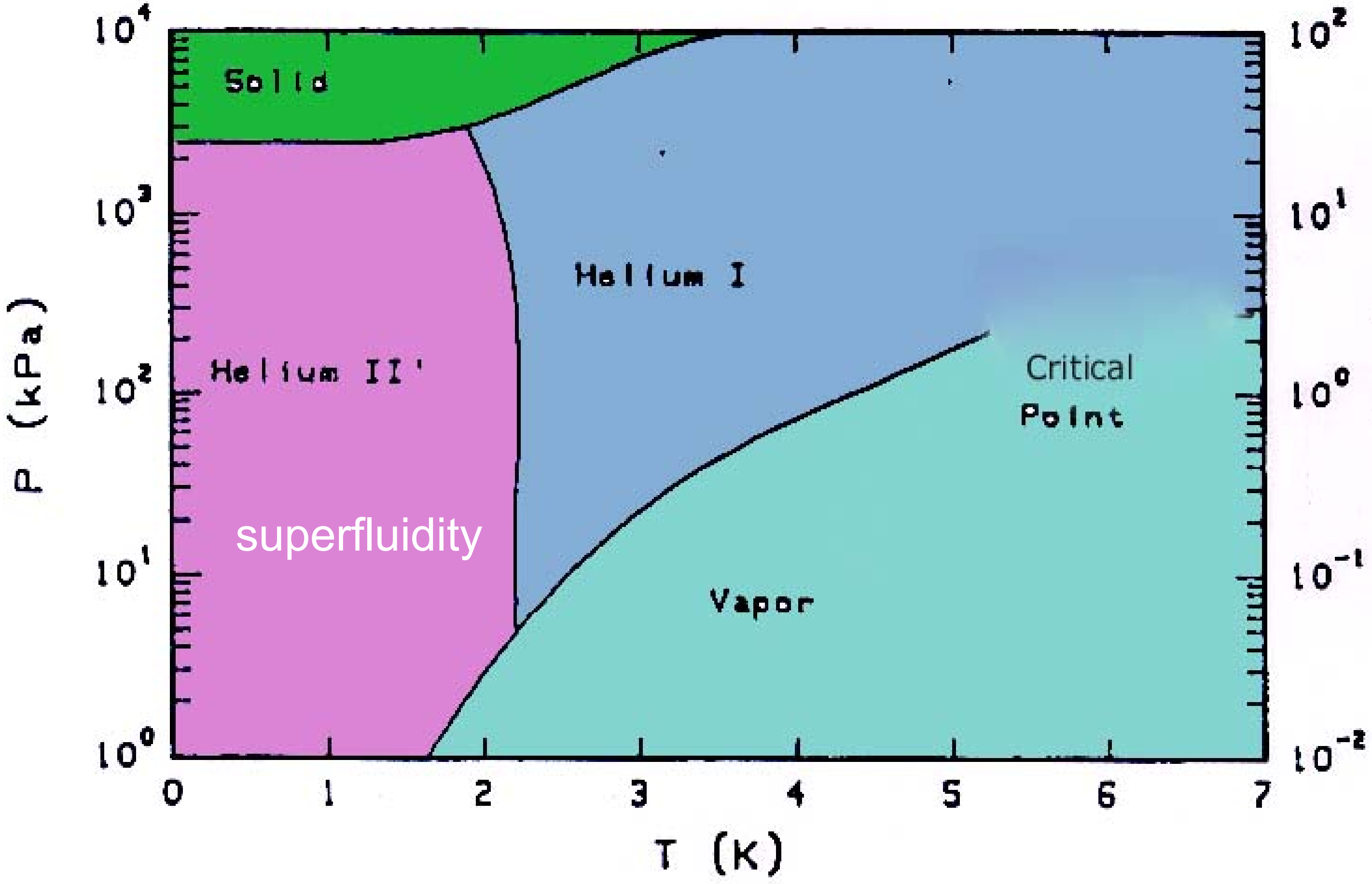
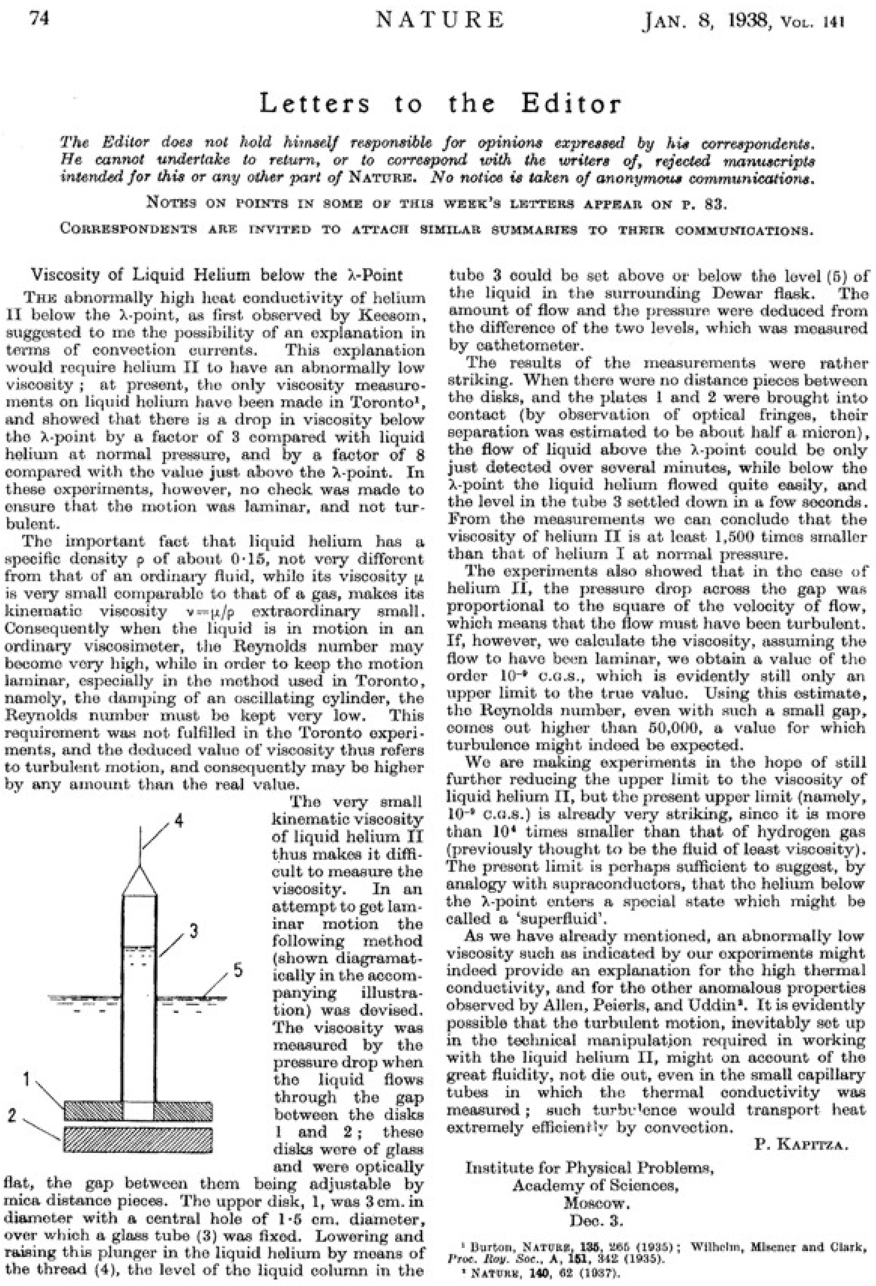
Vortices, particles and superfluidity

K.R. Sreenivasan



[Helium II] … possesses a number of peculiar properties, the most important of which is superfluidity discovered by P.L. Kapitza…

Landau (1941)



74 NATURE JAN. 8, 1938, VOL. 141

Letters to the Editor

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NOTES ON POINTS IN SOME OF THIS WEEK'S LETTERS APPEAR ON P. 83.

CORRESPONDENTS ARE INVITED TO ATTACH SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

Viscosity of Liquid Helium below the λ-Point

THE abnormally high heat conductivity of helium II below the λ-point, as first observed by Keesom, suggested to me the possibility of an explanation in terms of convection currents. This explanation would require helium II to have an abnormally low viscosity; at present, the only viscosity measurements on liquid helium have been made in Toronto[1], and showed that there is a drop in viscosity below the λ-point by a factor of 3 compared with liquid helium at normal pressure, and by a factor of 8 compared with the value just above the λ-point. In these experiments, however, no check was made to ensure that the motion was laminar, and not turbulent.

The important fact that liquid helium has a specific density ρ of about 0·15, not very different from that of an ordinary fluid, while its viscosity μ is very small comparable to that of a gas, makes its kinematic viscosity $\nu = \mu/\rho$ extraordinary small. Consequently when the liquid is in motion in an ordinary viscosimeter, the Reynolds number may become very high, while in order to keep the motion laminar, especially in the method used in Toronto, namely, the damping of an oscillating cylinder, the Reynolds number must be kept very low. This requirement was not fulfilled in the Toronto experiments, and the deduced value of viscosity thus refers to turbulent motion, and consequently may be higher by any amount than the real value.

The very small kinematic viscosity of liquid helium II thus makes it difficult to measure the viscosity. In an attempt to get laminar motion the following method (shown diagramatically in the accompanying illustration) was devised. The viscosity was measured by the pressure drop when the liquid flows through the gap between the disks 1 and 2; these disks were of glass and were optically flat, the gap between them being adjustable by mica distance pieces. The upper disk, 1, was 3 cm. in diameter with a central hole of 1·5 cm. diameter, over which a glass tube (3) was fixed. Lowering and raising this plunger in the liquid helium by means of the thread (4), the level of the liquid column in the tube 3 could be set above or below the level (5) of the liquid in the surrounding Dewar flask. The amount of flow and the pressure were deduced from the difference of the two levels, which was measured by cathetometer.

The results of the measurements were rather striking. When there were no distance pieces between the disks, and the plates 1 and 2 were brought into contact (by observation of optical fringes, their separation was estimated to be about half a micron), the flow of liquid above the λ-point could be only just detected over several minutes, while below the λ-point the liquid helium flowed quite easily, and the level in the tube 3 settled down in a few seconds. From the measurements we can conclude that the viscosity of helium II is at least 1,500 times smaller than that of helium I at normal pressure.

The experiments also showed that in the case of helium II, the pressure drop across the gap was proportional to the square of the velocity of flow, which means that the flow must have been turbulent. If, however, we calculate the viscosity, assuming the flow to have been laminar, we obtain a value of the order 10^{-9} c.g.s., which is evidently still only an upper limit to the true value. Using this estimate, the Reynolds number, even with such a small gap, comes out higher than 50,000, a value for which turbulence might indeed be expected.

