Flow of dense granular media
A peculiar liquid

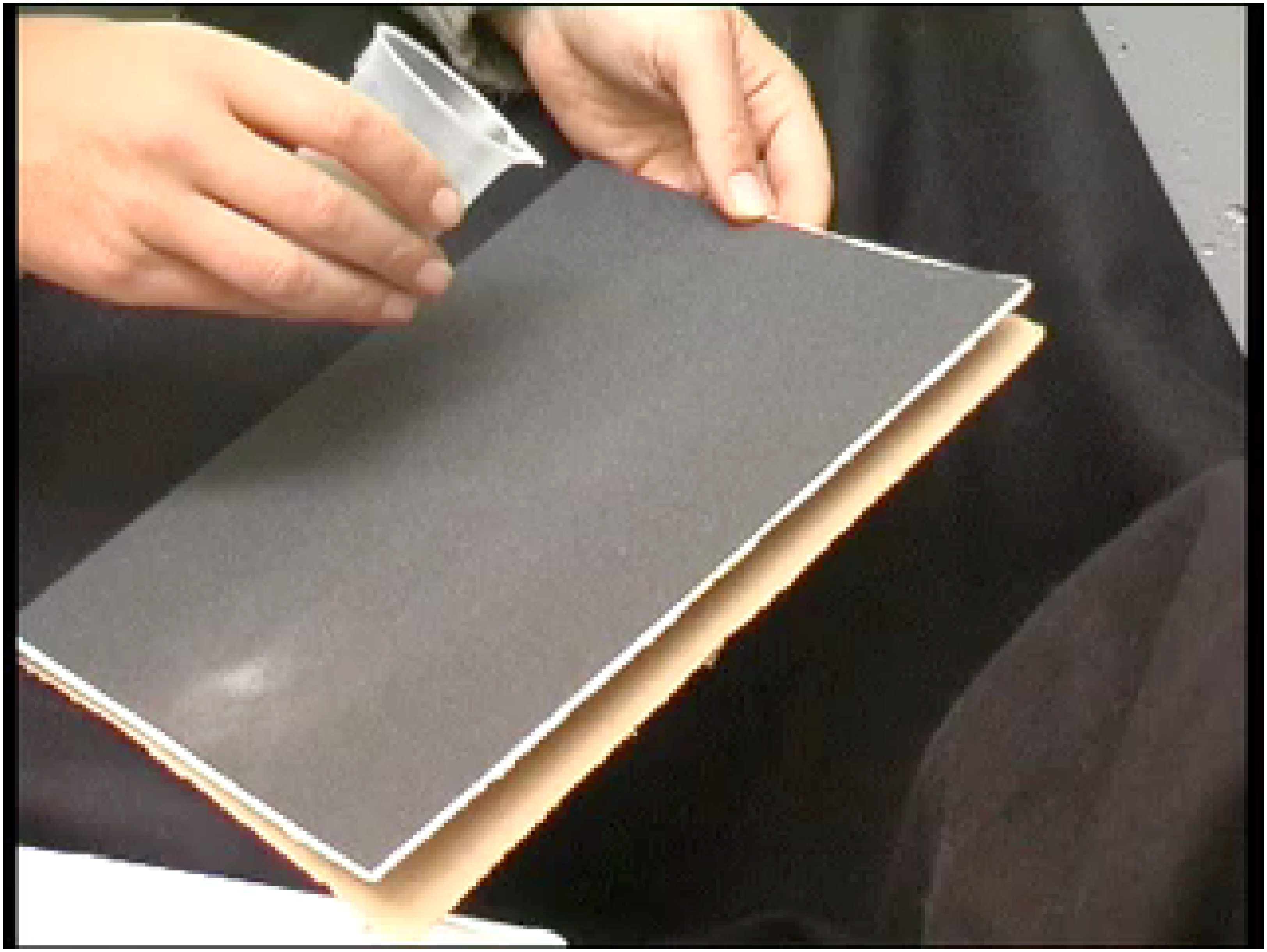
Pierre Jop,
Yoël Forterre,
Mickaël Pailha
Maxime Nicolas
Olivier Pouliquen

IUSTI-CNRS,
Université de Provence,
Marseille, France

Dry granular matter :

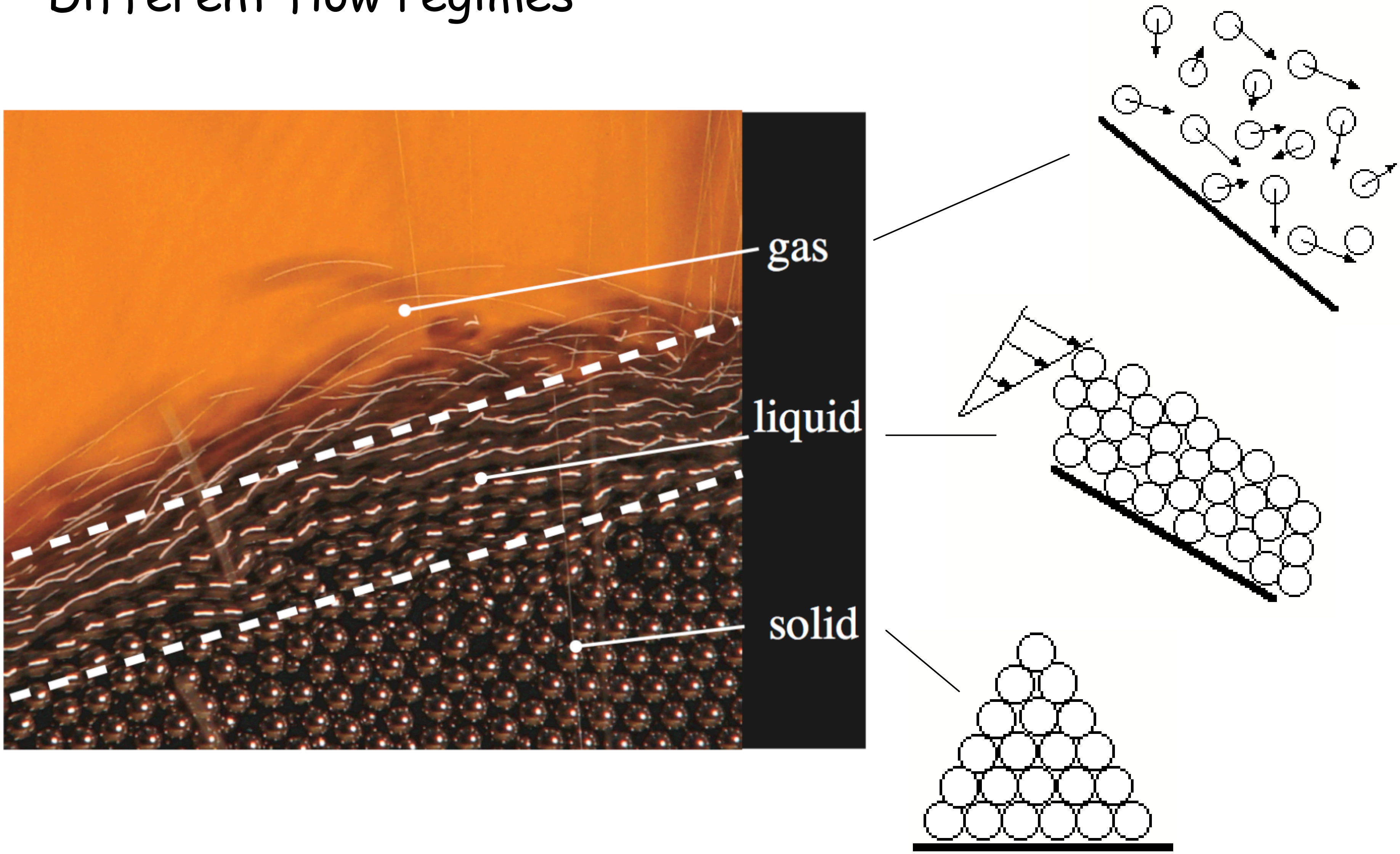
rigid (very stiff) particles with
-non brownian motion
-no cohesion
-no attraction

hard spheres at T=0

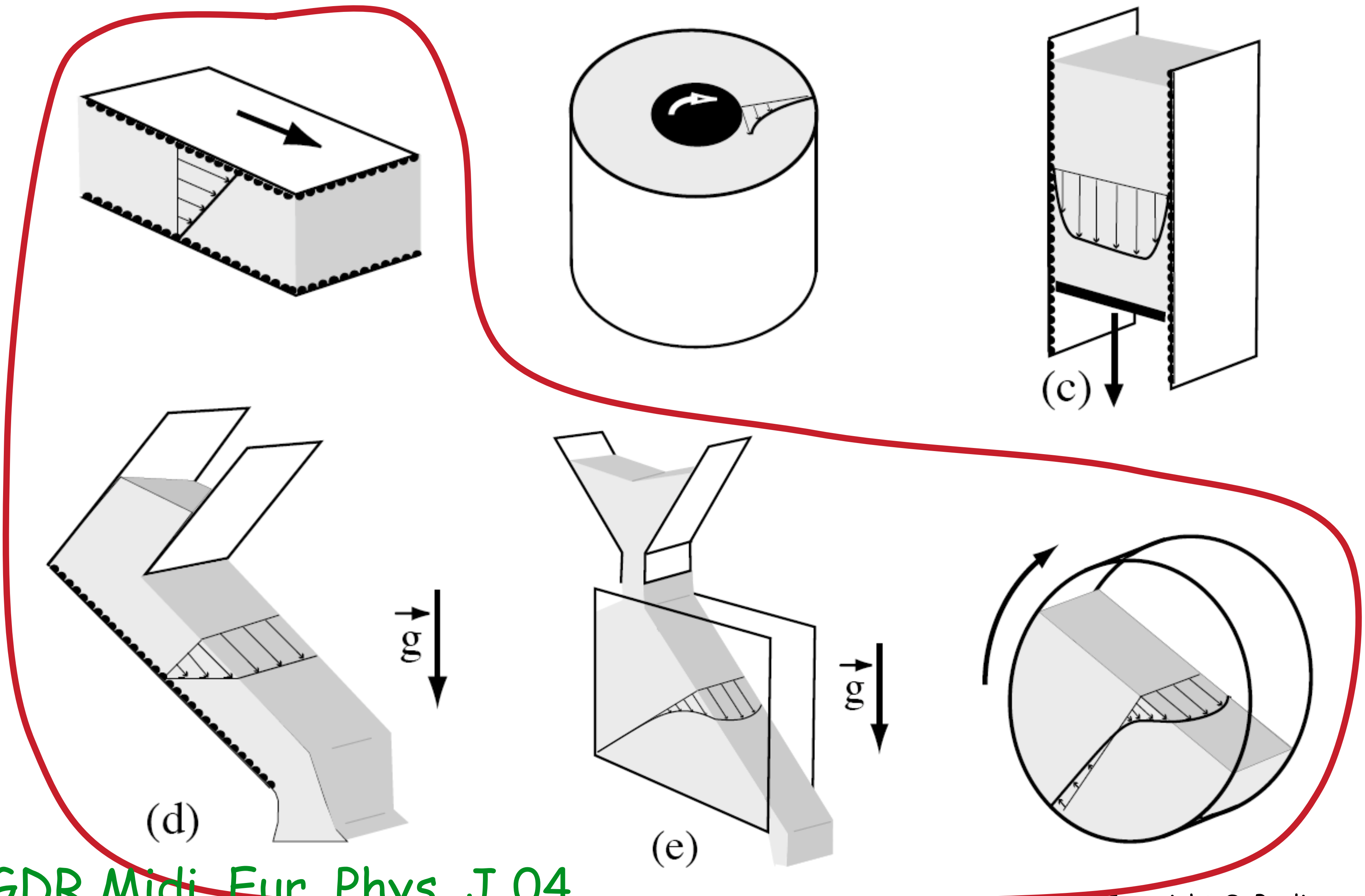


Dry granular flows
Different flow regimes

Kinetic theory
(Jenkins & Savage JFM 83
Goldhirsh ARFM 02)



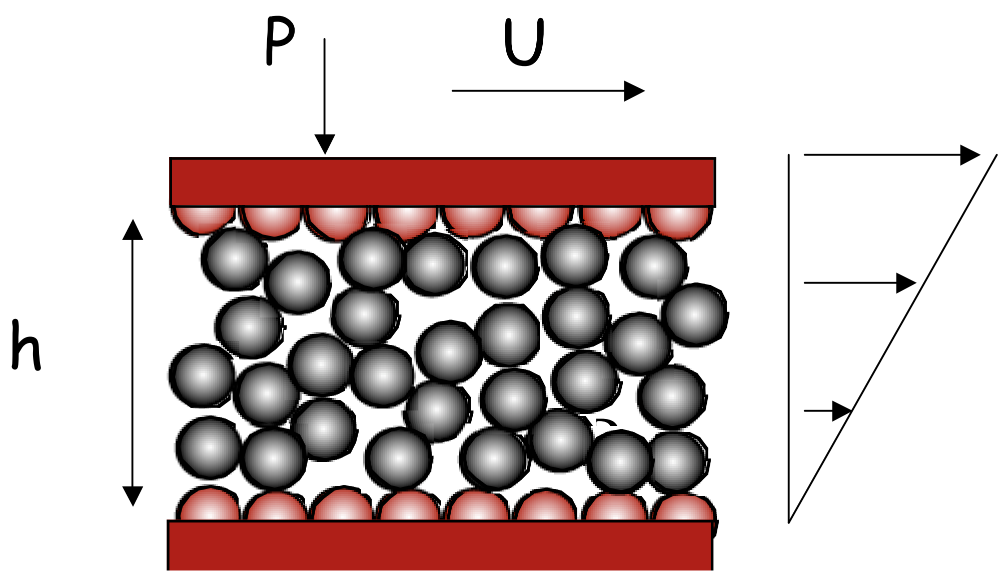
Different flow configurations studied both experimentally and numerically



GDR Midi, Eur. Phys. J 04

Numerical simulations (DEM) in plane shear

Da Cruz et al, PRE 05
GdR Midi Eur. Phys. J 04



*Linear Profile

$$\dot{\gamma} = U / h$$

One imposes P and $\dot{\gamma}$

Shear stress τ?
Volume fraction ϕ?

