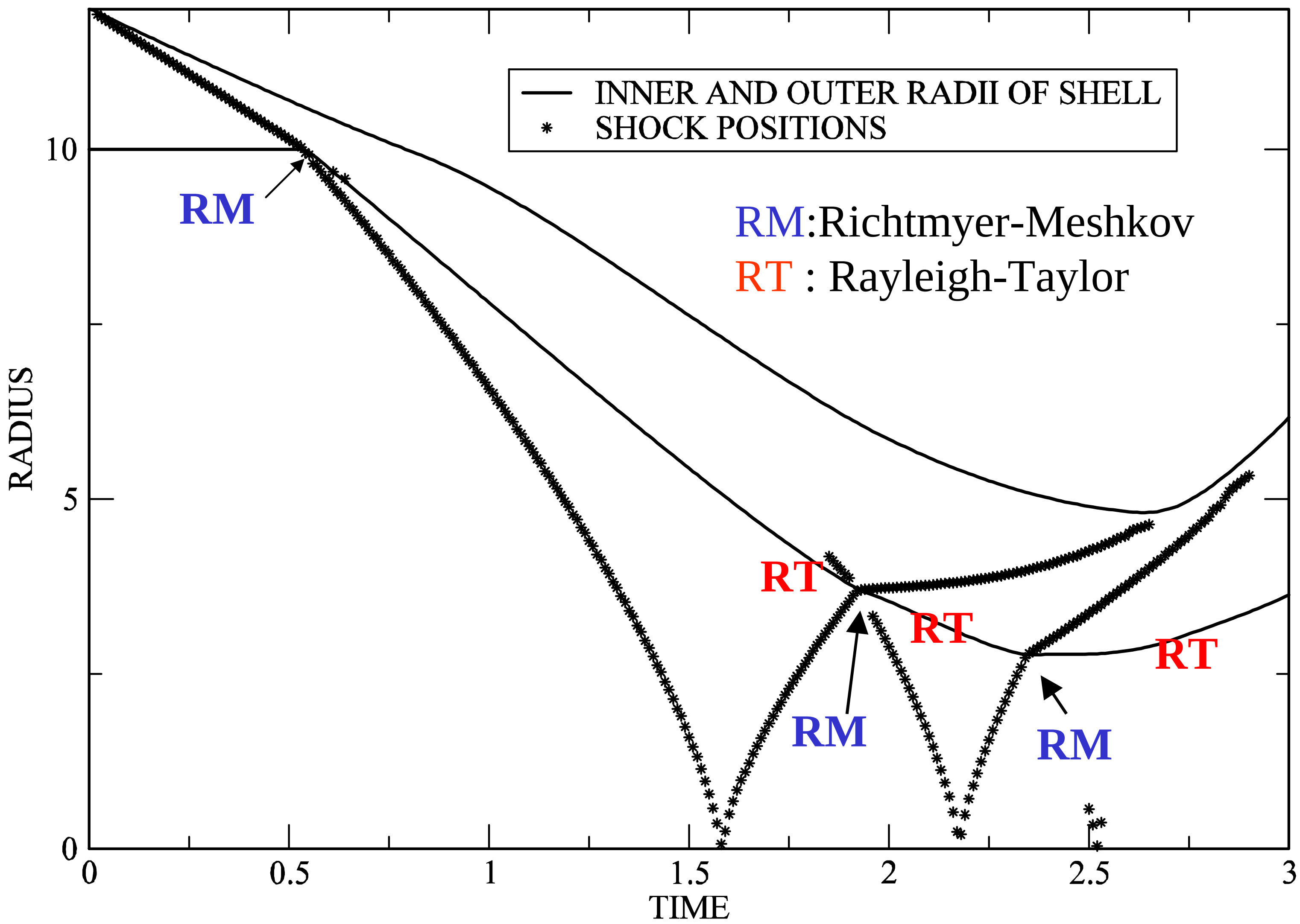
Outer radius= 10

Perfect gas equations of state $\gamma = 5/3$



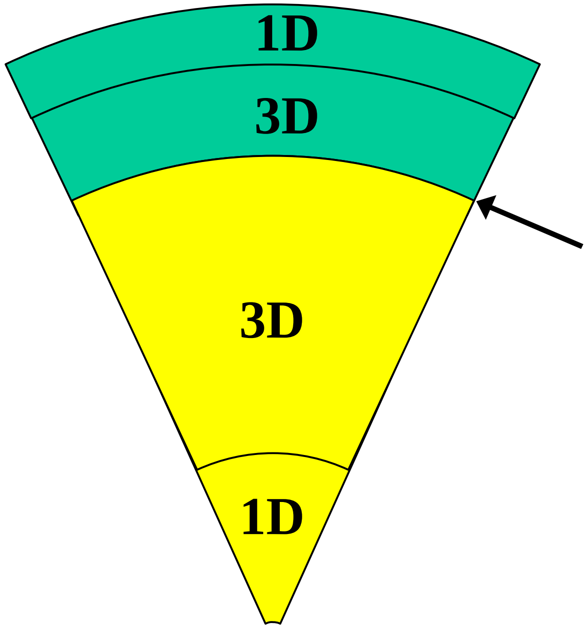
D.L.Youngs and R.J.R.Williams, (2008)