We are making experiments in the hope of still further reducing the upper limit to the viscosity of liquid helium II, but the present upper limit (namely, 10^{-9} c.g.s.) is already very striking, since it is more than 10^{4} times smaller than that of hydrogen gas (previously thought to be the fluid of least viscosity). The present limit is perhaps sufficient to suggest, by analogy with supraconductors, that the helium below the λ-point enters a special state which might be called a 'superfluid'.

As we have already mentioned, an abnormally low viscosity such as indicated by our experiments might indeed provide an explanation for the high thermal conductivity, and for the other anomalous properties observed by Allen, Peierls, and Uddin[2]. It is evidently possible that the turbulent motion, inevitably set up in the technical manipulation required in working with the liquid helium II, might on account of the great fluidity, not die out, even in the small capillary tubes in which the thermal conductivity was measured; such turbulence would transport heat extremely efficiently by convection.

P. KAPITZA.

Institute for Physical Problems,
Academy of Sciences,
Moscow.
Dec. 3.

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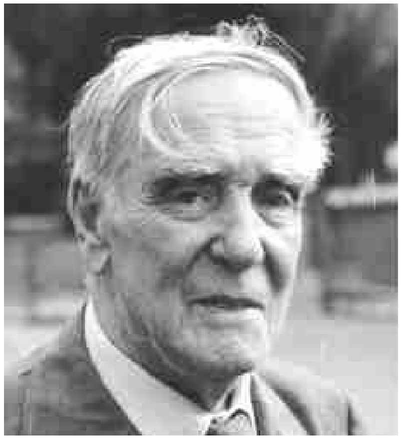
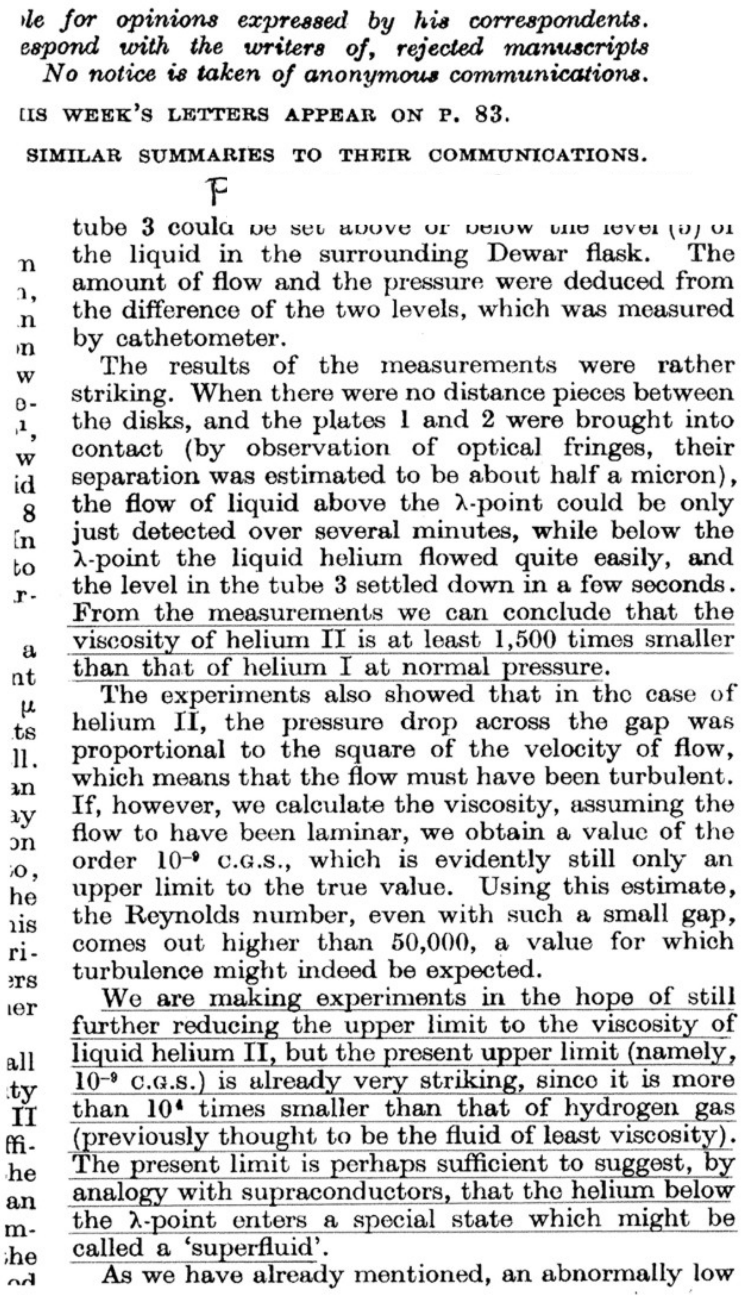
[1] Burton, NATURE, **135**, 265 (1935); Wilhelm, Misener and Clark, *Proc. Roy. Soc.*, A, **151**, 342 (1935).

[2] NATURE, **140**, 62 (1937).



Kapitza: 1894-1984



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IS WEEK'S LETTERS APPEAR ON P. 83.

SIMILAR SUMMARIES TO THEIR COMMUNICATIONS.

F

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As we have already mentioned, an abnormally low

Submitted
Dec 3, 1937
Published
Jan 8, 1938

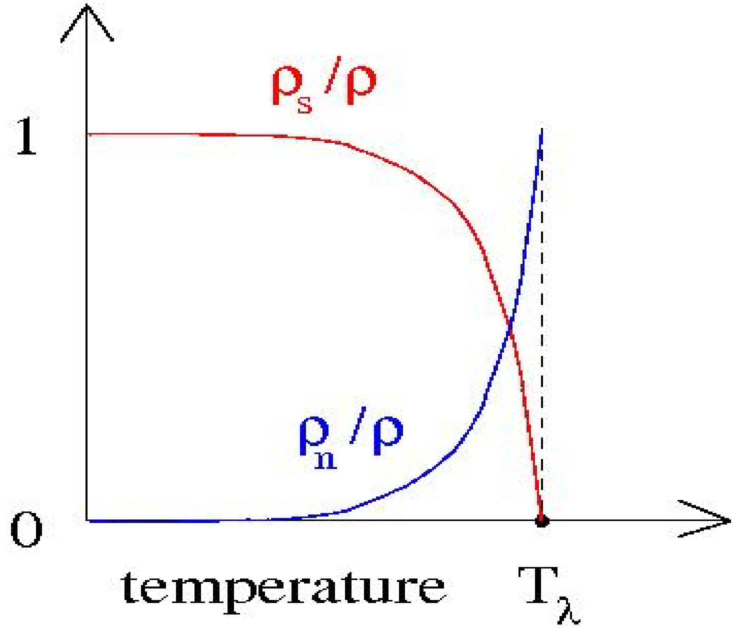
The two-fluid model

Superfluid: quantum ground state, density ρ_s, velocity v_s
no viscosity, no entropy, Euler fluid

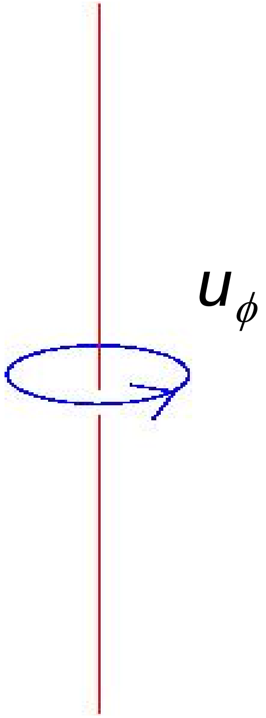
Normal fluid: thermal excitations, density ρ_n, velocity $v_{n,}$
carries viscosity and entropy, like a Navier Stokes fluid

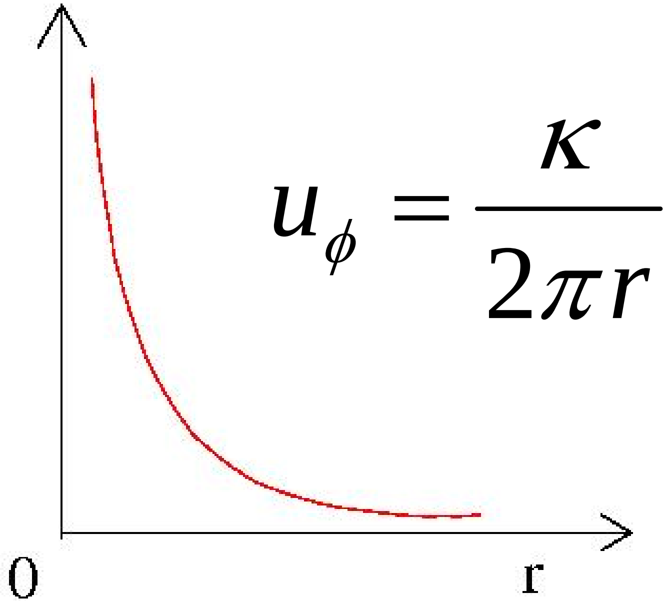
Landau: 1908-1968

Total density
$\rho=\rho_s+\rho_n$



vortex lines in helium II





$$u_\phi = \frac{\kappa}{2\pi r}$$

Onsager
1903-1976

"Thus, the well-known invariant called hydrodynamic circulation is quantized; the quantum of circulation is h/m."