Dimensional analysis:

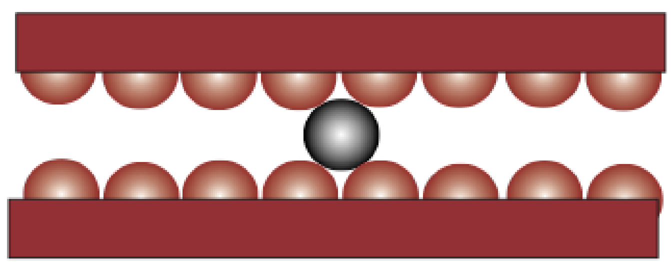
if one assumes that the system size plays no role

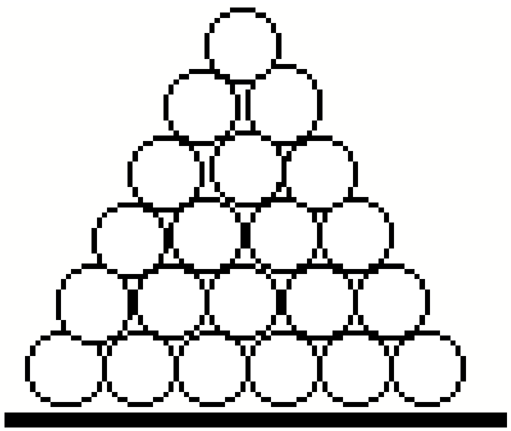
A single dimensionless number: $$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$ (Savage 84, Ancey et al 99)

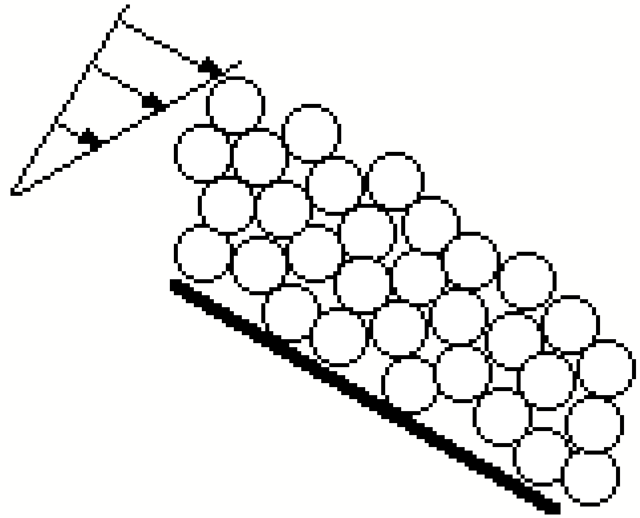
* I ratio between 2 times :

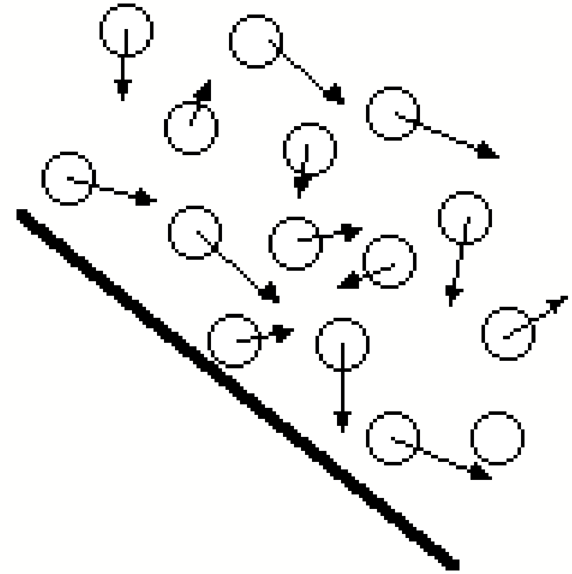
$1/\gamma$: time for the mean shear

$d\sqrt{\rho/P}$: microscopic time for rearrangement









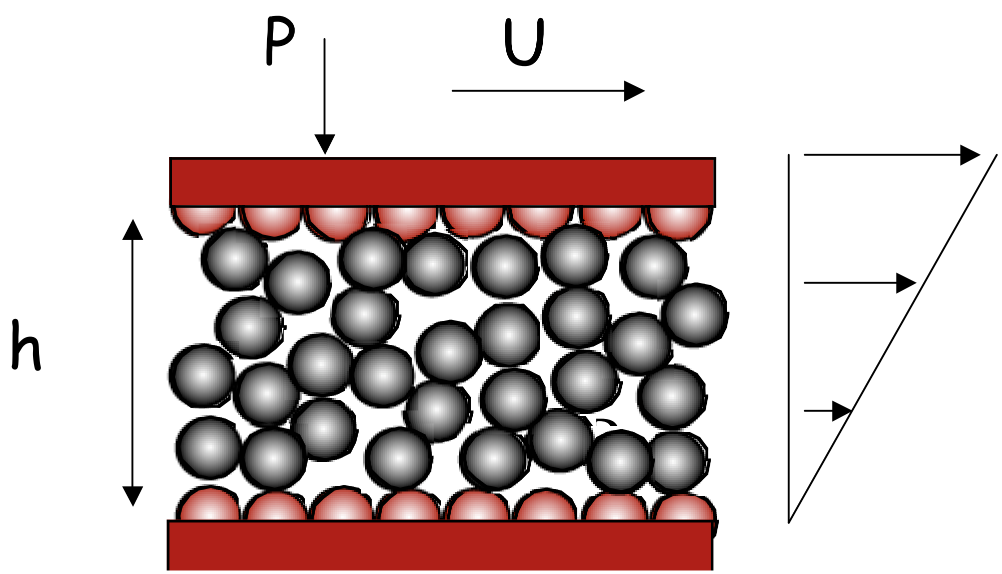
« quasi-static » « liquid » « gas »

0 → 1

$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$

Numerical simulations (DEM) in plane shear

Da Cruz et al, PRE 05
GdR Midi Eur. Phys. J 04



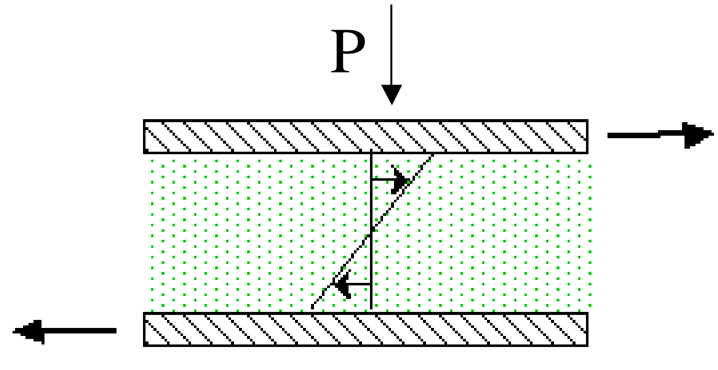
*Linear Profile

$$\dot{\gamma} = U/h$$

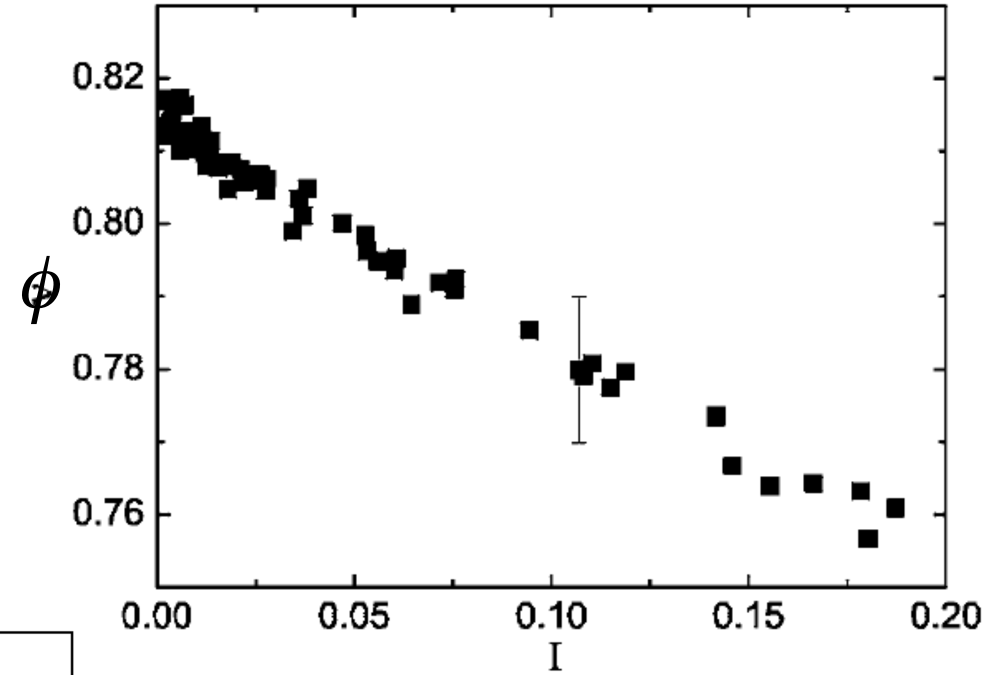
One imposes P and $\dot{\gamma}$

Shear stress τ?
Volume fraction ϕ?

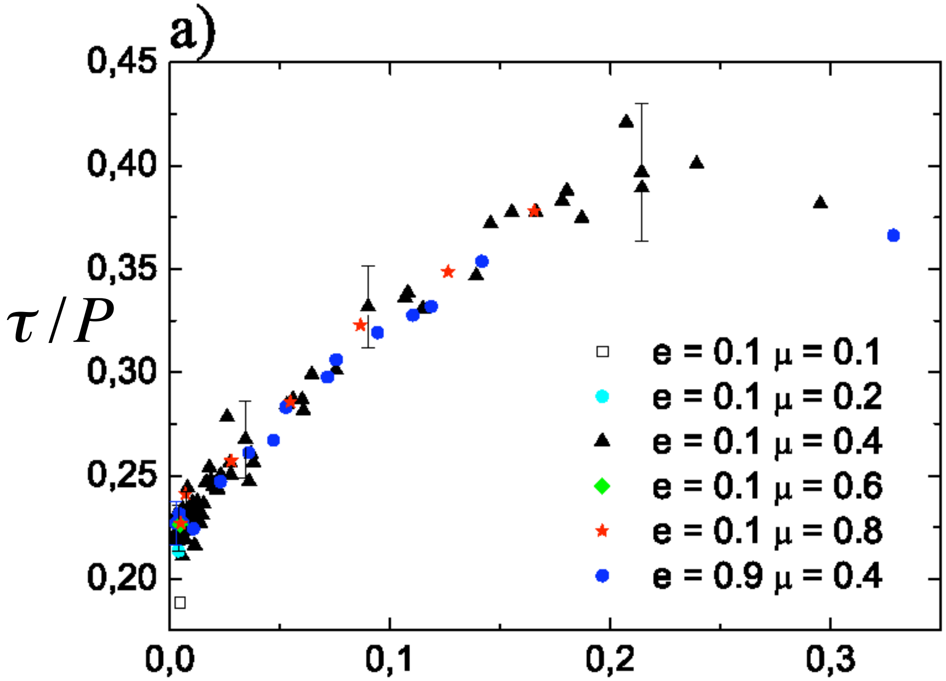
$$\tau = \mu(I)P$$
$$\phi = \phi(I)$$



$$\tau = \mu(I)P$$

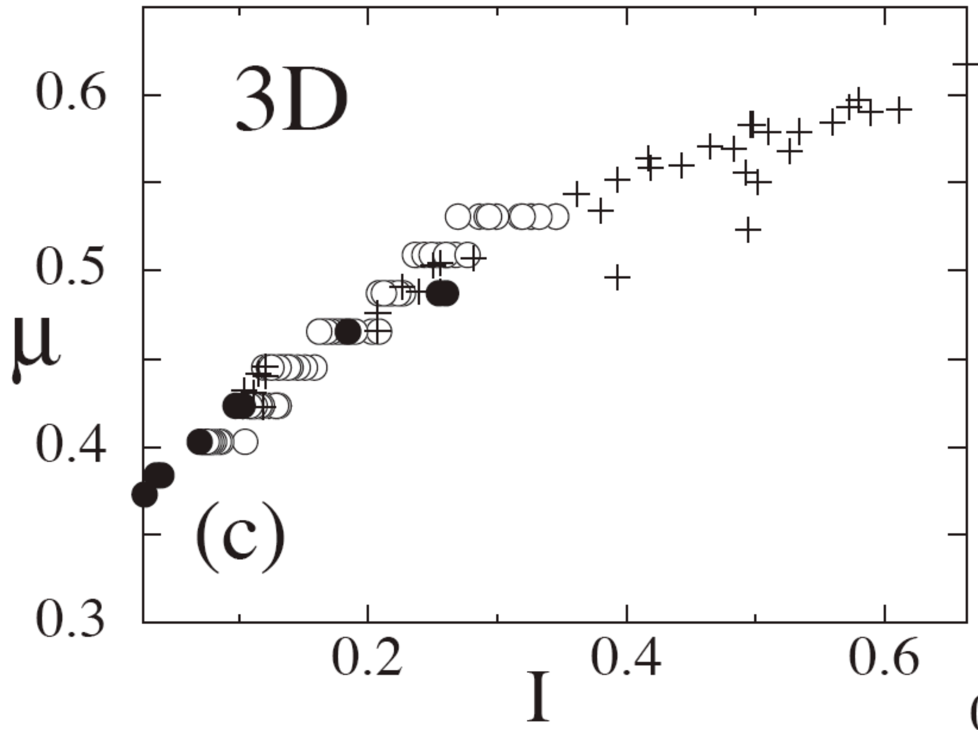


$$\phi = \phi(I)$$

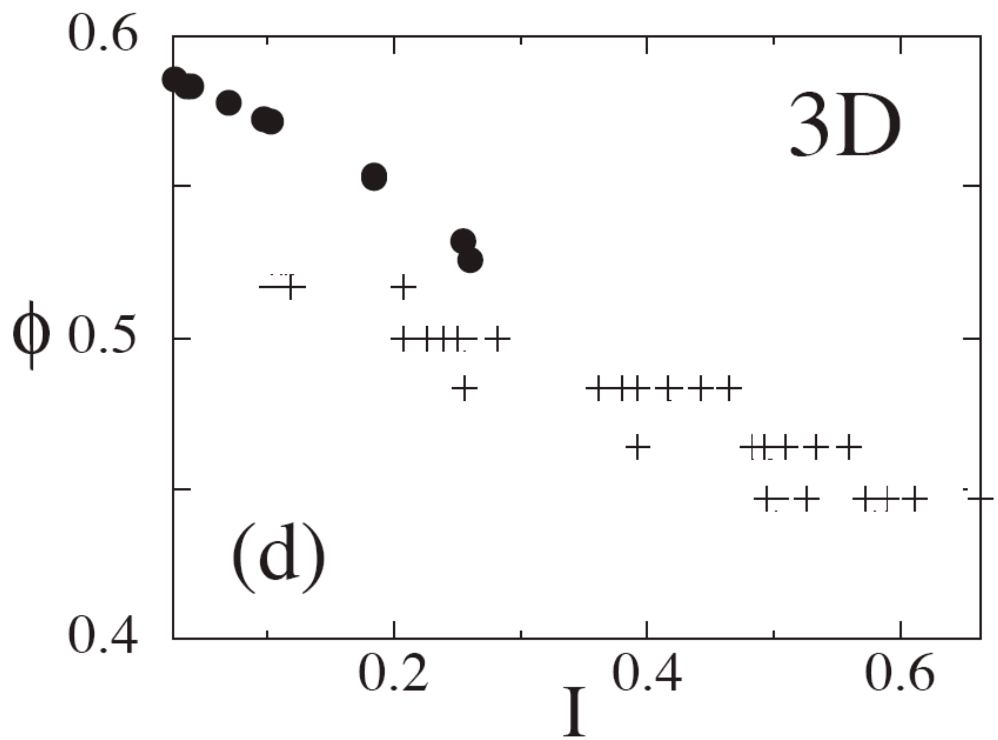


$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$

Forterre, Pouliquen ARFM 08



For spheres



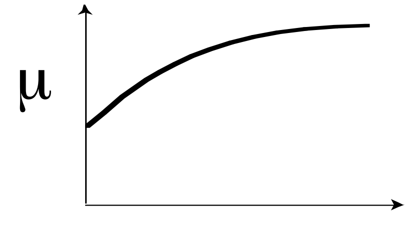
Data from

Inclined plane exp. (Pouliquen 99)
Inclined plane simulations
(Baran et al 2006)
Annular shear cell exp.
(Sayed, Savage JFM, 84)

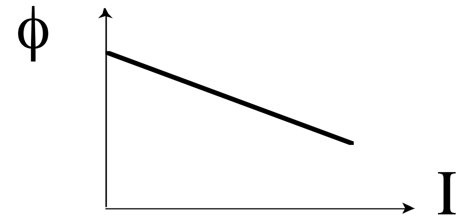
Constant pressure

$$I = \frac{\dot{\gamma}\, d}{\sqrt{P/\rho}}$$

$$\tau = \mu(I) P$$



$$\Phi = \Phi(I)$$



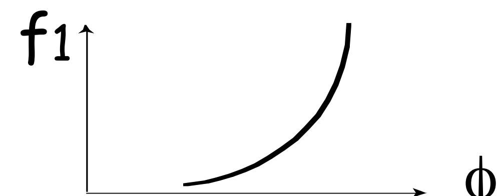
$$\Phi(I) = f_1^{-1}(1/I^2) \qquad \mu(I) = I^2 f_2(f_1^{-1}(1/I^2))$$

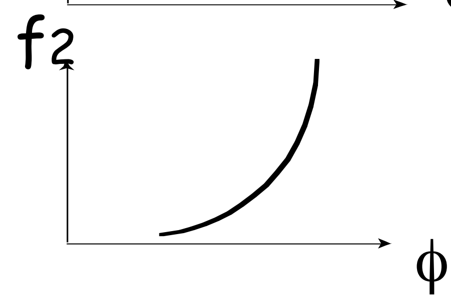
And shear at constant volume fraction ??