1D Lagrangian calculation

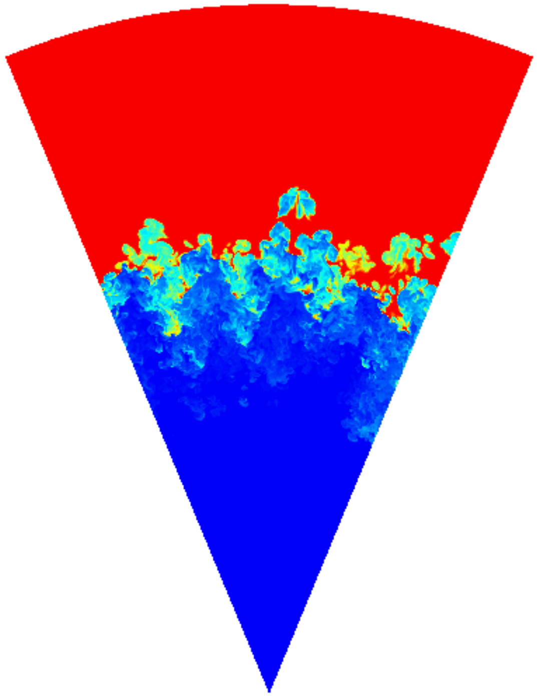


Note influence of initial conditions more complex: initial spectrum +amplification due to first shock + spherical convergence – set initial perturbations for late stage mixing

3D SIMULATION THE SPHERICAL IMPLOSION

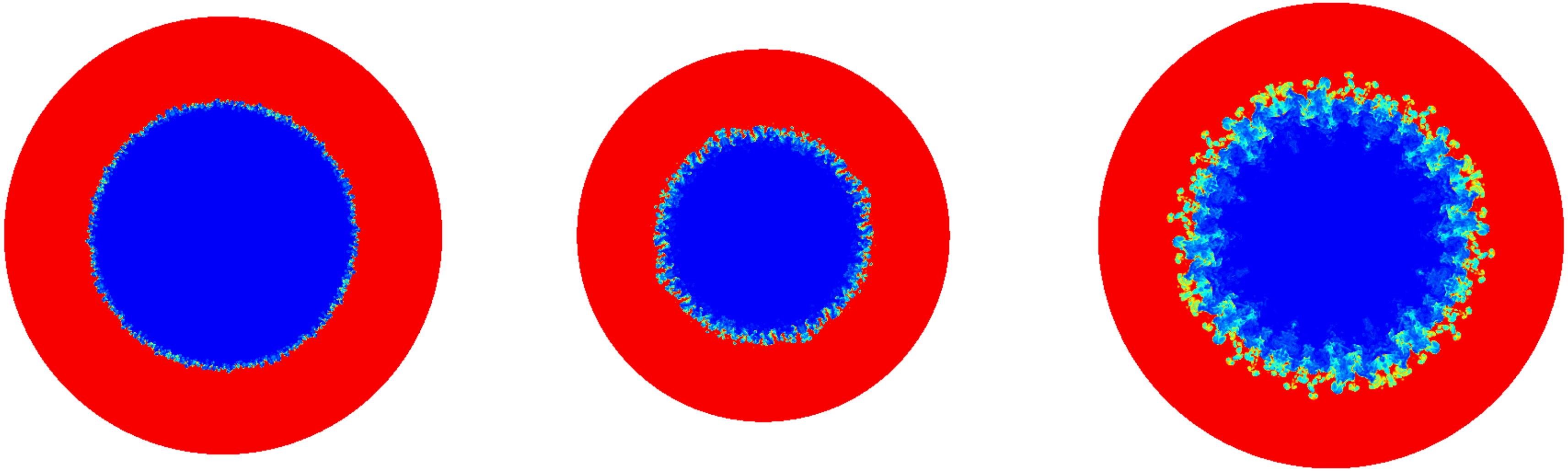


“square” patch perturbed by Fourier modes



Calculate sector $\frac{\pi}{2} - \frac{\pi}{8} < \theta, \phi < \frac{\pi}{2} + \frac{\pi}{8}$