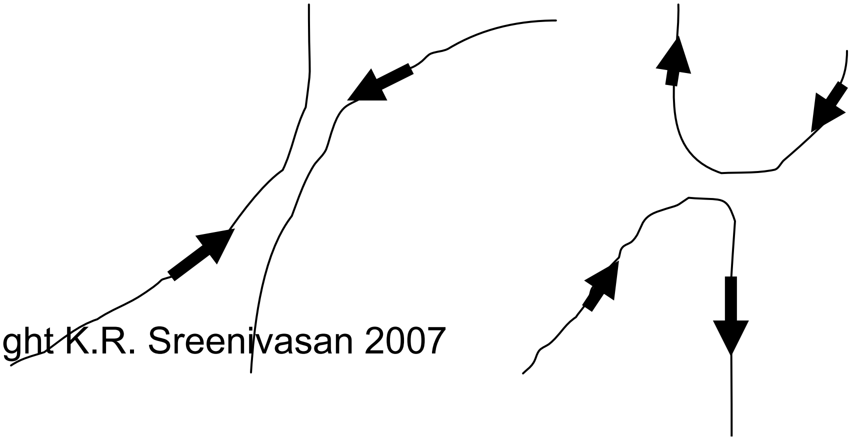
Onsager (1949)

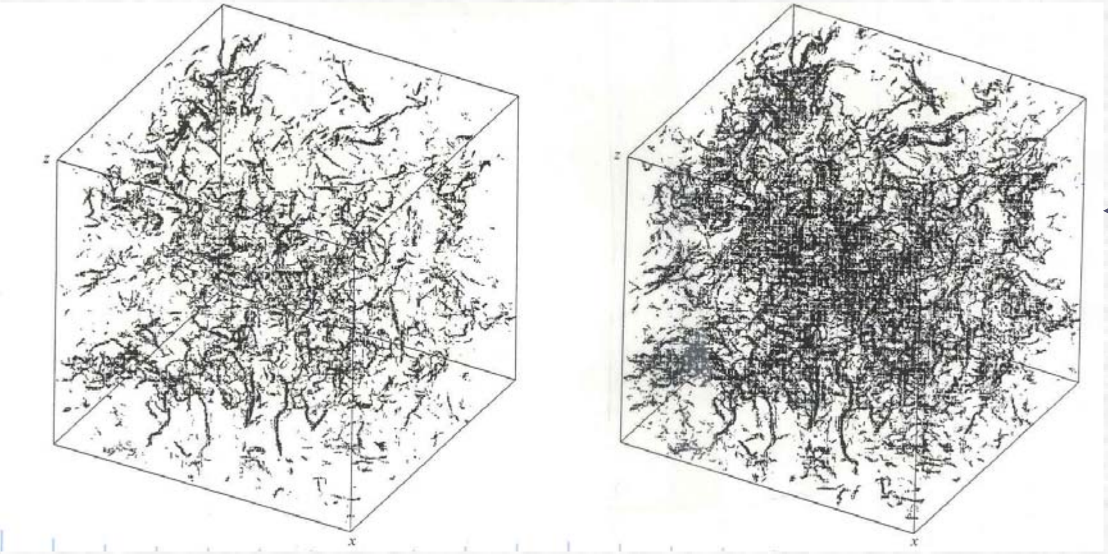
Except for a few angstroms from the center of the core, the laws obeyed are those of classical hydrodynamics [e.g., Biot-Savart].

R.P. Feynman: 1918-1988

Prog. Low Temp. Phys. **1, 17 (1955)**

If … two oppositely directed sections of [vortex] line approach closely, … the lines (which are under tension) may snap together and join connections a new way …





High-intensity vortex structures in homogeneous and isotropic turbulence (Vincenti & Meneguzzi 1991)

Vortex tangles (“**superfluid turbulence**”) by Tsubota, Araki & Nemirovskii 2000); pioneering simulations by K.W. Schwarz (1985)

Microscopic details of reconnection were explored by J. Koplik and H. Levine, *Phys. Rev. Lett.*, **71**, 1375 (1993), by solving the nonlinear Schrödinger equation with quadratic nonlinearity – which is a good model for wavefunction in BEC.

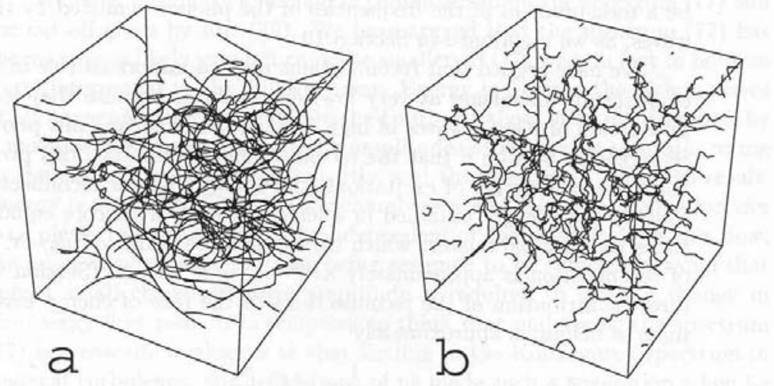
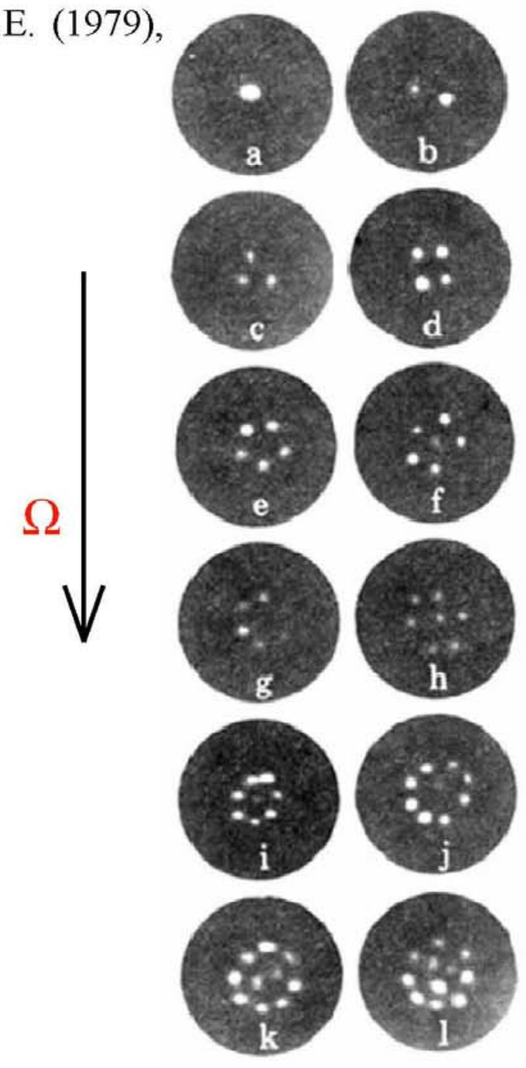


Fig. 10. Vortex tangles at (a) $T = 1.6$K and (b) $T = 0$K. From Tsubota *et al*[50].

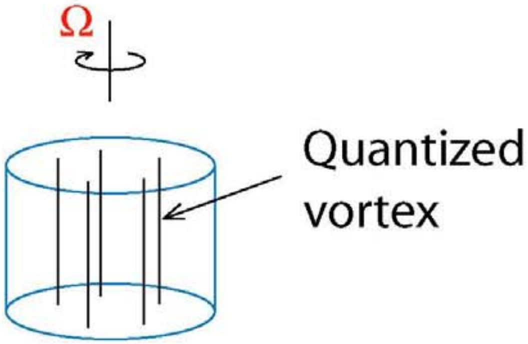
not observed until now experimentally

Previous observation of quantized vortices

Yarmchuk, E.J., Gordon, M.J.V. and Packard, R.E. (1979),
Phys. Rev. Lett. **43**, 214-217.



technique not suitable for visualizing tangled vortices



What kind of particles?


Greg Bewley

PIV using neutral particles of frozen mixtures of helium and hydrogen, of the order of a micron in size

Bewley, Lathrop & S, *Nature* **441**, 558 (2006); *Experiments in Fluids* (to appear, 2007)

Bewley, Poaletti, S and Lathrop, *PNAS* (submitted), 2007

A brief history of particle development for cryogenic helium

1. Chopra & Brown (1957)

Hydrogen and deuterium mixture into liquid helium through a heated nozzle, mm size particles which stick to end of walls

2. Chung & Crtichlow (1965)

Particle of the size of a few hundred microns

3. Kitchens, Steyert & Taylor (1965)

20-100 μm particles

4. Murakami & Ichikawa (1989) and Nakano & Murakami (1992)

Hydrogen particles of a few microns in size; somewhat uncontrolled; used for LDV

5. White, Karpetis & S (2002) and Donnelly, Karpetis, Niemela, S, Vinen, White (2002)

Polydispersed hollow glass spheres, using sedimentation to select particles of the size of a few microns for PIV measurements

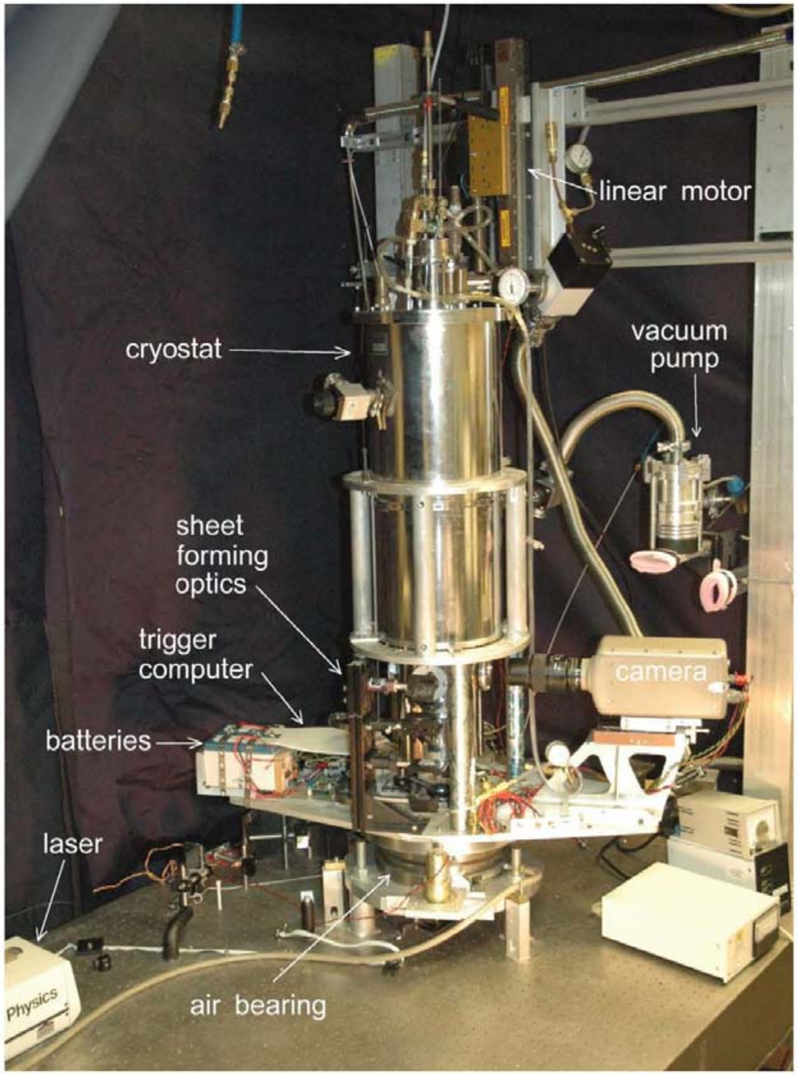
6. Celik & Van Sciver (2002); Zhang & Van Sciver (2005)

10μm particles; also micron sized particles for which clustering is a big problem