Bagnold Proc. R. Soc 54
Lois et al PRE 07
Lemaitre PRE 05
Da Cruz et al PRE 05

$$\tau = f_1(\Phi)\rho_s d^2 \dot{\gamma}^2$$

$$P = f_2(\Phi)\rho_s d^2 \dot{\gamma}^2$$





For more complex configurations

we need a real 3D tensorial constitutive law

A 3D generalisation of the friction law : introducing an effectif viscosity

assumptions: 1) P isotropic

2) $\dot{\gamma}_{ij}$ and τ_{ij} are co-linear

(Savage 83, Goddard 86, Schaeffer 87,...)

$$\tau_{ij} = \frac{\mu(I)P}{\|\dot{\gamma}\|}\dot{\gamma}_{ij}$$

effective viscosity

$$\eta(I) = \frac{\mu(I)P}{\|\dot{\gamma}\|}$$

$$\dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}$$

$$I = \frac{\|\dot{\gamma}\| d}{\sqrt{P/\rho}}$$

$$\|\dot{\gamma}\| = \sqrt{\frac{1}{2}\dot{\gamma}_{ij}\dot{\gamma}_{ij}}$$

Flow threshold : $\|\tau\| < \mu_s P$

$$\|\tau\| = \sqrt{\frac{1}{2}\tau_{ij}\tau_{ij}}$$

Mass and momentum equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial z} = 0,$$

$$\rho_s \phi \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial z} \right) = \rho_s \phi g \sin\theta - \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z},$$

$$\rho_s \phi \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial z} \right) = -\rho_s \phi g \cos\theta - \frac{\partial P}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zz}}{\partial z},$$

$$\tau_{ij} = \frac{\mu(I) P}{\dot\gamma} \dot\gamma_{ij}$$

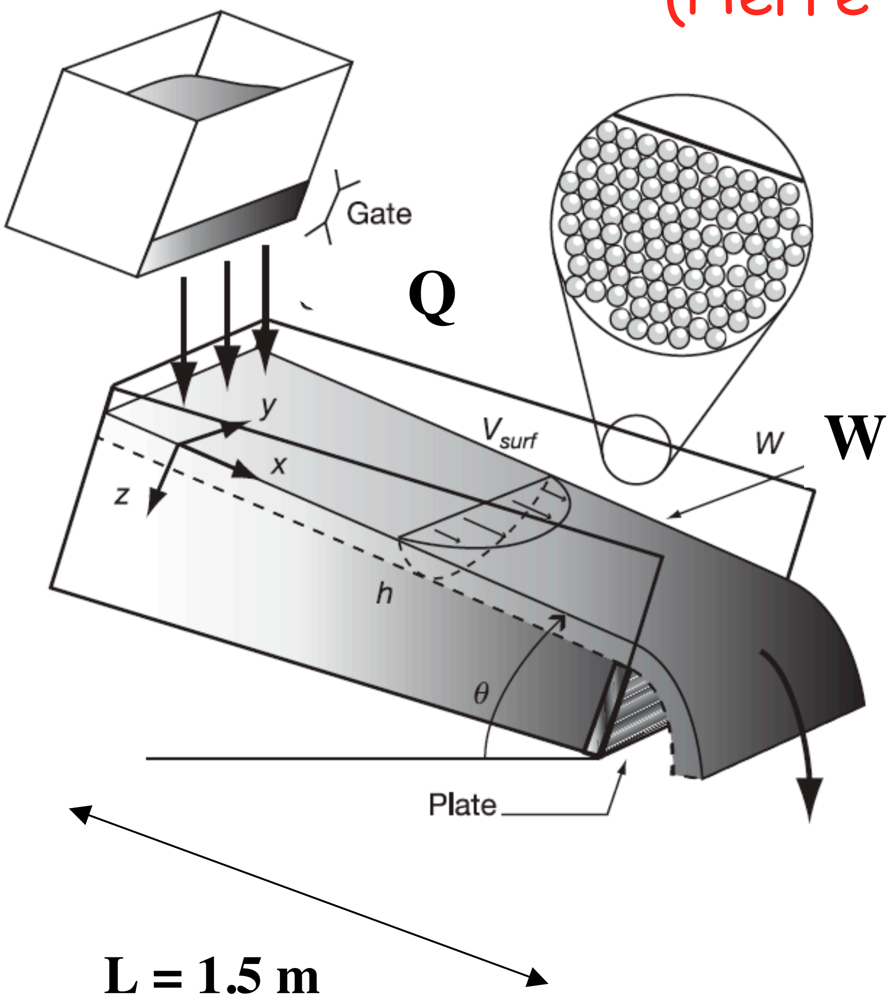
3 quantitative tests where 3D effects exist:

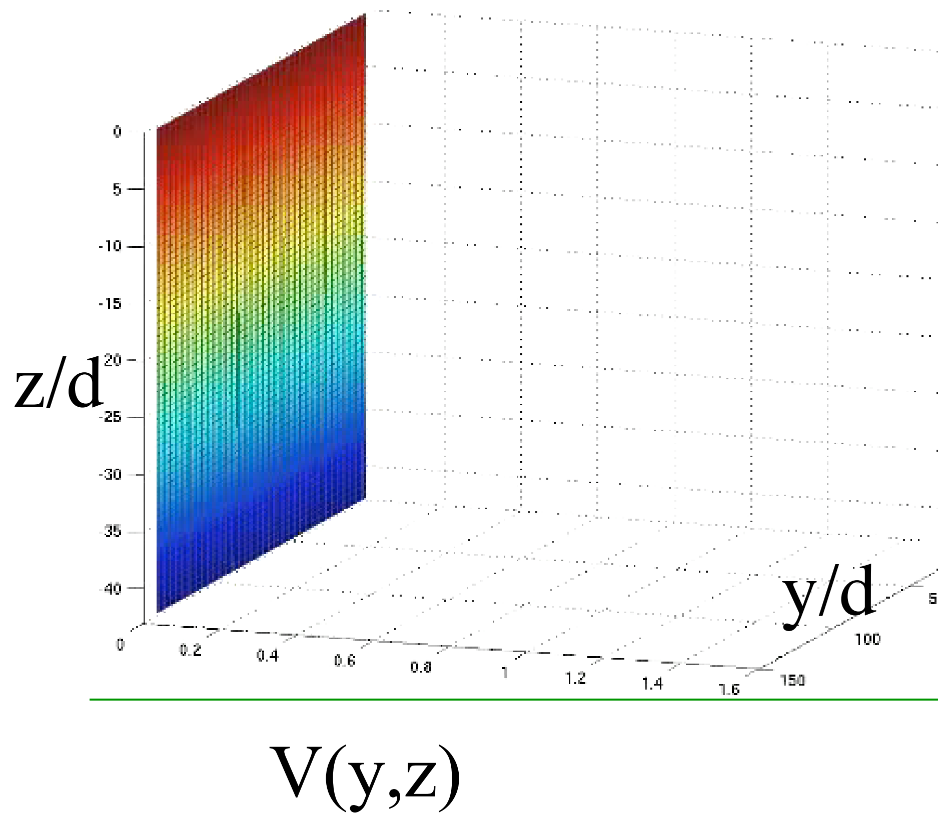
-steady flow in a rough channel
-avalanches
-roll wave instability

We keep the same material (glass beads .5mm) for which the friction law has been calibrated on steady uniform flows down inclined plane…

No free parameter….

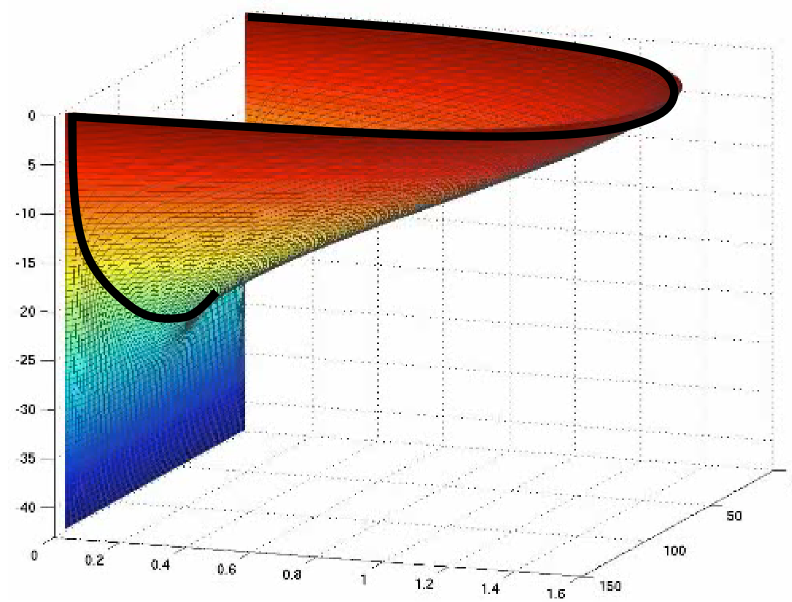
1st test:
flows on a heap : a full 3D problem
(Pierre Jop's PhD)

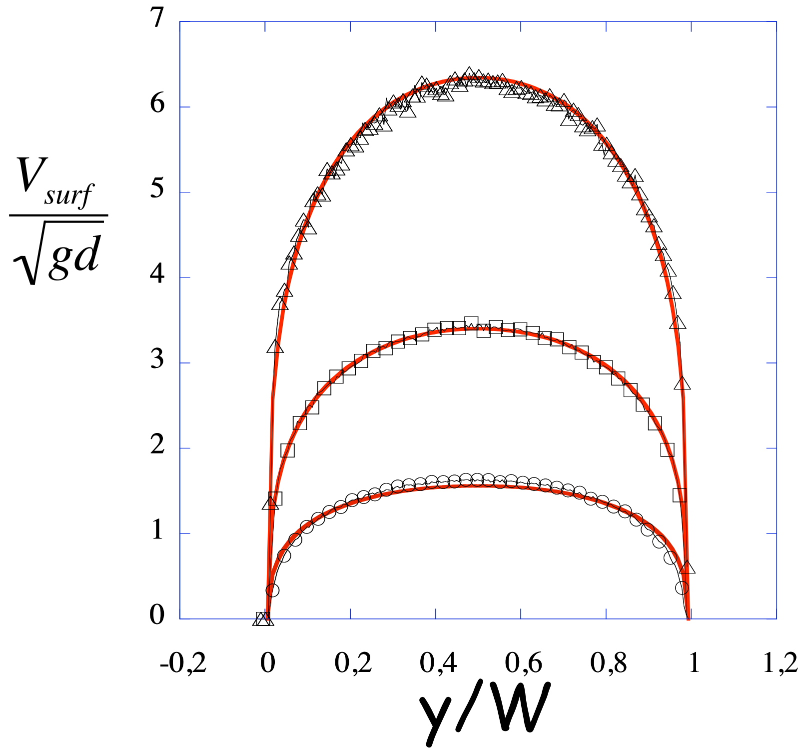


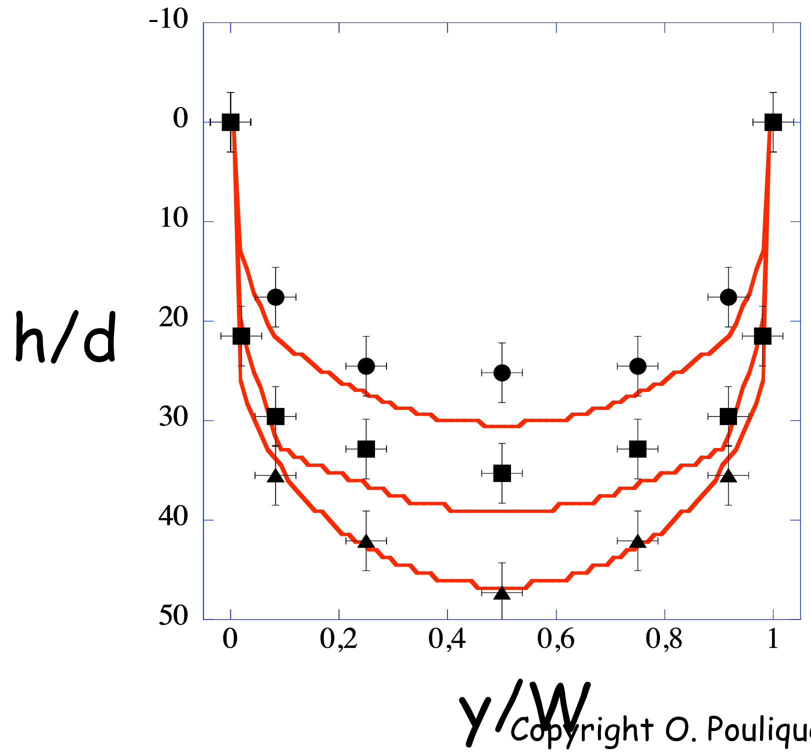


Flow between rough lateral walls:

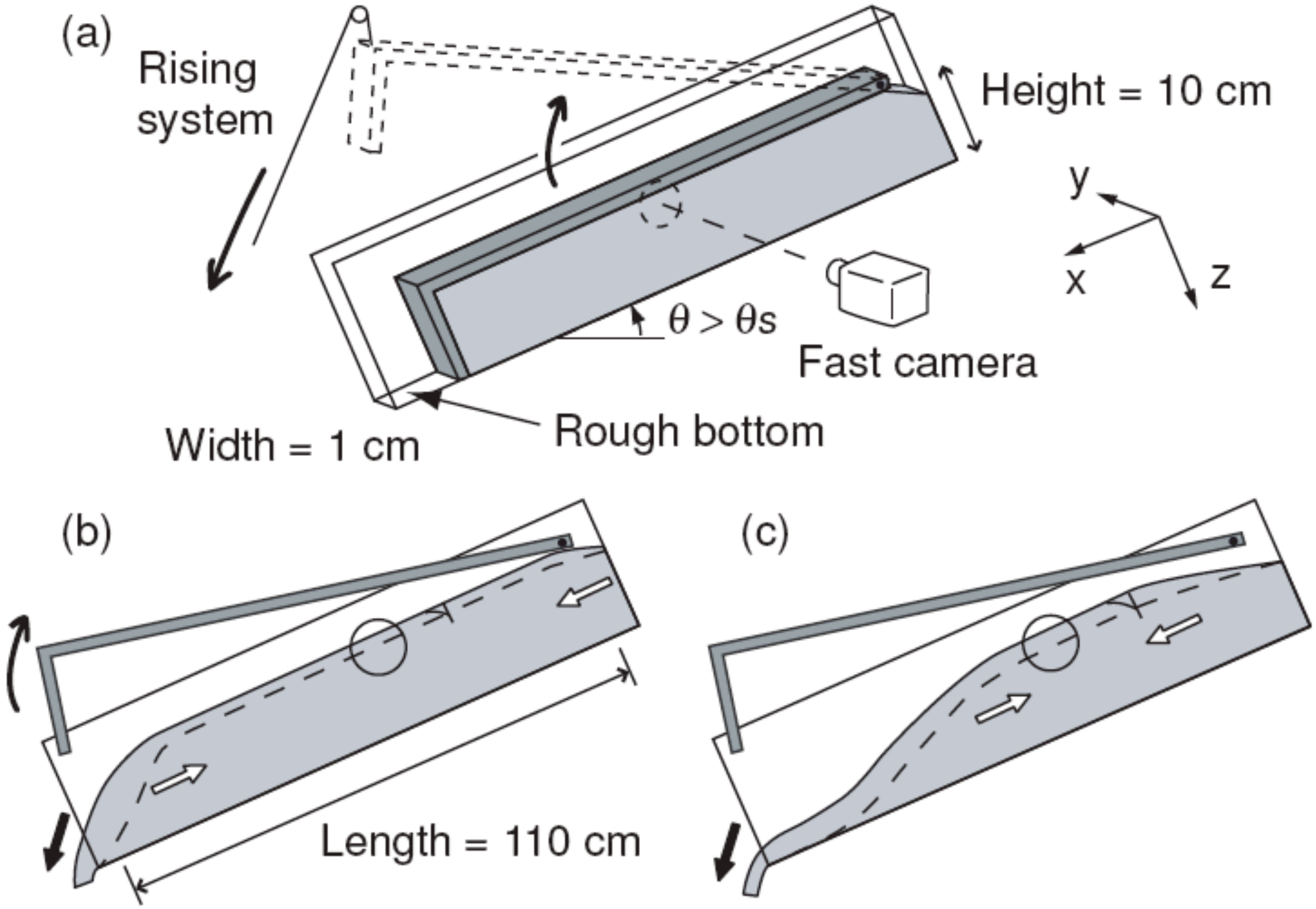
Jop et al
Nature 2006



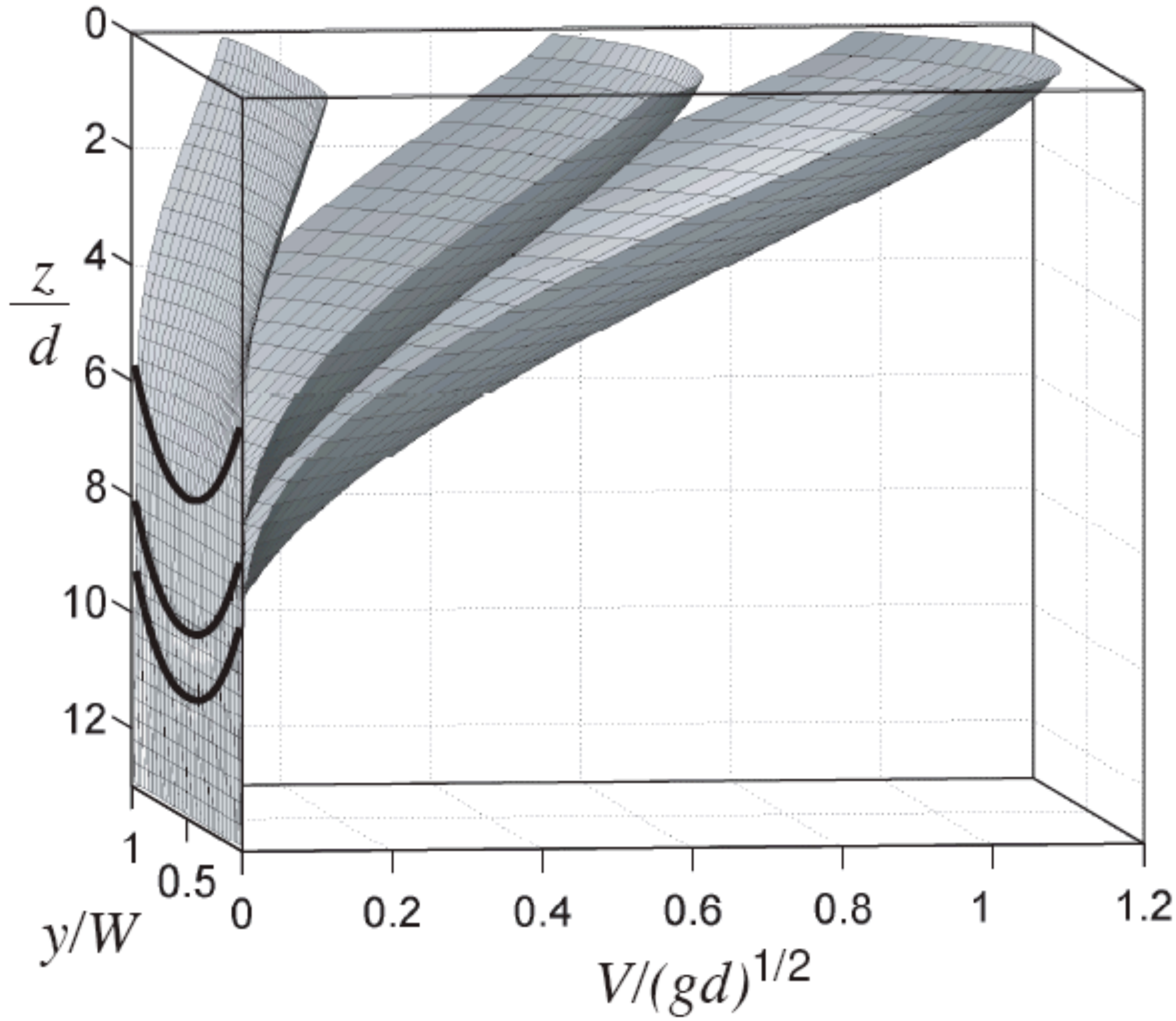




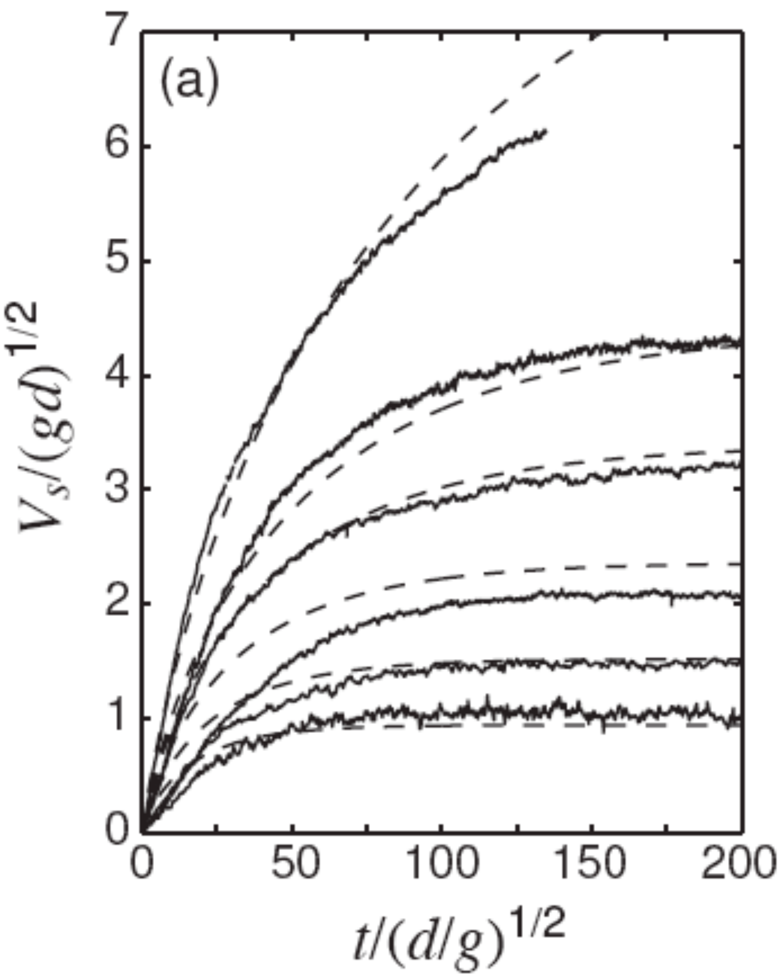
2 test: how an avalanche starts?
(Jop et al, Phys. Fluids 07)



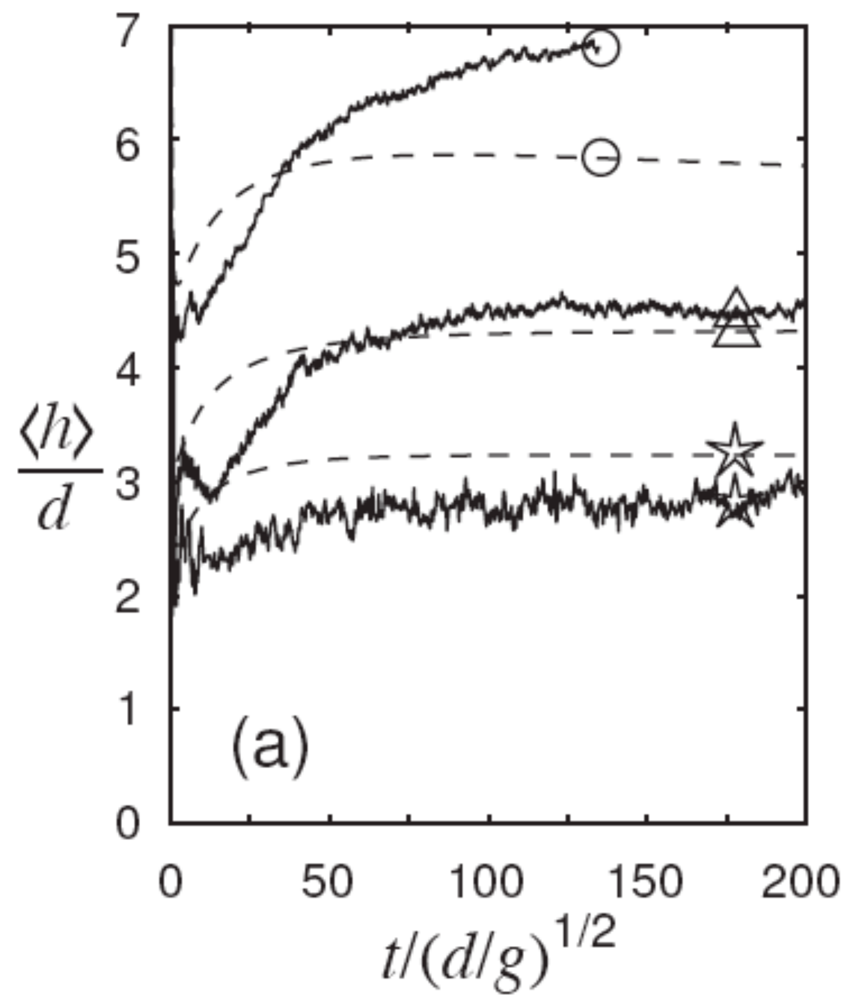
Acceleration of the layer (V(t) ↗), and erosion (h(t) ↗)