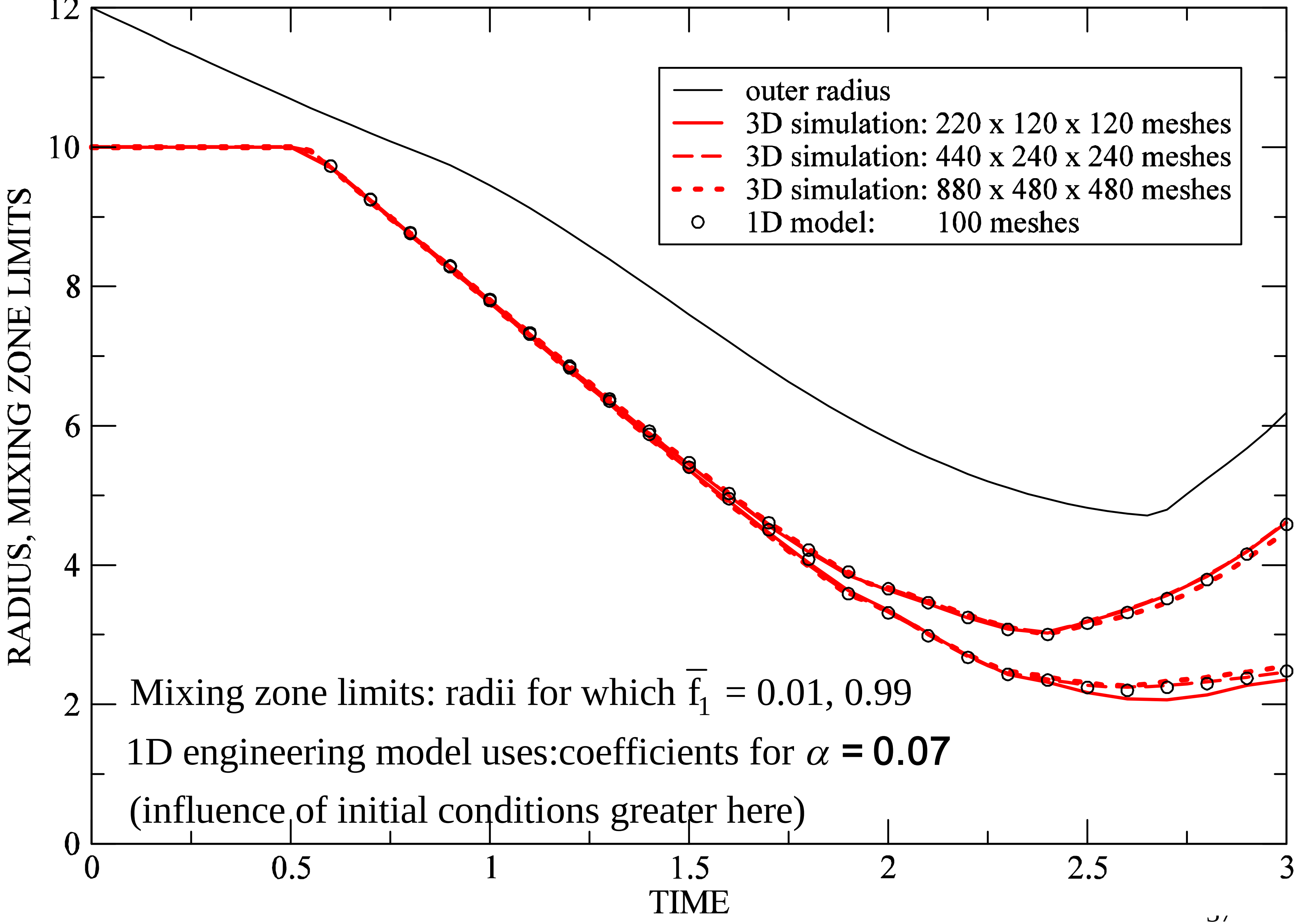
Spherical polar mesh, Lagrangian in r-direction, 1D Lagrangian regions at origin and at outer boundary.