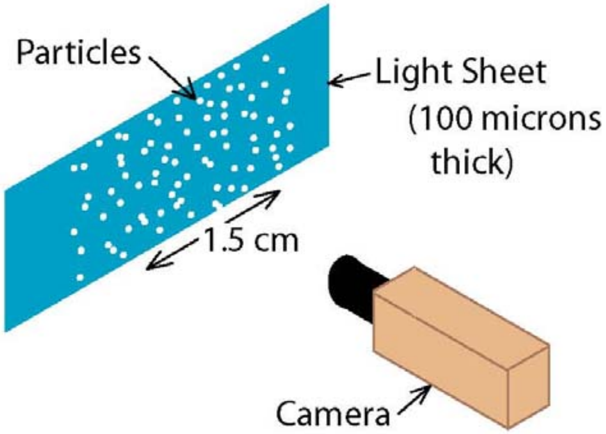
7. Boltnev, Frossati, Gordon, Krushinskaya, Popov & Usenko (2002)

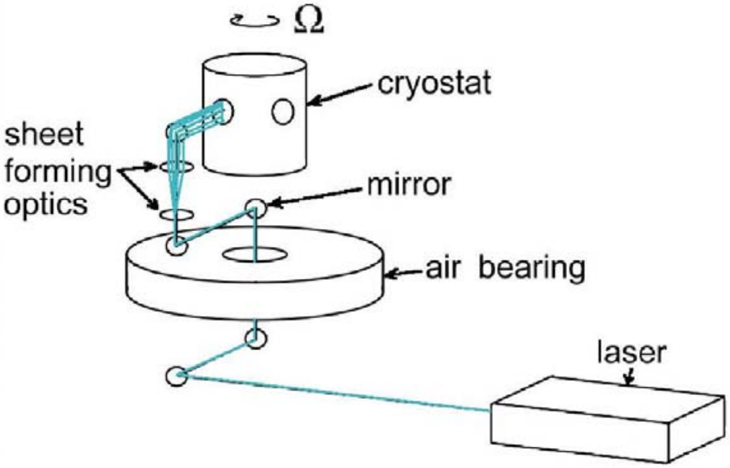
Dilution of hydrogen with large amounts of helium; can yield submicron particles

Apparatus



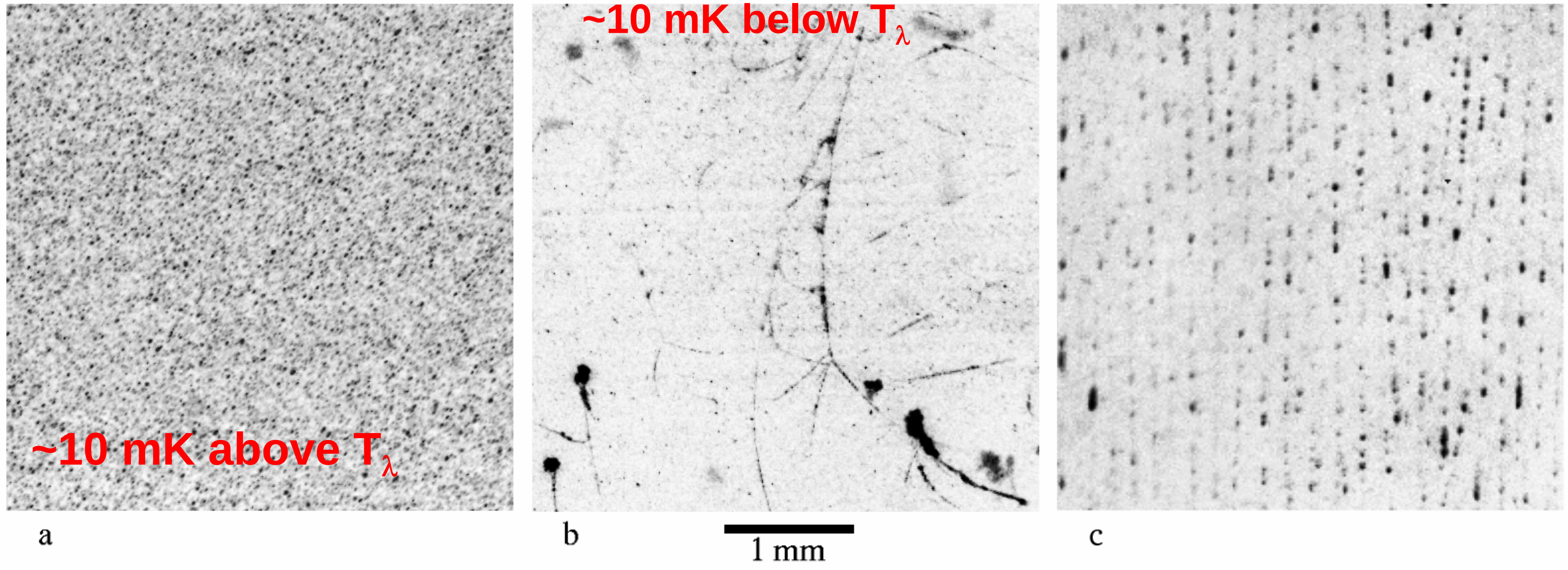
(with advice from W.F. Vinen, J.J. Niemela)





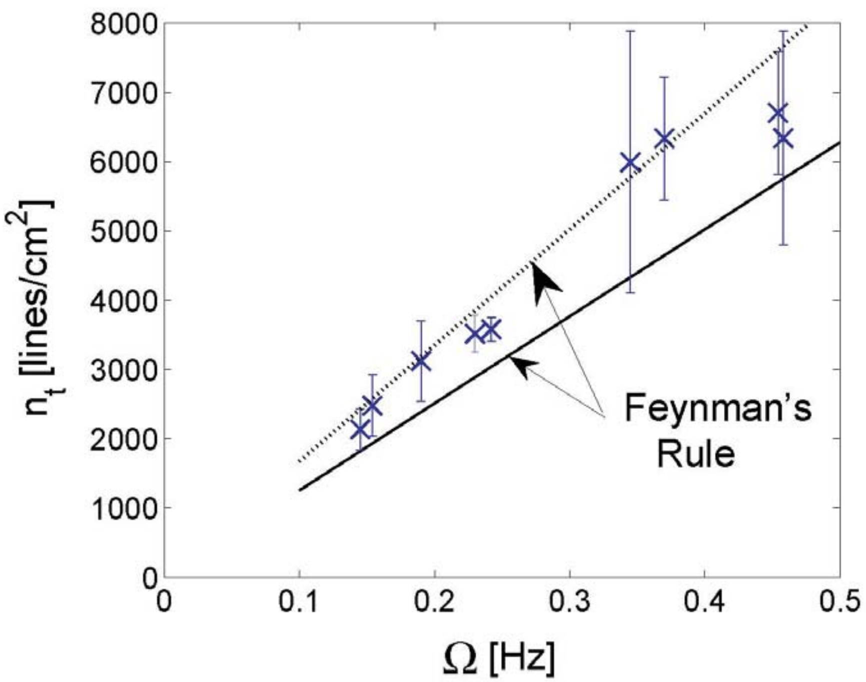
Laser Beam routing

50 years later…

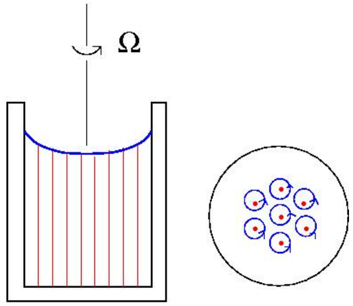


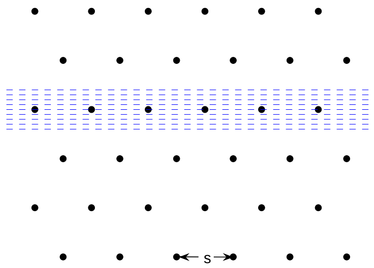
Panel (a) shows a suspension of hydrogen particles just above the transition temperature. Panel (b) shows similar hydrogen particles after the fluid is cooled below the lambda point. Some particles have collected along branching filaments, while other are randomly distributed as before. Fewer free particles are apparent in (b) only because the light intensity is reduced to highlight the brighter filaments in the image. Panel (c) shows an example of particles arranged along vertical lines when the system is rotating steadily about the vertical axis. The spacing of lines is remarkably uniform, although there are occasional distortions of the lattice and possible points of intersection. G.P. Bewley, D.P. Lathrop & K.R.S., *Nature* **441**, 558 (2006). Volume fraction $\cong 3\text{X}10^{-5}$