0
2
4
z/d
6
8
10
12
1
0.5
y/W
0
0.2
0.4
0.6
0.8
1
1.2
V/(gd)^1/2



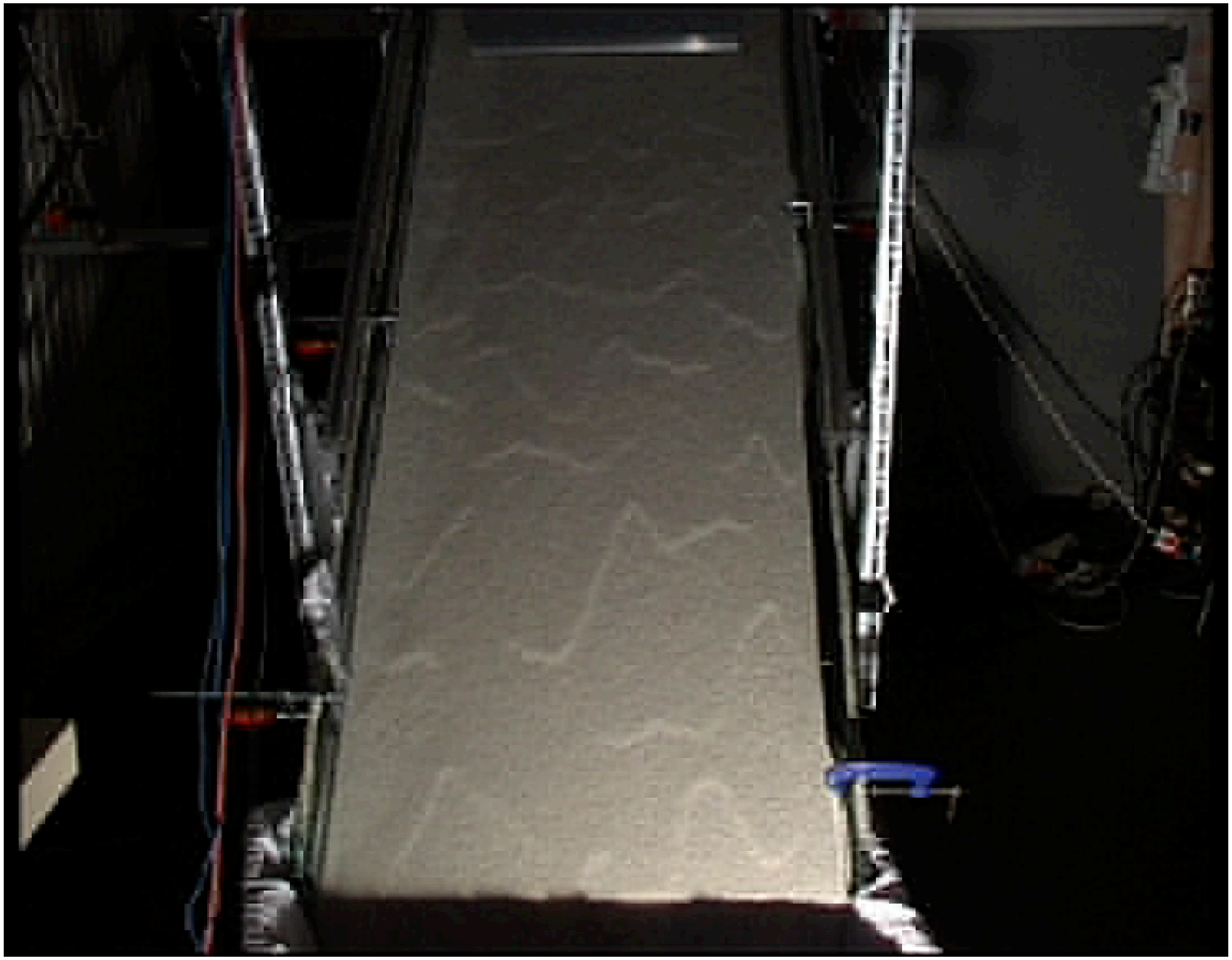
(a)
$V_s/(gd)^{1/2}$
$t/(d/g)^{1/2}$



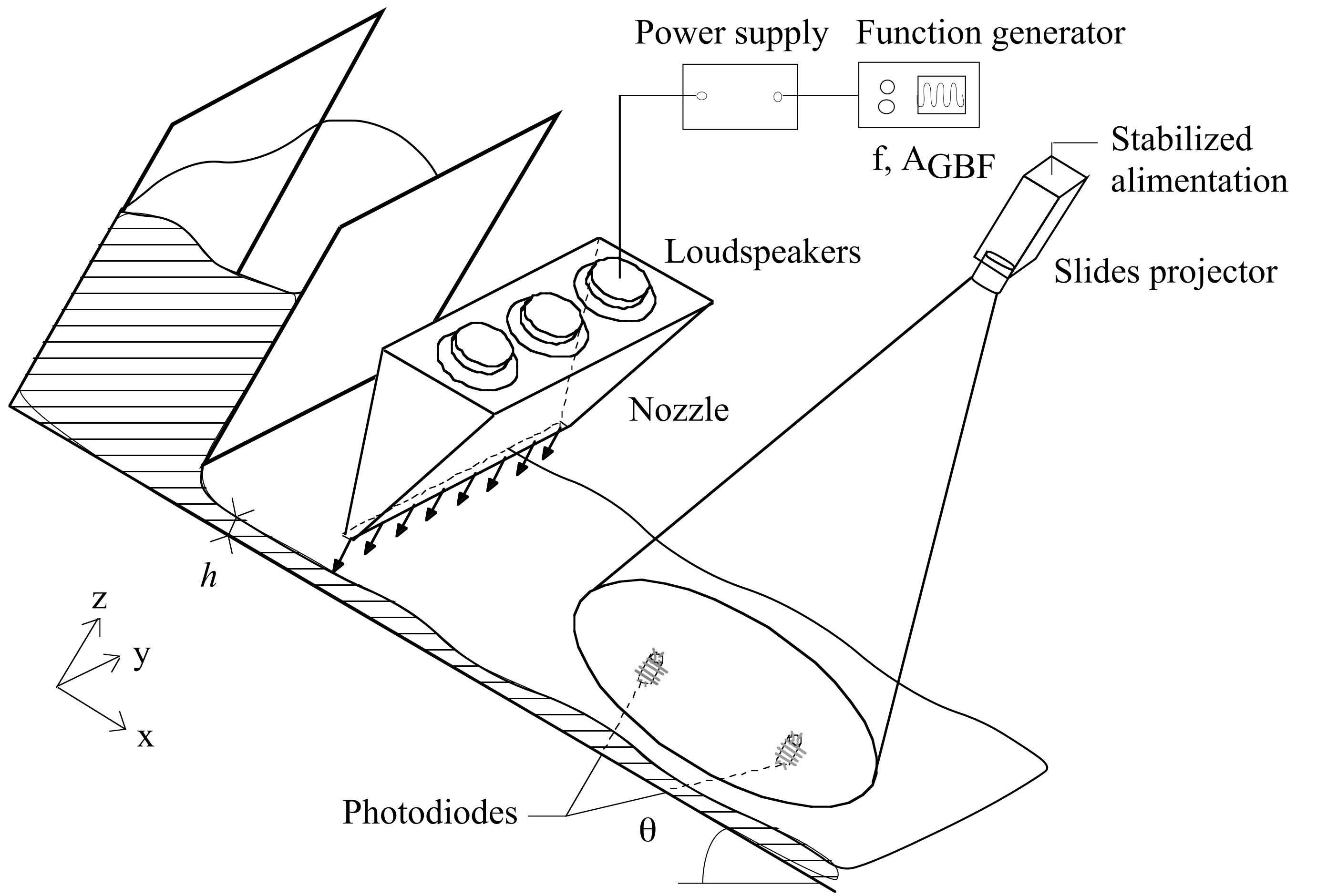
(a)
$\frac{\langle h \rangle}{d}$
$t/(d/g)^{1/2}$

3D test : Long wave instability in granular flows

By Yoel Forterre (Forterre, JFM 06)



Experimental Setup :
forcing at a given frequency

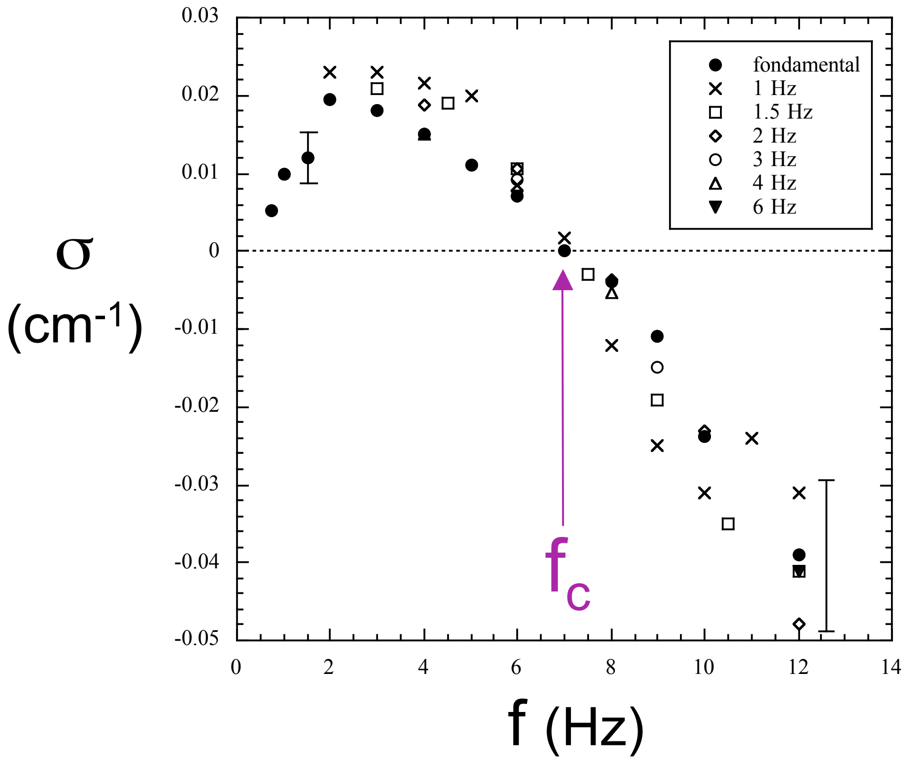


Forterre and Pouliquen JFM 02

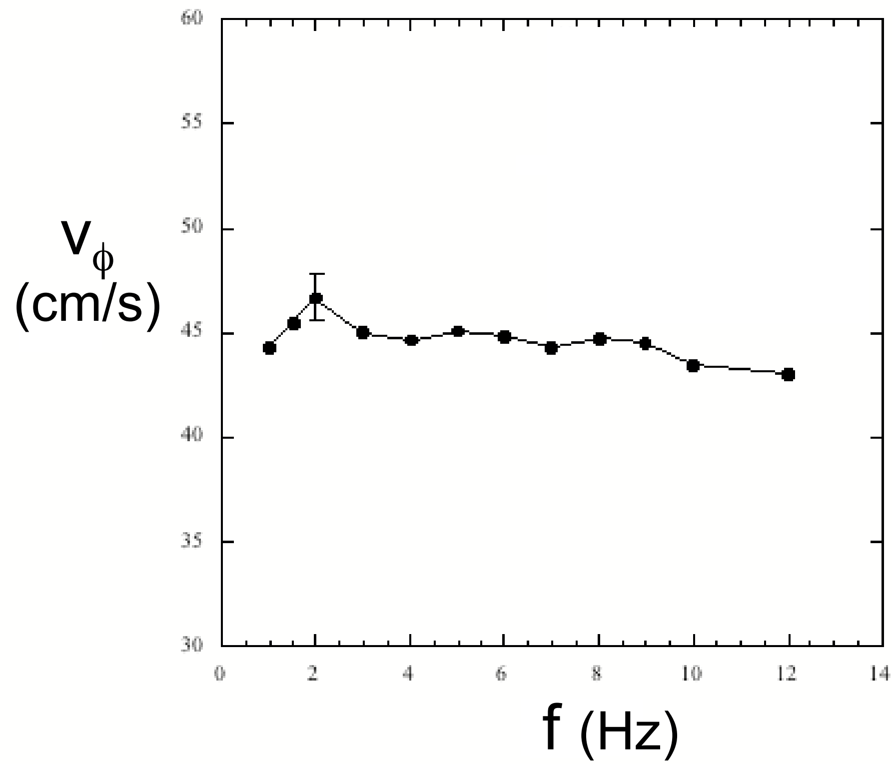
Dispersion relation

(glass beads, α=29°, h=5.3 mm)

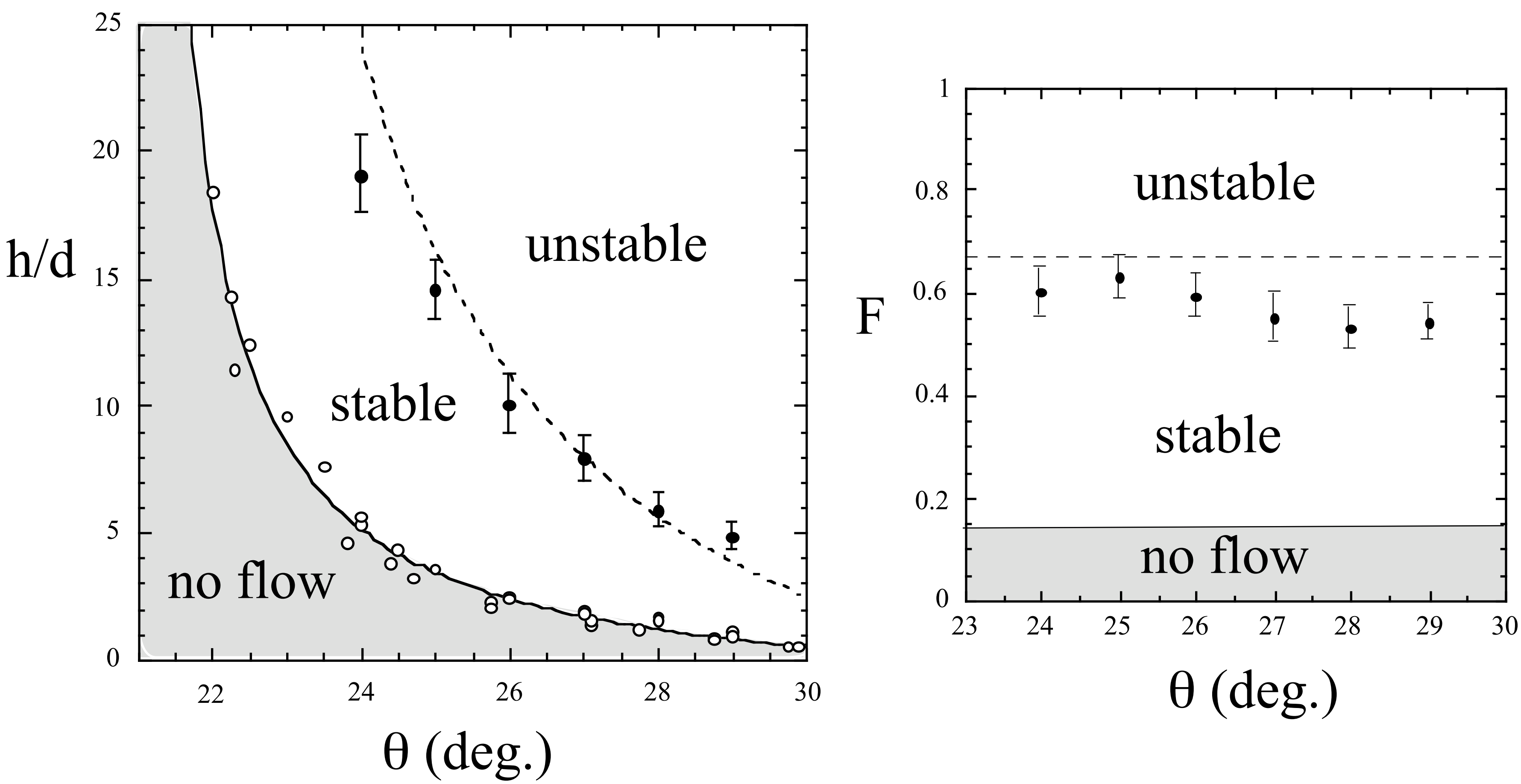
Growth rate



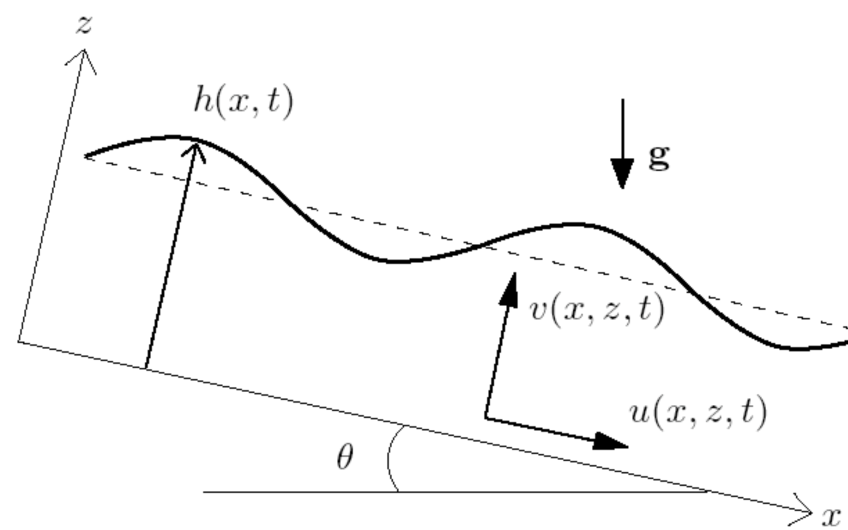
Phase velocity



Stability diagram
(glass beads)



Full 3D linear stability analysis:



$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial z}=0,$$

$$\rho_s\phi\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial z}\right)=\rho_s\phi g\sin\theta-\frac{\partial P}{\partial x}+\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{xz}}{\partial z},$$

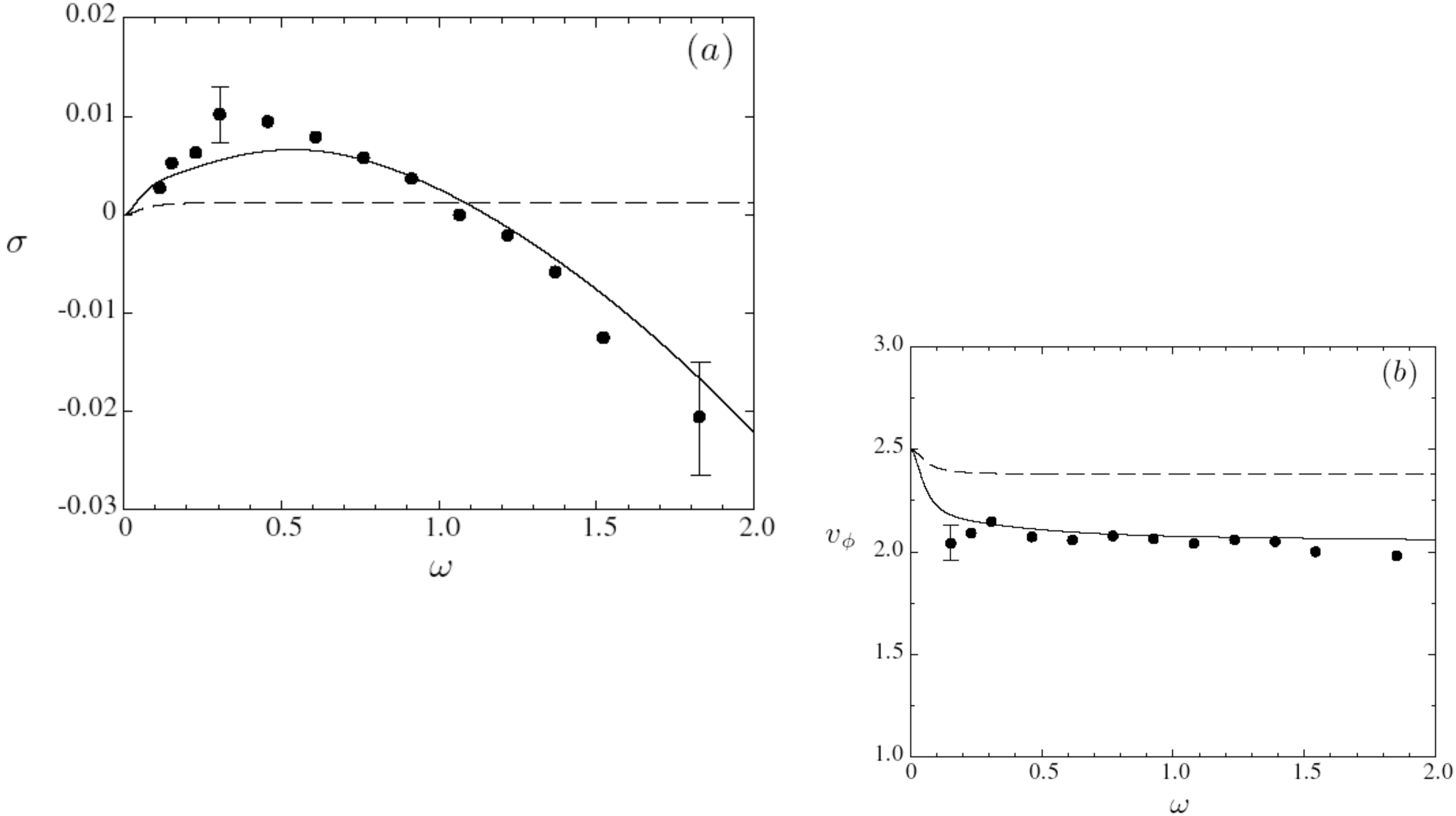
$$\rho_s\phi\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial z}\right)=-\rho_s\phi g\cos\theta-\frac{\partial P}{\partial z}+\frac{\partial \tau_{zx}}{\partial x}+\frac{\partial \tau_{zz}}{\partial z},$$

$$\tau_{ij}=\frac{\mu(I)P}{\dot\gamma}\dot\gamma_{ij}$$

No fit parameter...