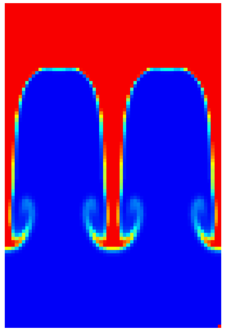
t = 2.0 t = 2.4(~max compression) t = 2.8

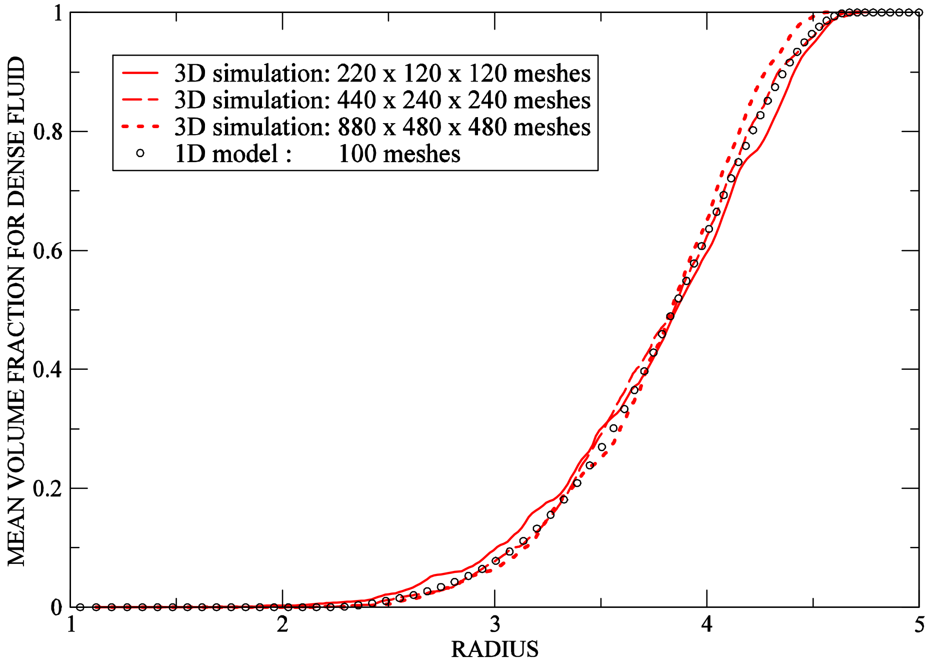
2D sections through the 3D simulation (standard mesh)

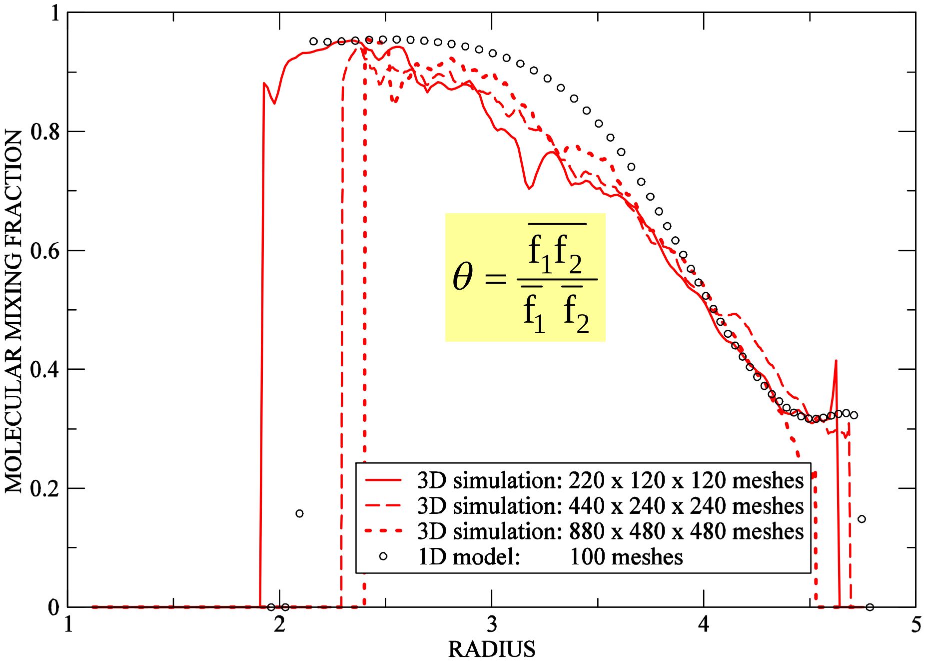
Comparison of 3D results with 1D model



Distributions at time=3







3D simulation for a simplified problem like this is used to "tune" the engineering model constants for a more complex application

CONCLUDING REMARKS

- During the last decade 3D simulation and more detailed experimental diagnostics has led to major advances in our understanding of RT mixing. Interesting range of self-similar cases for further investigation – enhanced growth due to longwavelength perturbations, variable acceleration, effect of density ratio.

- Influence of initial conditions is an extremely important issue. Needs to be allowed for both in the engineering modelling and comparison with experiment.

- The related process RM mixing has been less well studied via 3D simulation. For self-similar mixing $h = a(Ut)^{\theta}$, where U = change in interface velocity due to a shock. Many of the same issues arise as for RT mixing.

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QUESTIONS?

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