Lattice density

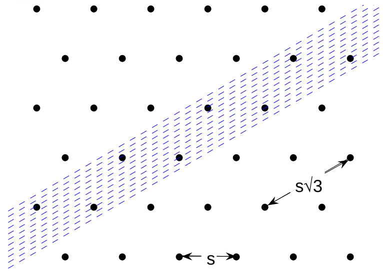


Feynman's rule
$n_t \cong 2000\Omega$

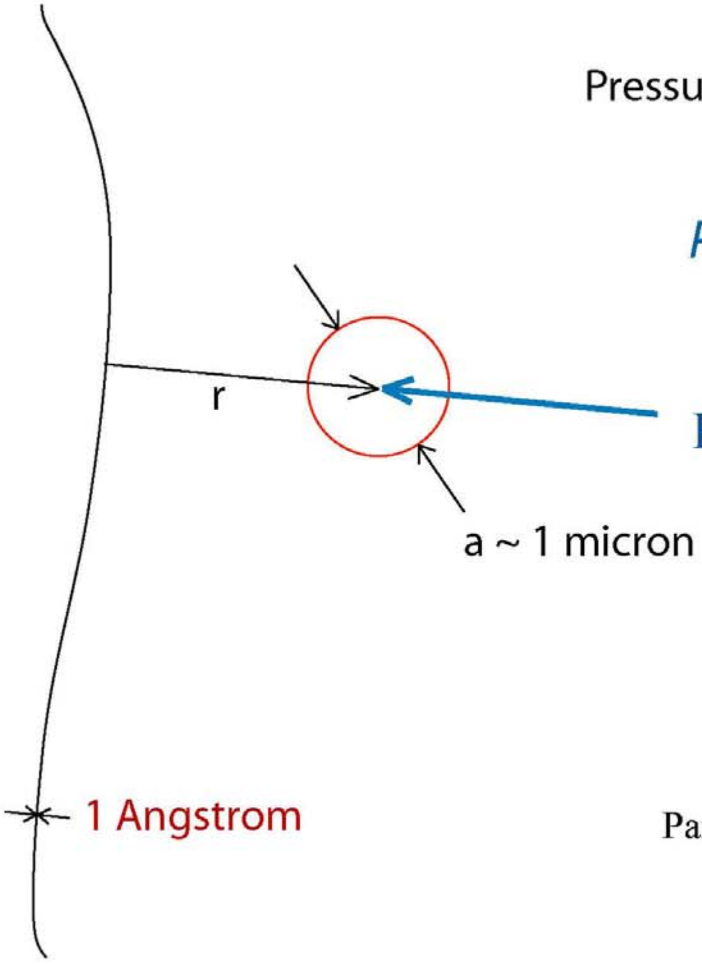




$n_o \approx 2000\Omega$ lines/cm^2



Particle Trapping



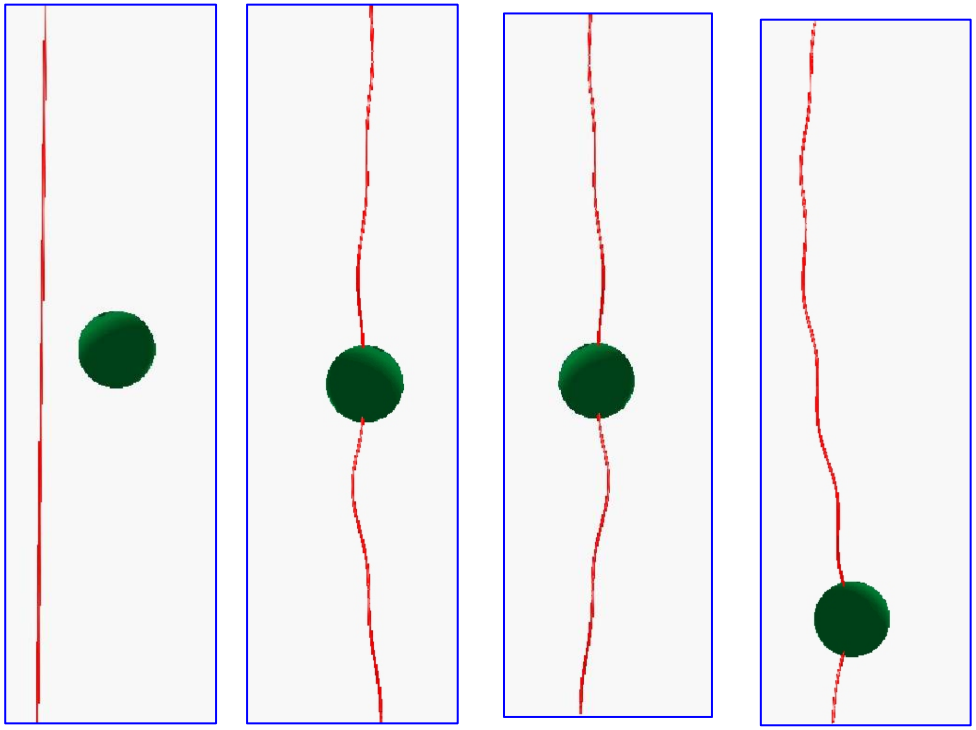
Pressure gradient:

$$P = \frac{-\rho_s \kappa^2}{8\pi^2 r^2}$$

$$F = \frac{4}{3}\pi a^3 \nabla P \sim \frac{1}{r^3}$$

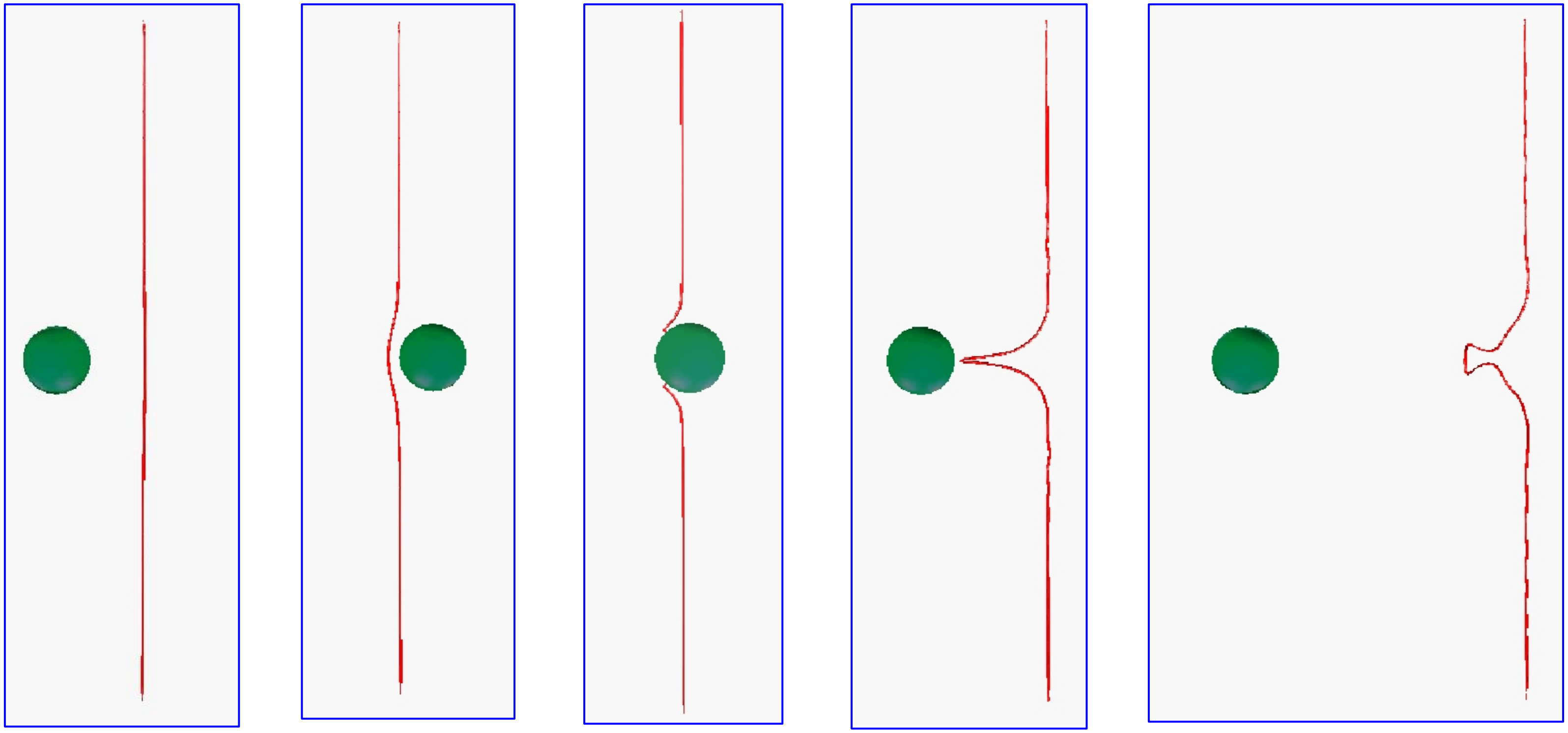
Parks, P.E. and Donnelly, R.J. (1966), *Phys. Rev. Lett.* **16**, 45–48.