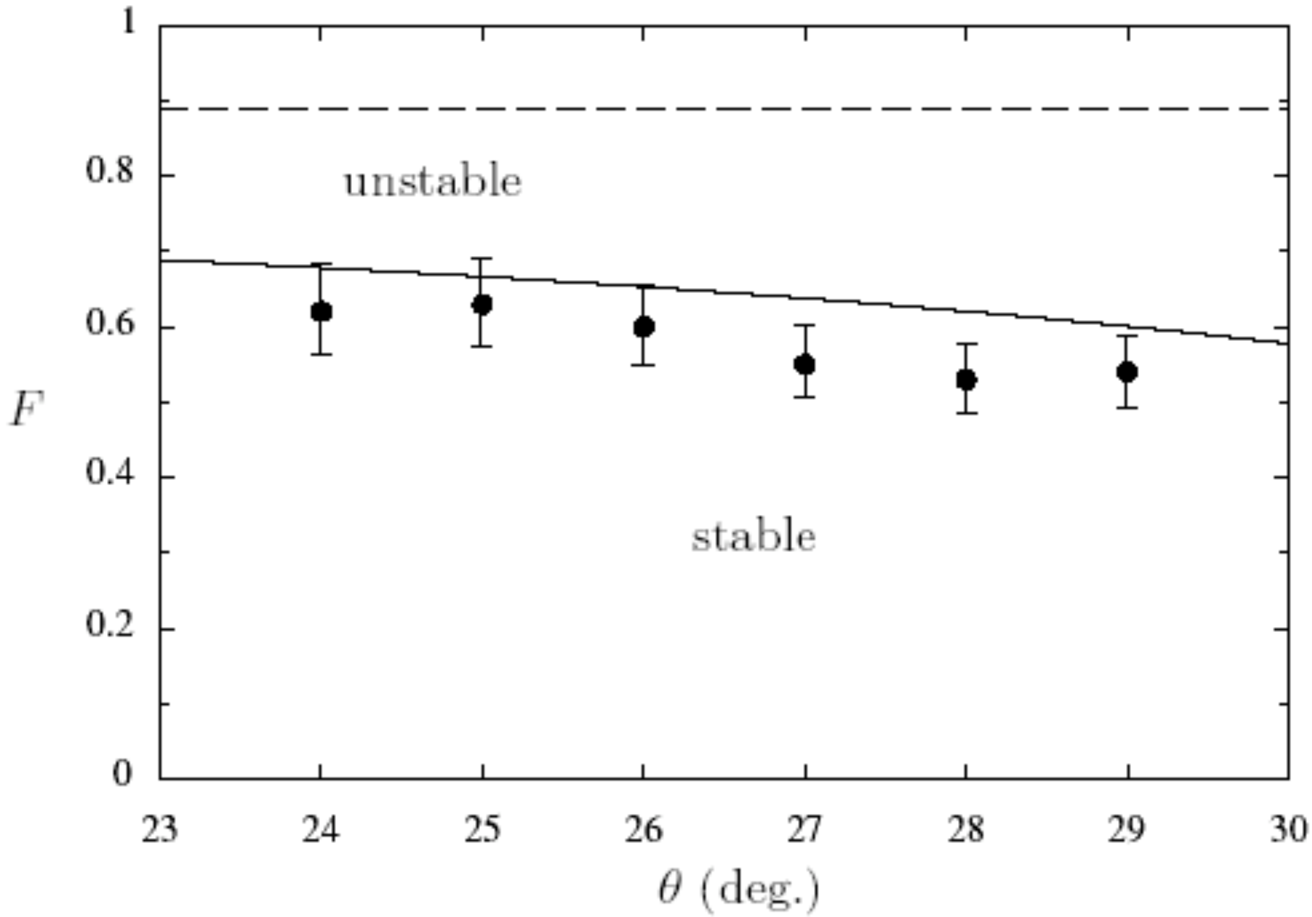
Dispersion relation

Forterre JFM 06



Forterre JFM 06

Instability threshold



The simple visco-plastic approach:

$$\tau_{ij} = \eta \gamma_{ij} \qquad \eta = \frac{\mu(I)P}{\|\gamma\|}$$

captures the viscous nature of granular flows and gives quantitative predictions in 3D geometries

??? Other configuration ???
??? other material ???

But ...

... serious limits !!!

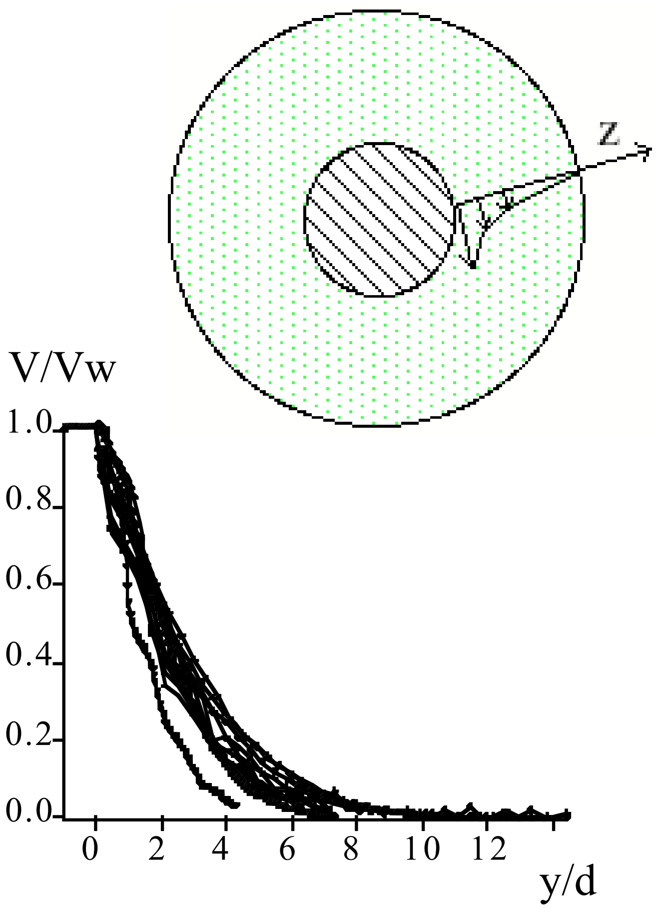
- Quasi-static regime
- Flow threshold

Limits in the quasi-static limit when I-> 0

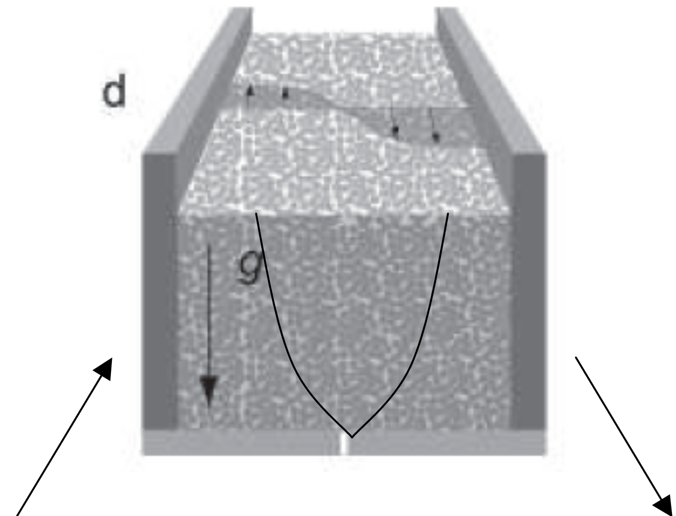
Shear bands in quasi-static flow not captured

(C .Cawthorn APS DFD 07, Forterre Pouliquen ARFM 08)

« narrow shear band »



« wide shear bands »



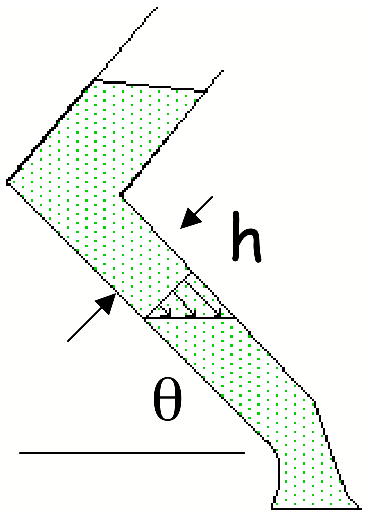
Depken et al PRE 06

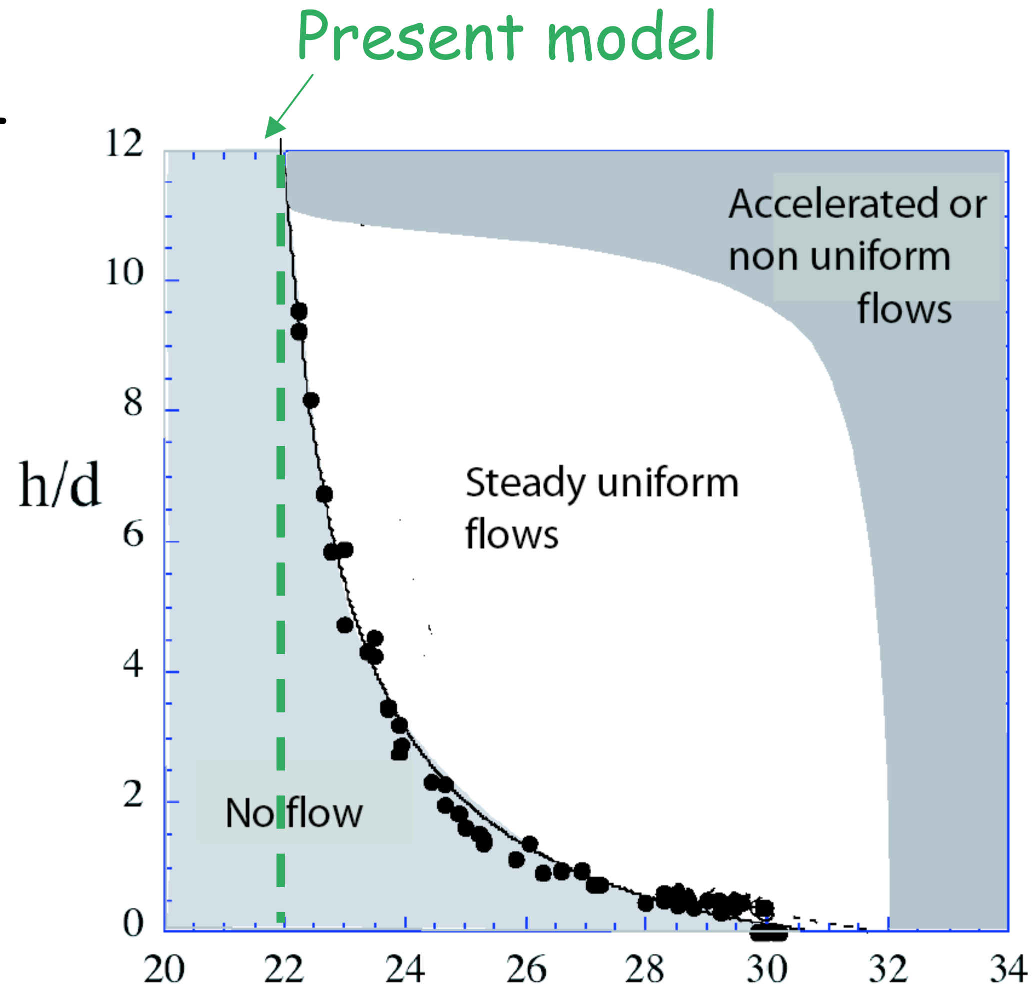
Howell et al PRL 99
Mueth et al Nature 00
Bocquet et al PRE 02
...

flow threshold not well captured

-no hysteresis

-no finite size effect





Pouliquen, Phys. Fluids 99
Daerr, Douady Nature 99
Ecke, Borzsonyi APS DFD 07

link with the microstructure?

Role of the fluctuations ?

Aranson and Tsimring PRE,01,

Louge Phys. Fluids 03,

Josserand et al 06

Lemaitre 02

Role of the correlations ?

Pouliquen et al 01,

Ertas and Halsey 03,

Mills et al 99, 00, 04

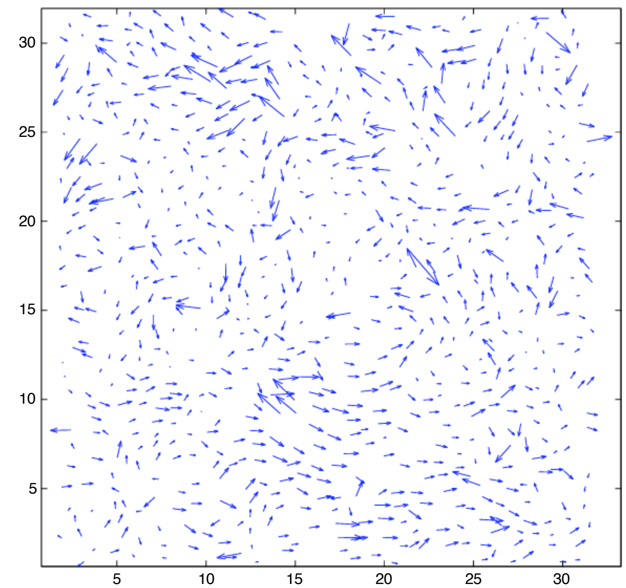
Jenkins and Chevoir 01,

Jenkins Phys. Fluids 06,

...

Link with other glassy systems?

Radjai and Roux PRL 02



Pouliquen PRL 04

Conclusion:

The visco-plastic model gives good predictions
when the granular material flows in the inertial regime
but fails close to quasi-static regime

Perspectives:

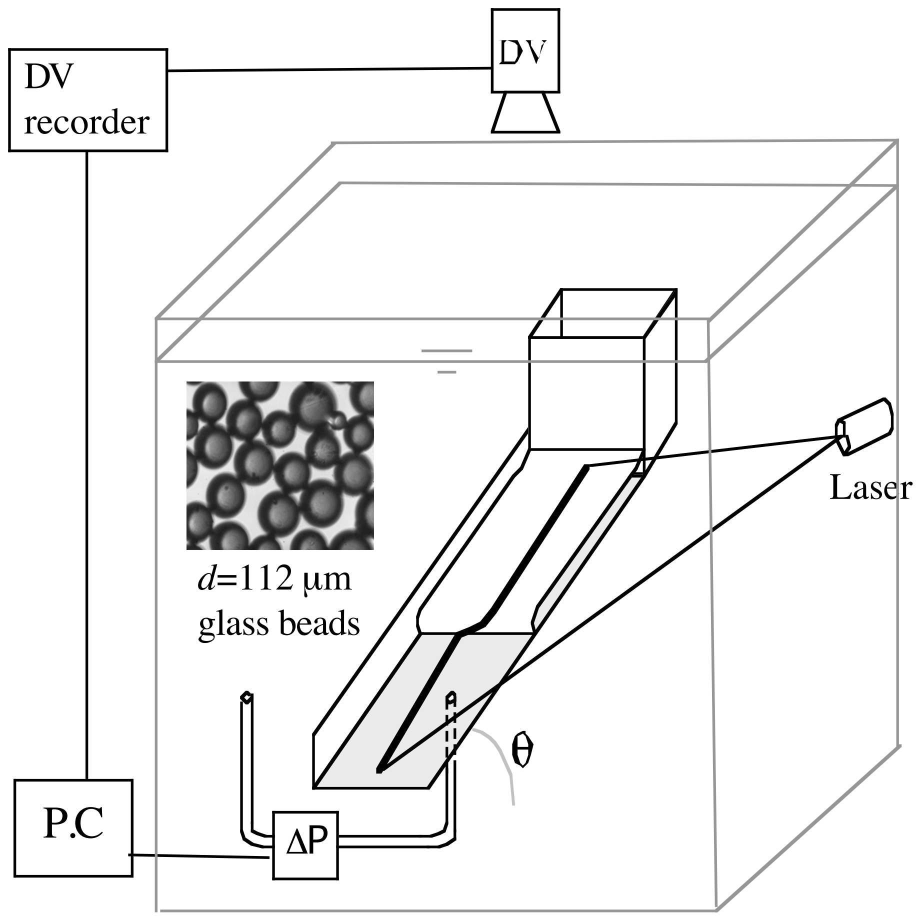
Link with the microstructure

More complexe materials:

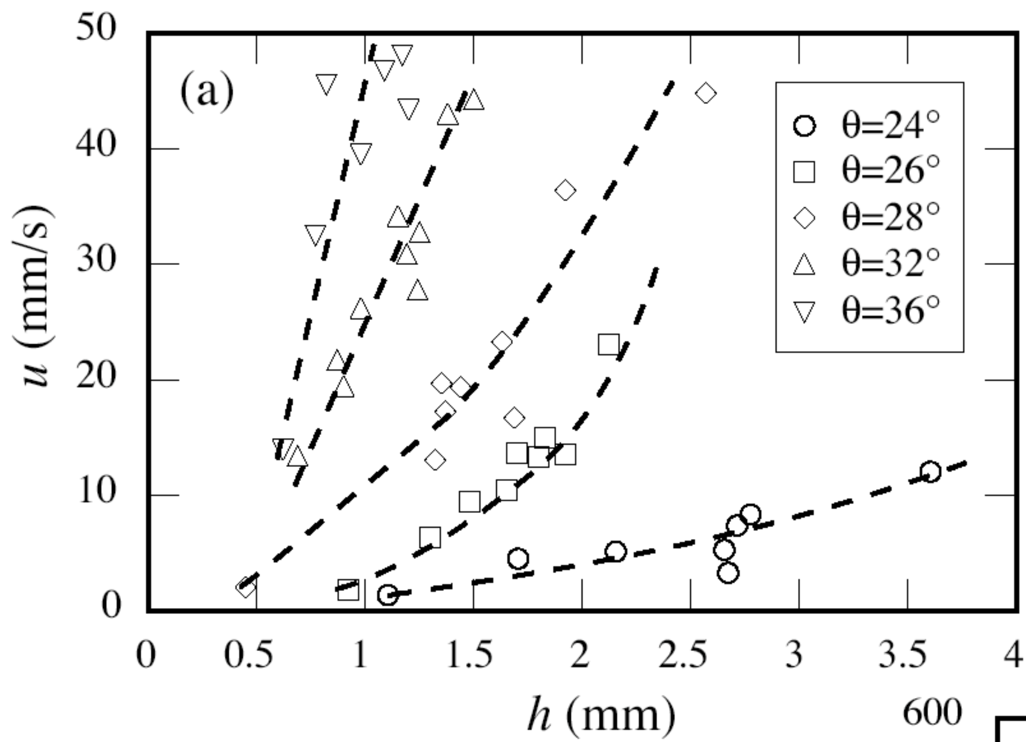
cohesion? Rognon et al PRE 06

role of the interstitial fluid ? Cassar et al Phys. Fluids 06

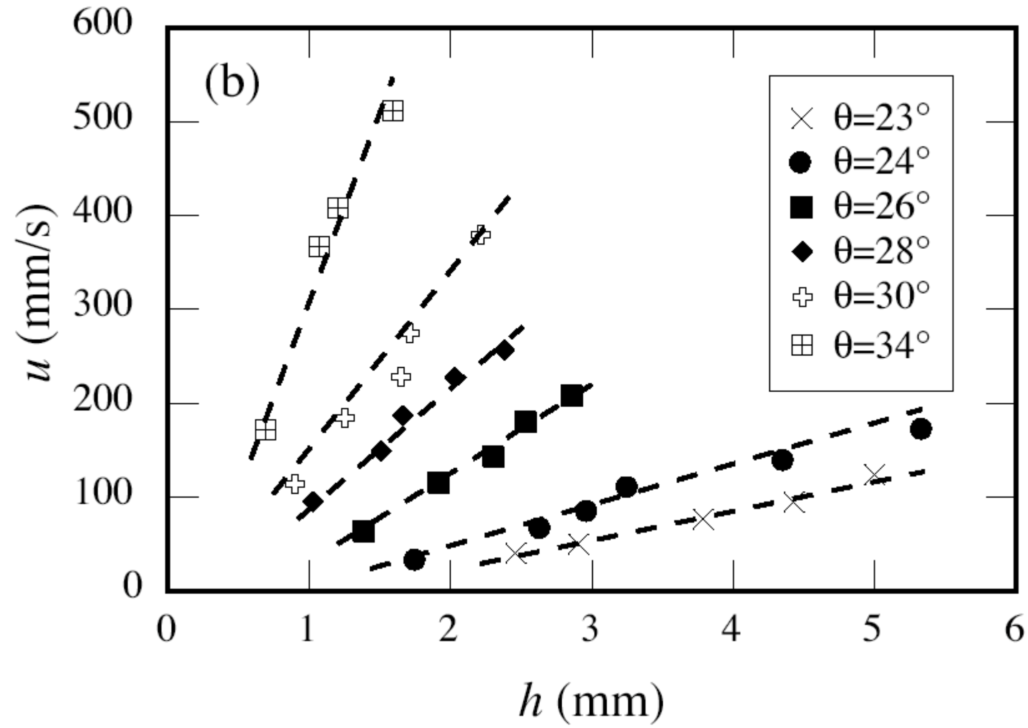
Changing time scales...
by putting the granular material in water



Immersed



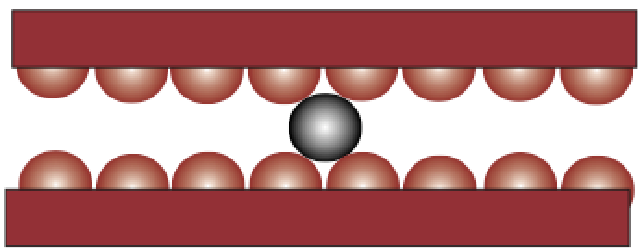
Dry



A naive idea :

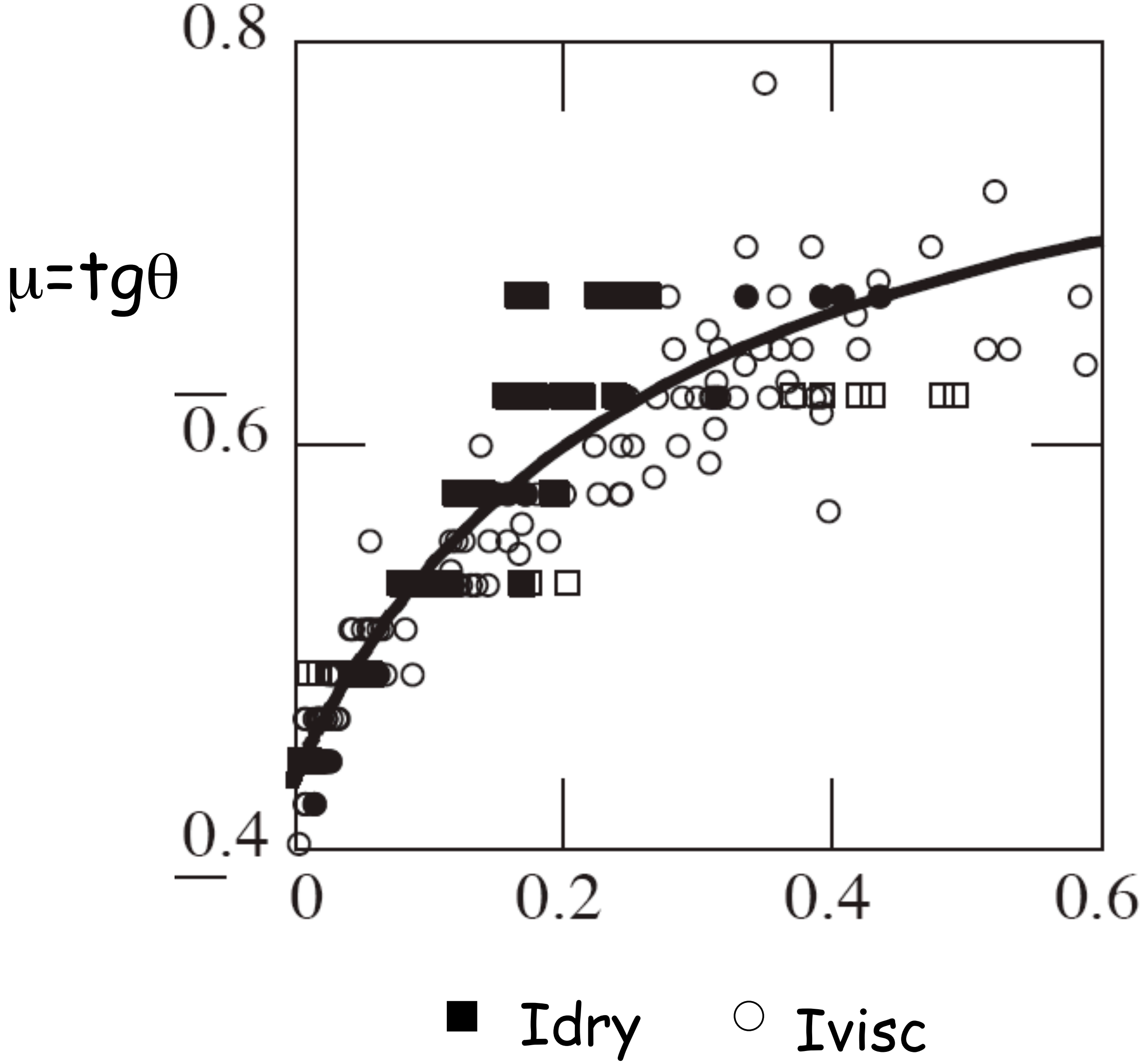
fluid only plays a role by changing the time scale of rearrangements

$$\tau = P\,\mu(I) \qquad \text{with} \qquad I = \dot{\gamma}\, t_{micro}$$



viscous : $$t_{micro} = \frac{\eta_f}{\alpha P}$$

dry : $$t_{micro} = \frac{d}{\sqrt{P/\rho}}$$


0.8
0.6
0.4
0
0.2
0.4
0.6
μ=tgθ
Idry
Ivisc

Suggestion for immersed granular media:

$$\tau = P\,\mu(I)$$

$$\Phi = \Phi(I)$$

$$I = \frac{\eta\dot{\gamma}}{\alpha P}$$

Or

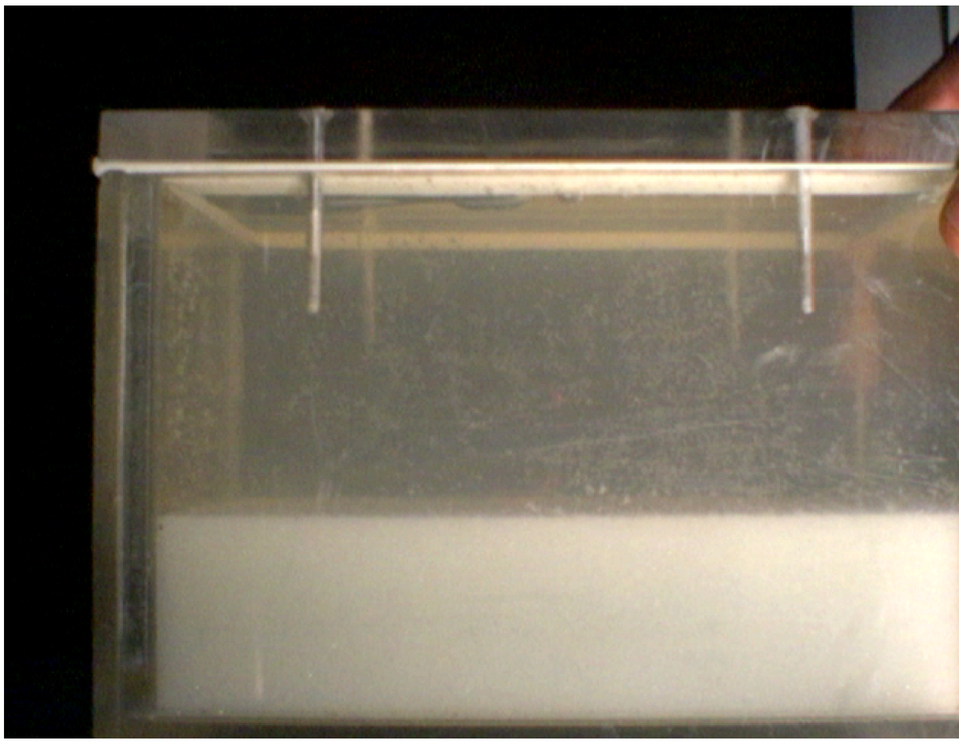
$$\tau = f_1(\Phi)\eta\dot{\gamma}$$

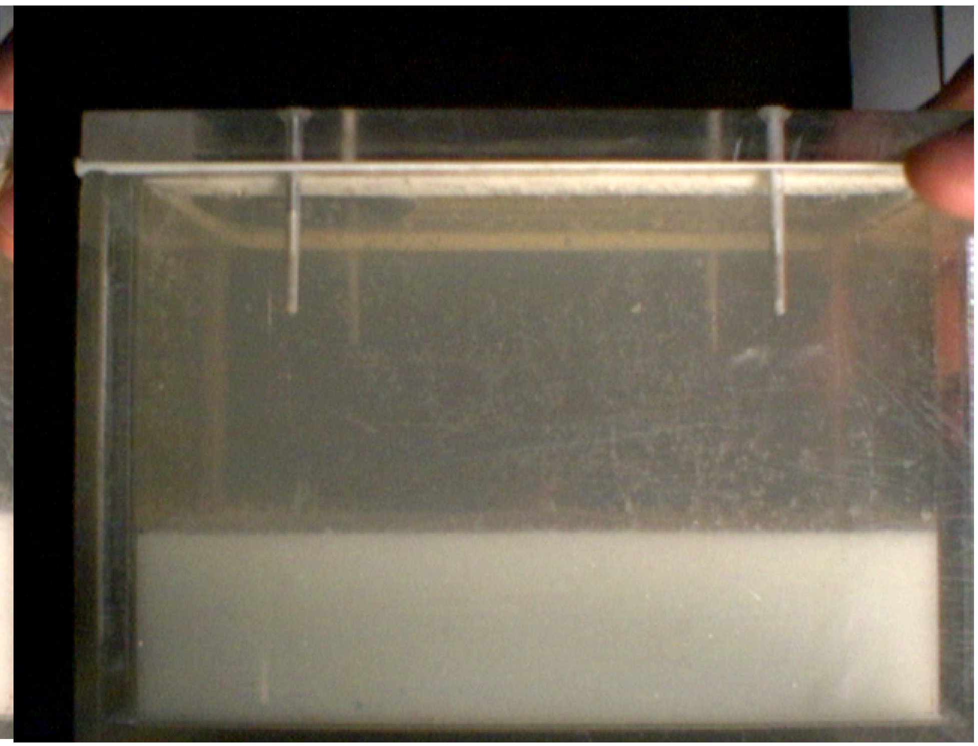
$$P = f_2(\Phi)\eta\dot{\gamma}$$

Similar to rheology of dense suspensions....