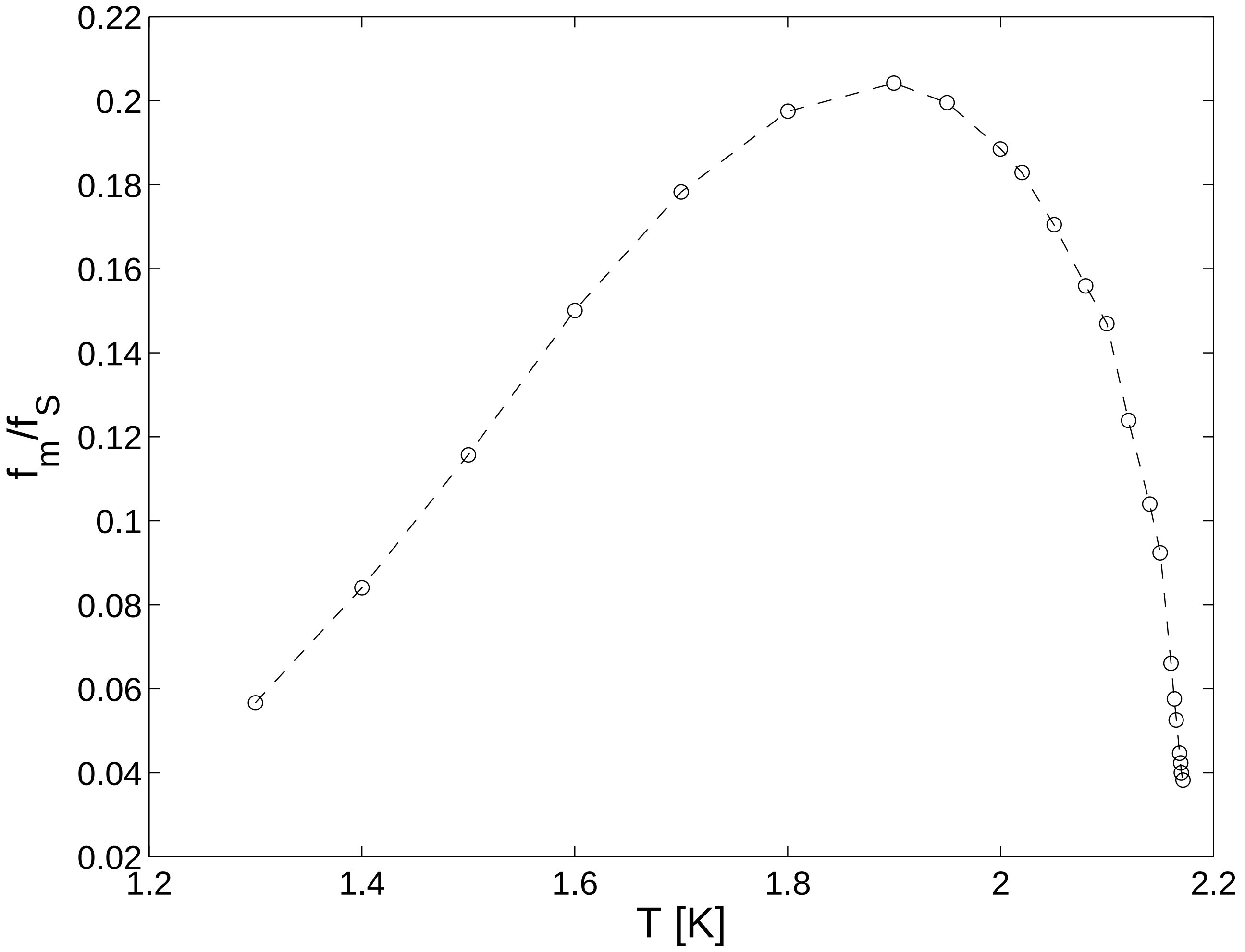
sphere is trapped by vortex



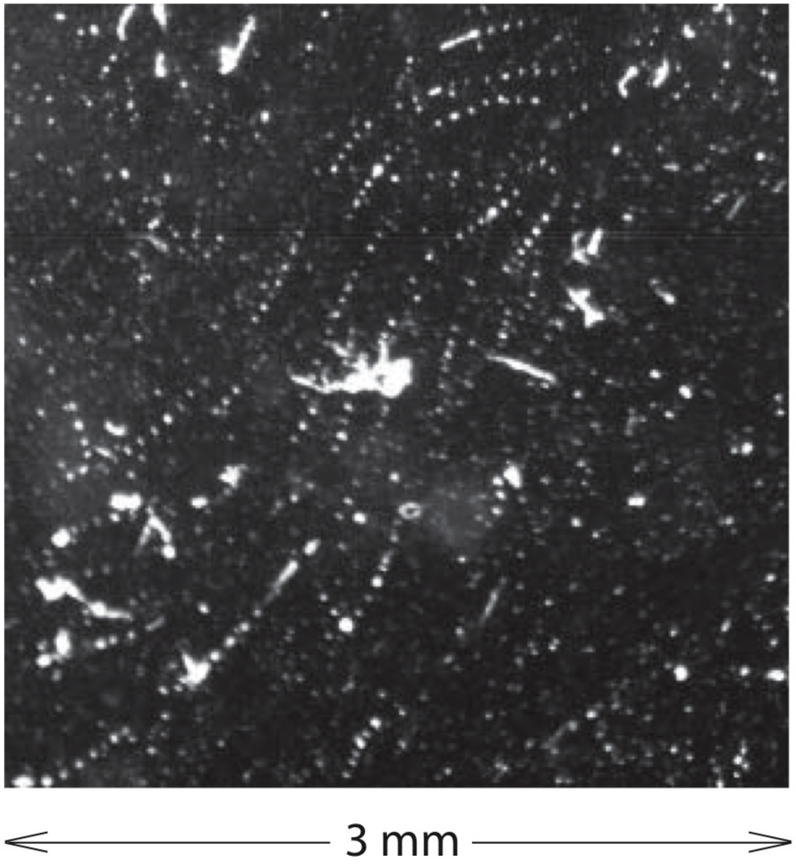
For a discussion of interaction between the fluid and particles in He II, see Sergeev, Barenghi & Kivotides, *Phys. Rev. B* **74**,184506 (2006); the simulations shown are by these authors.

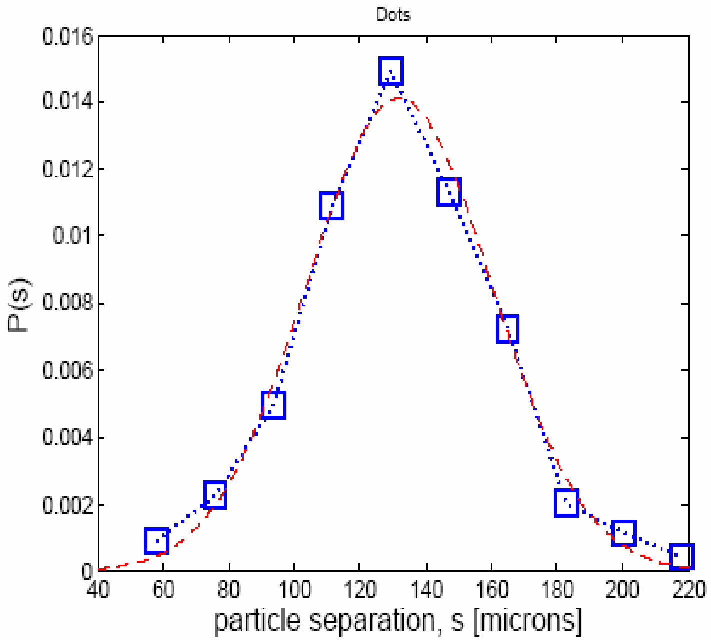
sphere escapes vortex



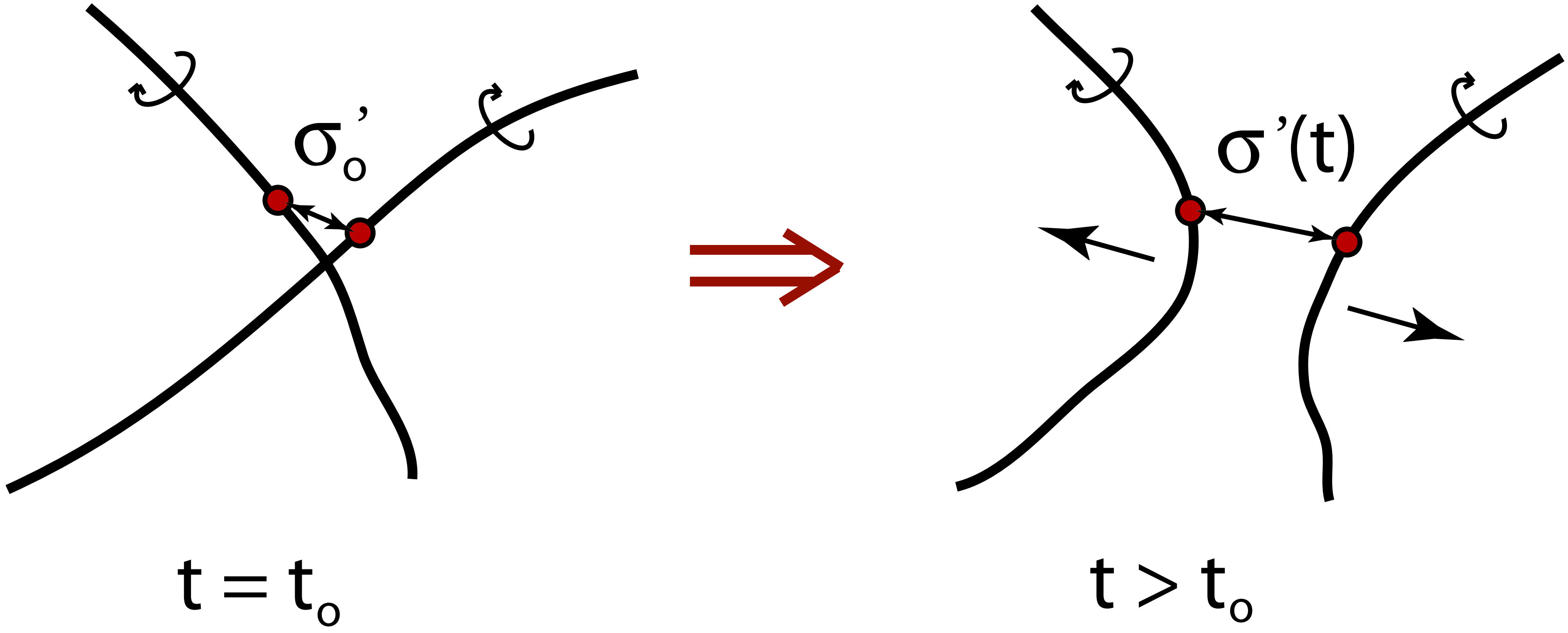


The ratio of the mutual friction force per unit length of a vortex to the drag on a particle trapped on the line. At about 2.17 K, the particle drag is equal to mutual friction if neighboring particles are about ten diameters apart.

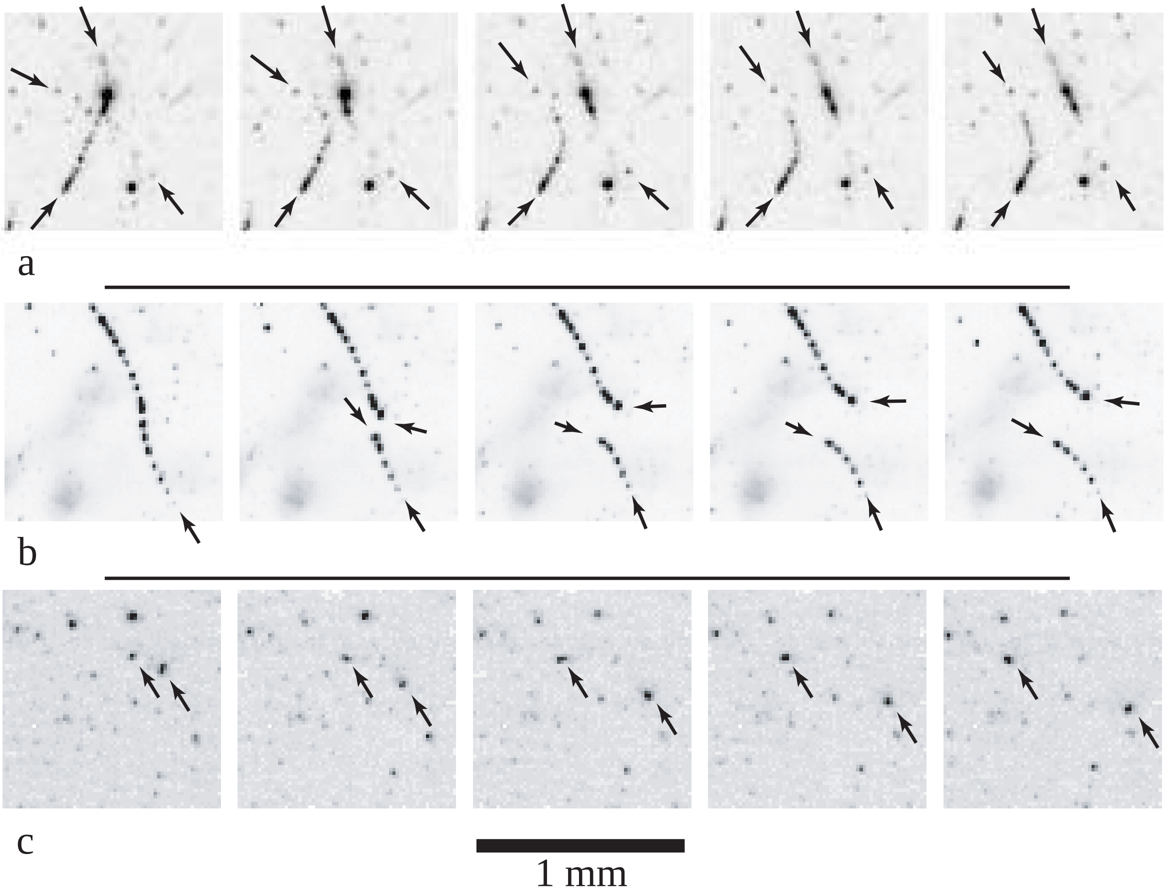




~100 diameters apart,
a few percent effect!