Morris and Boulay 99
Ovarlez et al 06...

A simple experiment:

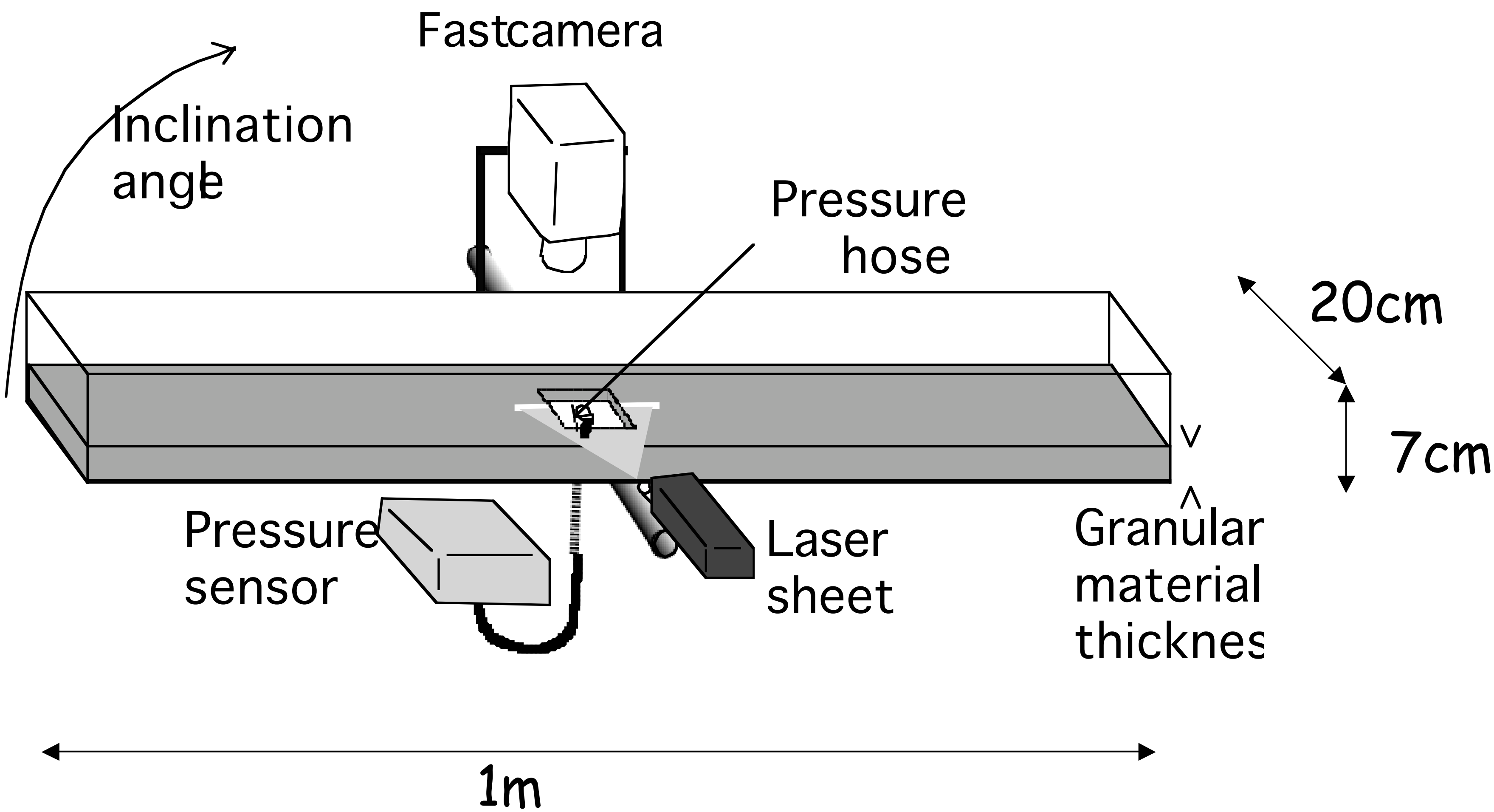




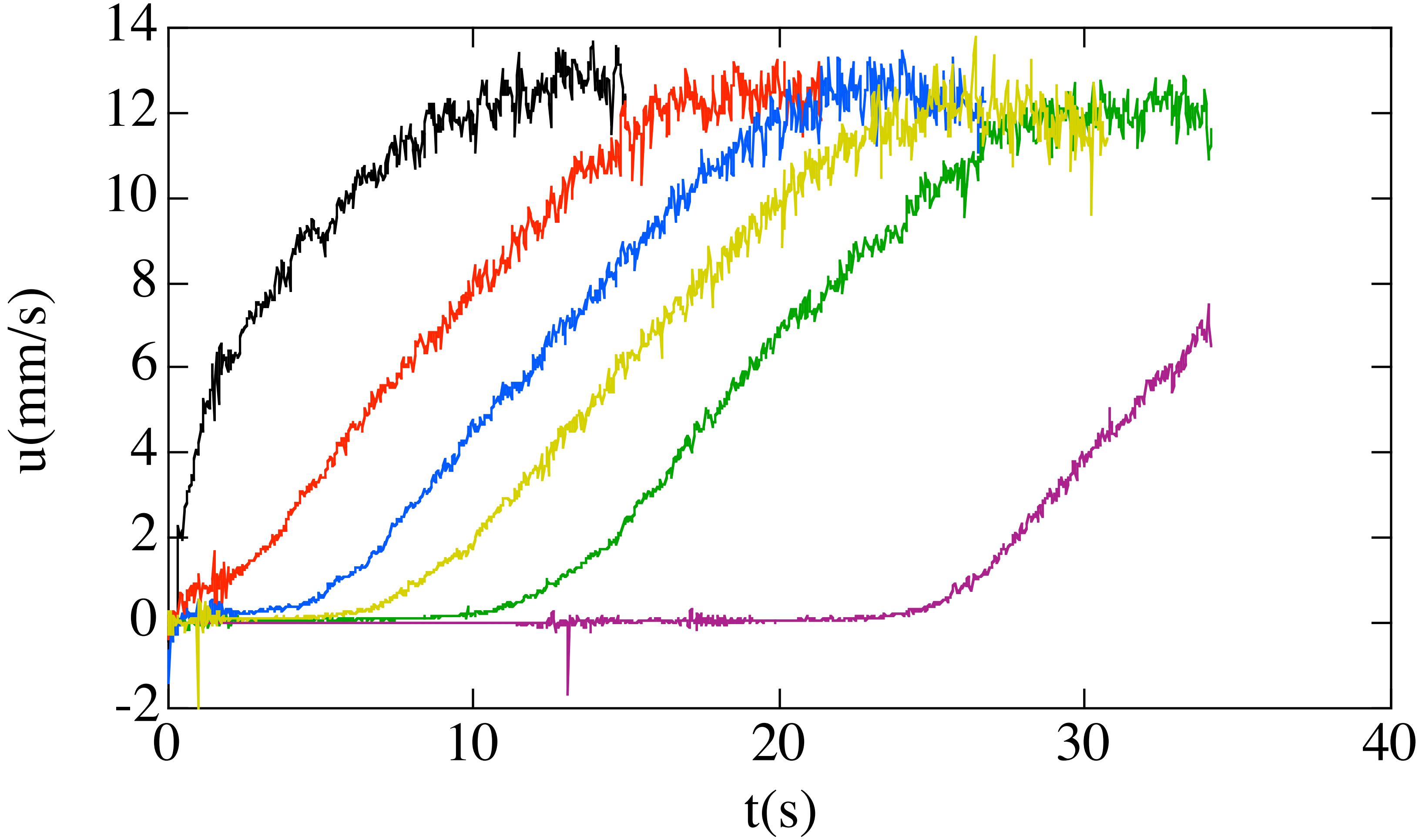
Loose sample

Dense sample

Experimental setup (Mickael Pailha PhD)



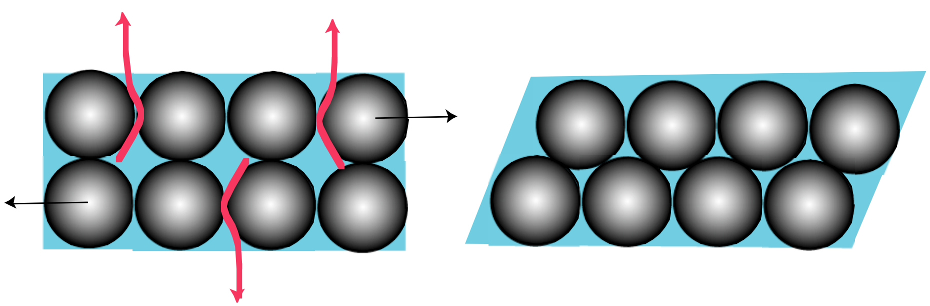
Free surface Velocity starting at different initial volume fraction



Hand waving argument

(Iverson Rev. Geo. 97, Huppert 04)

Loose case



$\Phi \nearrow$

$\Rightarrow$ Fluid expelled

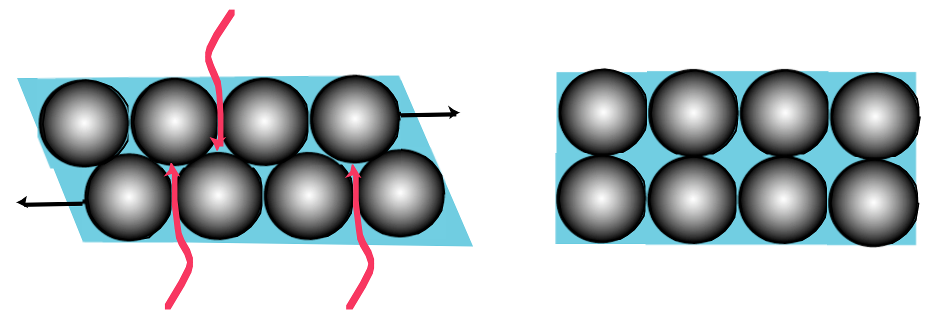
$\Rightarrow P_{fluid} \nearrow$

$\Rightarrow P_{grain} \searrow$

$\Rightarrow$ Friction $\searrow$

$\Rightarrow$ Less friction between grains

Dense case



$\Phi \searrow$

$\Rightarrow$ Fluid sucked

$\Rightarrow P_{fluid} \searrow$

$\Rightarrow P_{grain} \nearrow$

$\Rightarrow$ Friction $\nearrow$

$\Rightarrow$ higher friction between grains

We need to put together

Volume fraction variation at small deformations

Soils mechanics

Rheology of Immersed granular flows

This work

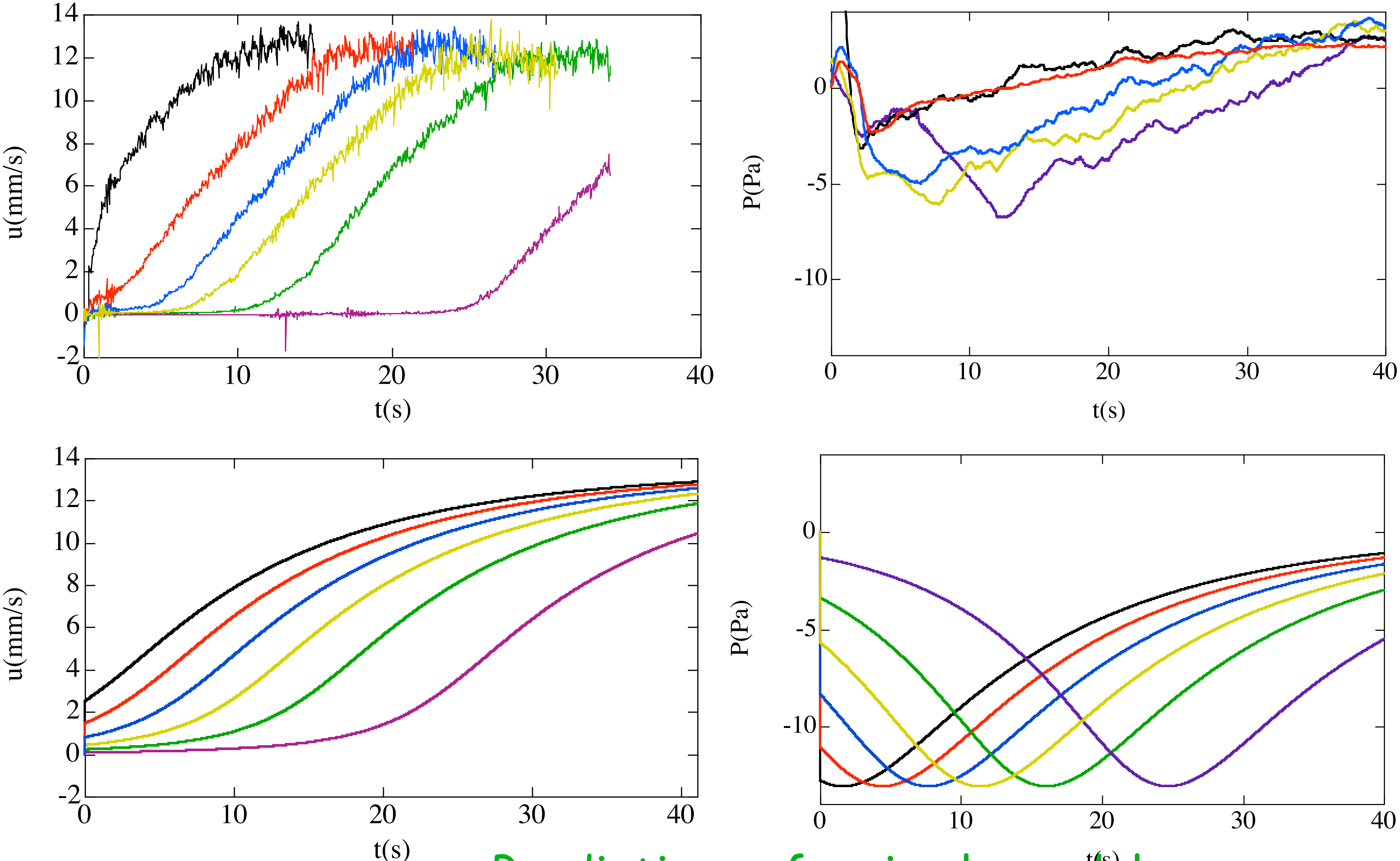
Two-phase flows equations

fluid mechanics

experiments:

Velocity

Pressure



Predictions of a simple model:

Conclusions

A visco-plastic description of granular flows is a first step towards a granular rheology (quantitative predictions in 3D)

⇒Other configuration (Pb of simulation of a peculiar visco-plastic rheologie…)

⇒Transition to the quasi-static regime
Jamming transition

⇒ transition to the kinetic gaseous regime

⇒Link with the micro-structure

The granular rheology gives a new point of view for immersed granular media: link with suspension rheology ?

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