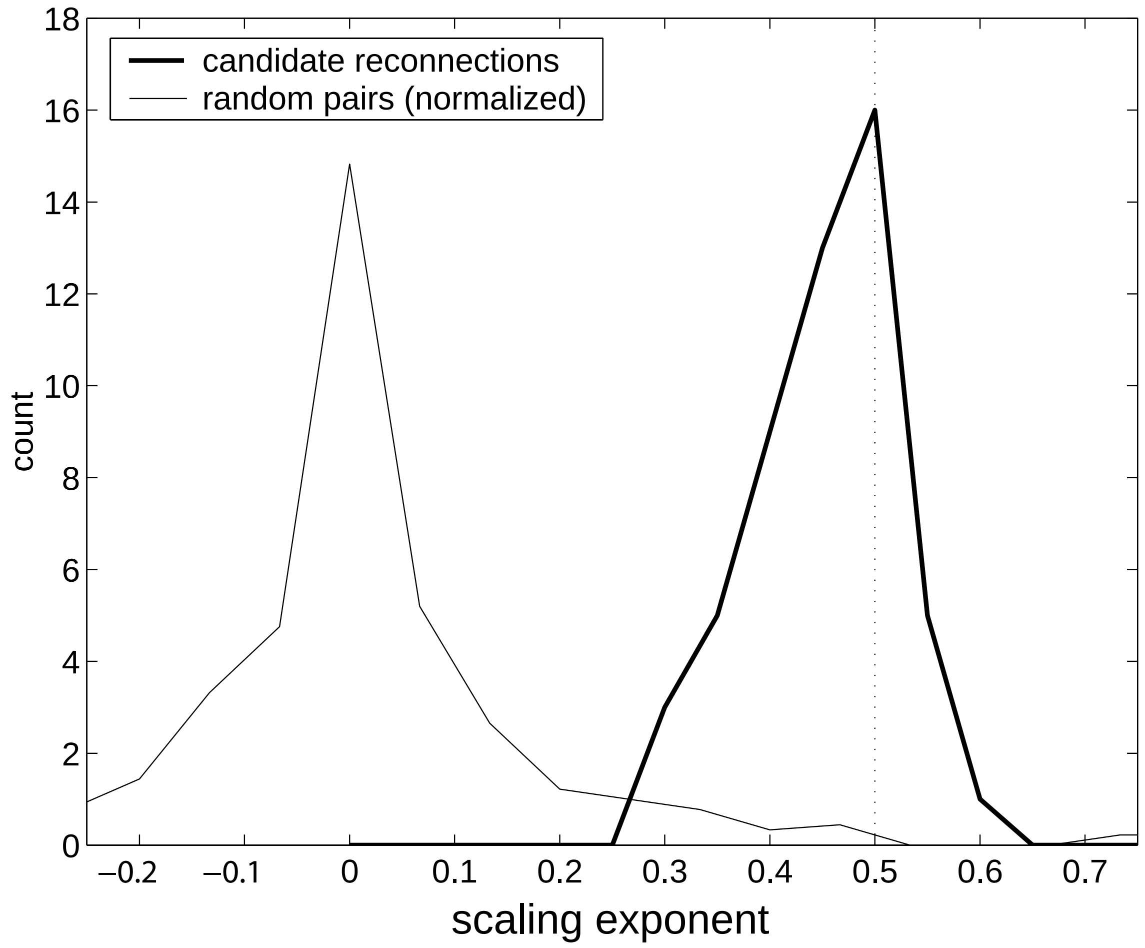


The cores of reconnecting vortices at the moment of reconnection, t_0, and after reconnection, $t > t_o$. The small circles mark the positions of particles trapped on the cores of the vortices. The arrows indicate the motion of the vortices and particles. We measure the distance between two particles at different times. Critical element in numerical simulations; for the classical case, see Kerr & Hussain, *Physica* D **37**, 474 (1989).



Each series of frames in (a), (b) and (c) are images of hydrogen particles suspended in liquid helium, taken at 50 *ms* intervals. Some of the particles are trapped on quantized vortex cores, while others are randomly distributed in the fluid. Before reconnection, particles drift collectively with the background flow in a configuration similar to that shown in the first frames of (a), (b) and (c). Subsequent frames show reconnection as the sudden motion of a group of particles. In (a), both vortices participating in the reconnection have several particles along their cores. In projection, the approaching vortices in the first frame appear crossed. In (b), particles make only one vortex visible, the other vortex probably has not yet trapped any particles. In (c), we infer the existence of a pair of reconnecting vortices from the sudden motion of pairs of particles recoiling from each other.

movie

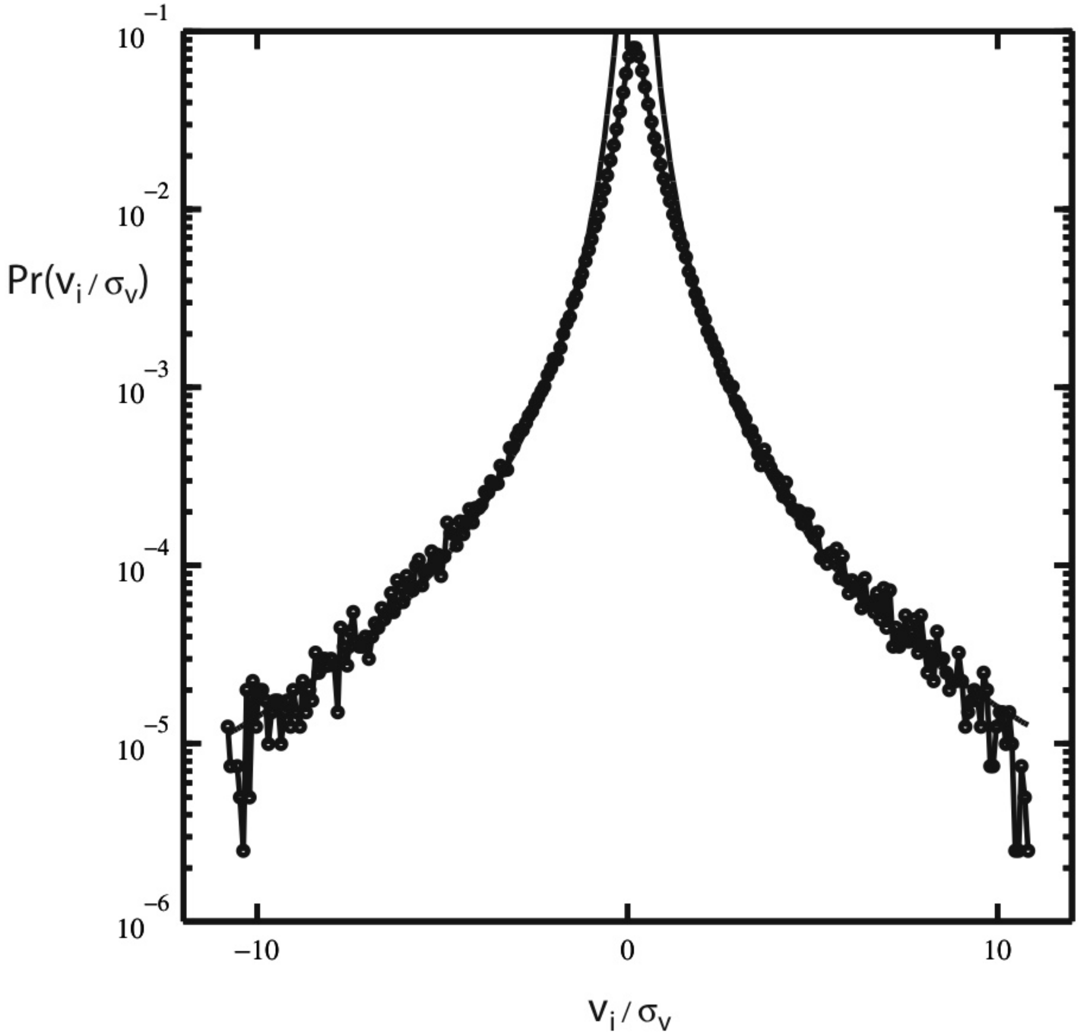


$\sigma'(t) \sim (t-t_0)^{\alpha}$

dimensional analysis
$\alpha = 0.5$

Histogram of the scaling exponents for the data in previous figure, as well as those found for randomly chosen particle trajectories. We normalized the histogram for the random-pair data to have the same area beneath it as the histogram for the experimental data. The mean value of the scaling exponents for the candidate reconnections is ½, as indicated with a vertical dotted line.

Probability distribution of velocity (v_x and v_z combined)



from AR06 by Paoletti

$$P(v)dv = P(t)dt$$

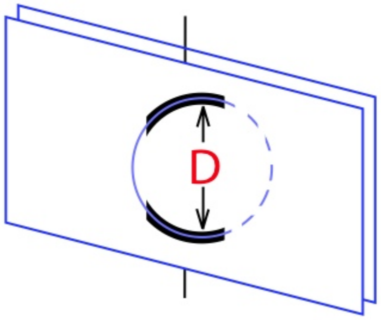
$$v = \kappa\,(t\text{-}t_o)^{\alpha}$$

P(v

$$v = \kappa(t - t_0)^{0.5}$$

$$p(v) \sim |v|^{-3}$$

T = 2.05 K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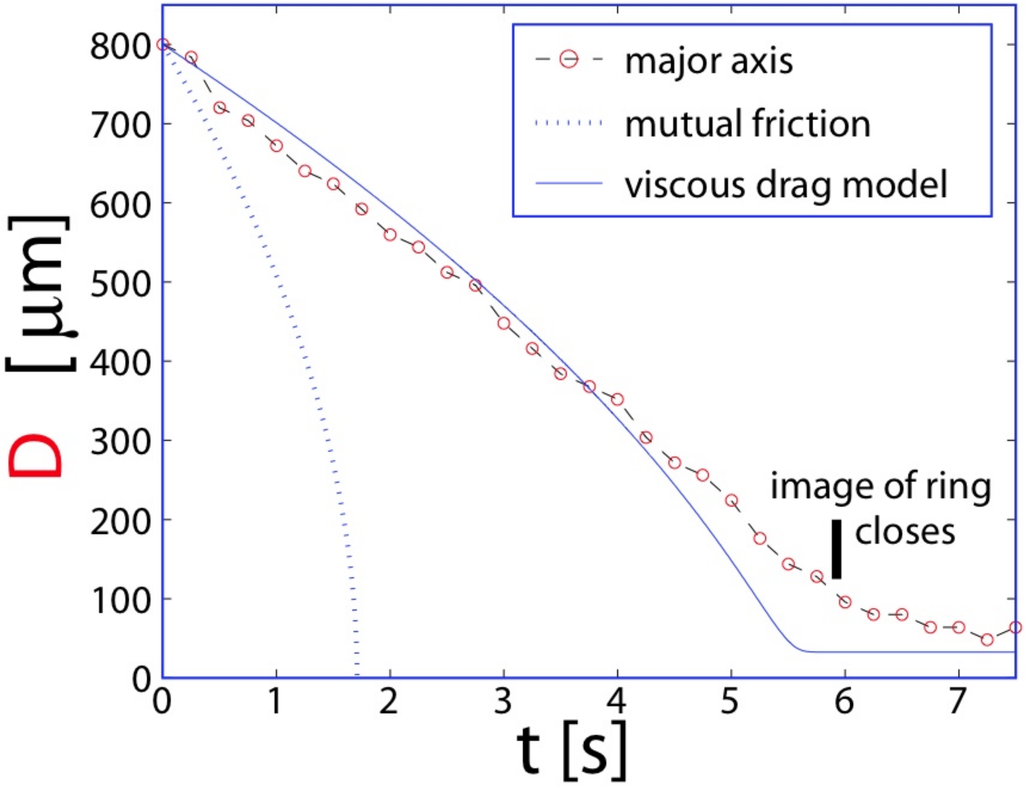
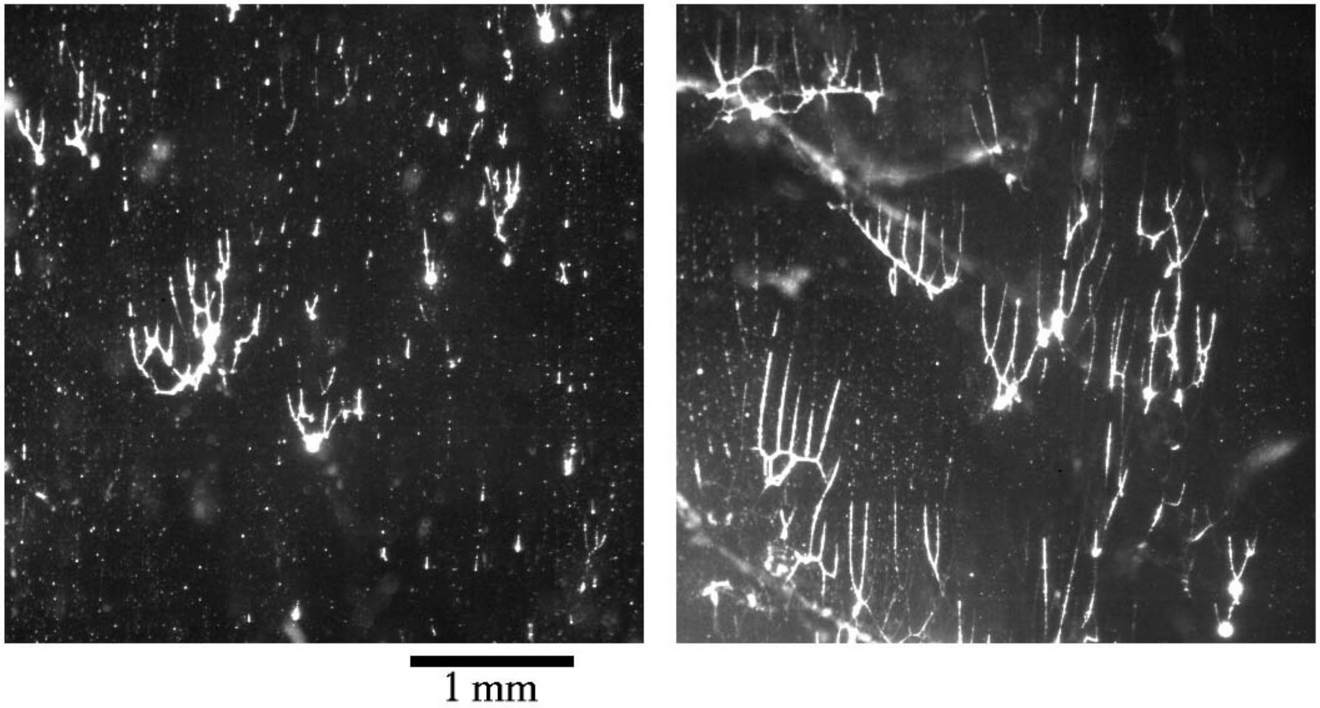
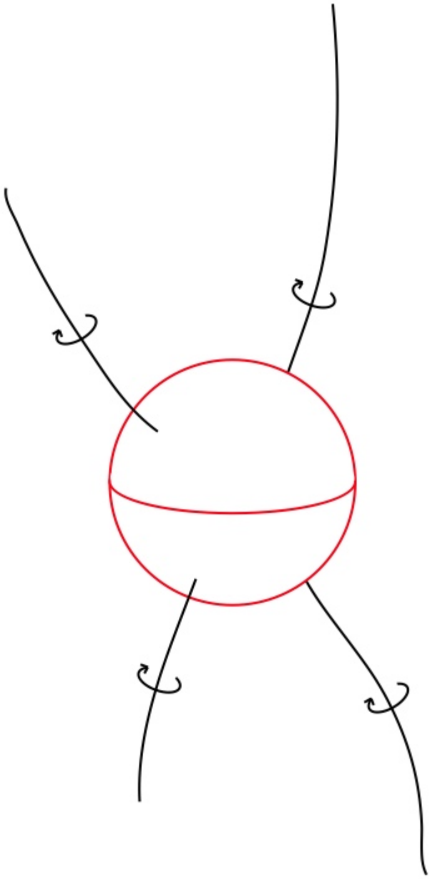

D [μm]
t [s]
0
100
200
300
400
500
600
700
800
0
1
2
3
4
5
6
7
major axis
mutual friction
viscous drag model
image of ring closes

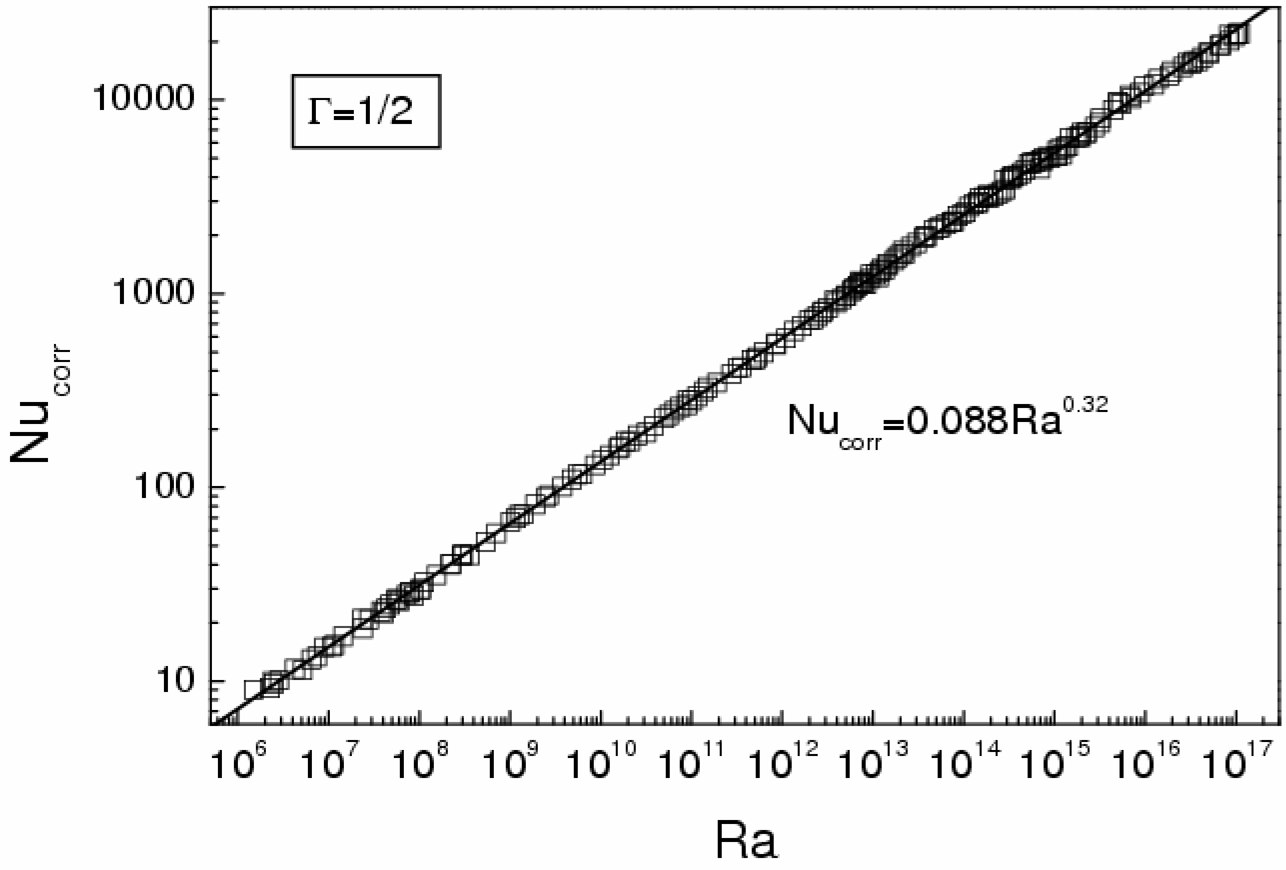
Particles are not always passive tracers!

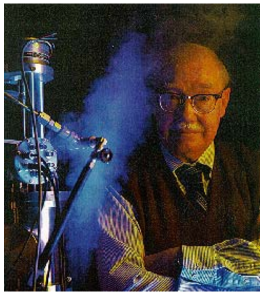




Summary remarks

1. Using helium as working fluid, it has been possible to extend parameter regimes of classical fluid dynamic experiments as never before.
2. Using neutral particles of hydrogen-helium mixture, it has been possible to visualize superfluid vortex lines and rings, and study their properties such as reconnection and decay.
3. These particles are not always passive so there is more work to be done. Interesting problems of particle-vortex interactions need to be studied further.





Niemela, Skrbek, S & Donnelly, *Nature* **404**, 837 (2000)
Slightly revised: Niemela & S, *J. Low Temp. Phys.* **143**, 163 (2006)

[Pioneers: Threlfall (Cambridge); Libchaber, Kadanoff and coworkers (Chicago)]

Latest theoretical bound for the exponent (X. Wang, 2007): 1/3 for Pr/Ra = O(1)

(Previous work on bounds: Howard, Malkus, Constantin, Doering, …)

What next?

1. Dynamics of superfluid turbulence
(with Donnelly, Niemela, Skrbek, Vinen)

2. Superfluidity of He3 and turbulence
3. Turbulence of condensates

Thank you for your attention