Nonlinear Dynamics of Fluid Motion

- "Nonlinear Dynamics" has deepened our understanding of fluid dynamics, and taught us many lessons about nonlinear phenomena.

- No movies in this version; some may be found at www.haverford.edu/physics-astro/Gollub/lab.html

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Thanks to Collaborators

- **Postdocs and graduate students:** **G. Voth**, JC Tsai, W. Losert, A. Kudrolli, D. Vallette, B. Gluckman, J. Liu, O. Mesquita, T. Solomon, F. Simonelli, A. Dougherty, M. Heutmaker, M. Lowe, S. Ciliberto.
- **Undergraduate collaborators:** E. Henry, B. Pier, Antony Bak, David G.W. Cooper, David Rothstein, David Schalk, Mark Buckley, and Ben Bigger and many previous students.

NSF - DMR-0072203 (Haverford) and DMR-0072203 (Penn)

Central Ideas from Nonlinear Dynamics

- **Using phase space to describe fluid systems**: mode amplitudes and their evolution.
- **Geometrical thinking:** attracting and repelling fixed points; multiple attractors; limit cycle attractors; chaotic attractors representing nonperiodic flows.
- **Understanding instabilities as bifurcations** or qualitative changes in the phase space of a system, e.g. birth or death of fixed points.

Central ideas - II

- **Common features shared by disparate systems:**
  - Supercritical vs. subcritical transitions
  - Bifurcation scenarios
  - The central role of symmetry
  - Patterns and spatiotemporal chaos described by generic amplitude equations in large systems

A Few Examples

- The role of stretching in fluid mixing
- Clustering of particles suspended in a liquid.
- Nonlinear waves on fluid interfaces
- Multiple stability in granular flow

1. Stretching and Fluid Mixing

Measuring fluid stretching can illuminate mixing and illustrate nonlinear dynamics.

Greg Voth, George Haller, Greg Dobler, Tim Saint.

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Types of Fluid Mixing

- **Turbulent mixing:** Random structures produced by fluid instability at high Reynolds number Re stretch and fold fluid elements.
- **Chaotic mixing:** Some laminar flows at modest Re can produce complex distributions of material. Fundamental process: Non-reversible stretching and folding of fluid elements, with diffusion at small scales. Chaos in real space.

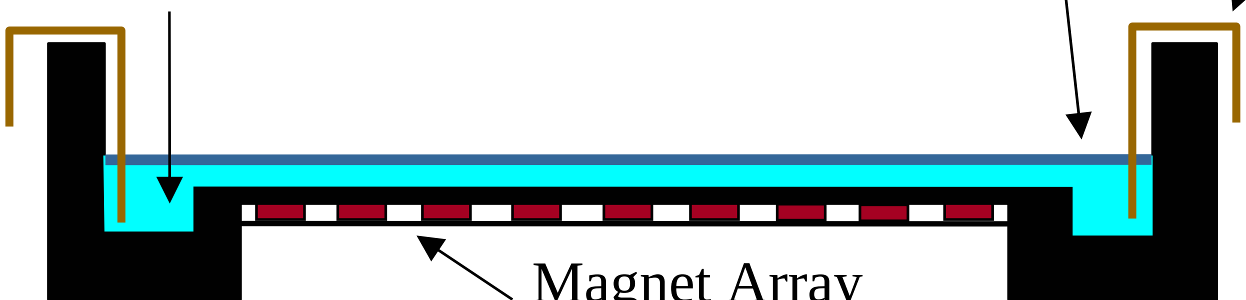
Background on Stretching in Chaotic Mixing

- Chaotic mixing has been well studied, and the importance of stretching was emphasized and investigated by J. Ottino and colleagues Khakhar, Swanson, and Muzzio.
- Others (Rom-Kedar, Leonard, Wiggins) have illuminated the connection between nonlinear dynamics and mixing theoretically.
- What is new in our work is the experimental measurement of space and time resolved stretching fields.

Apparatus:
2D Magnetically Driven Fluid Layer

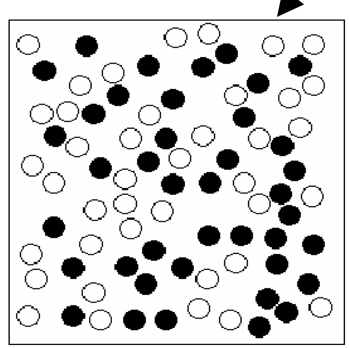
Glycerol and water a few mm thick, containing fluorescent dye.

Electrodes



Magnet Array

Top View:



Periodic forcing:

$$I(t) = I_0 \sin(2\pi f t)$$

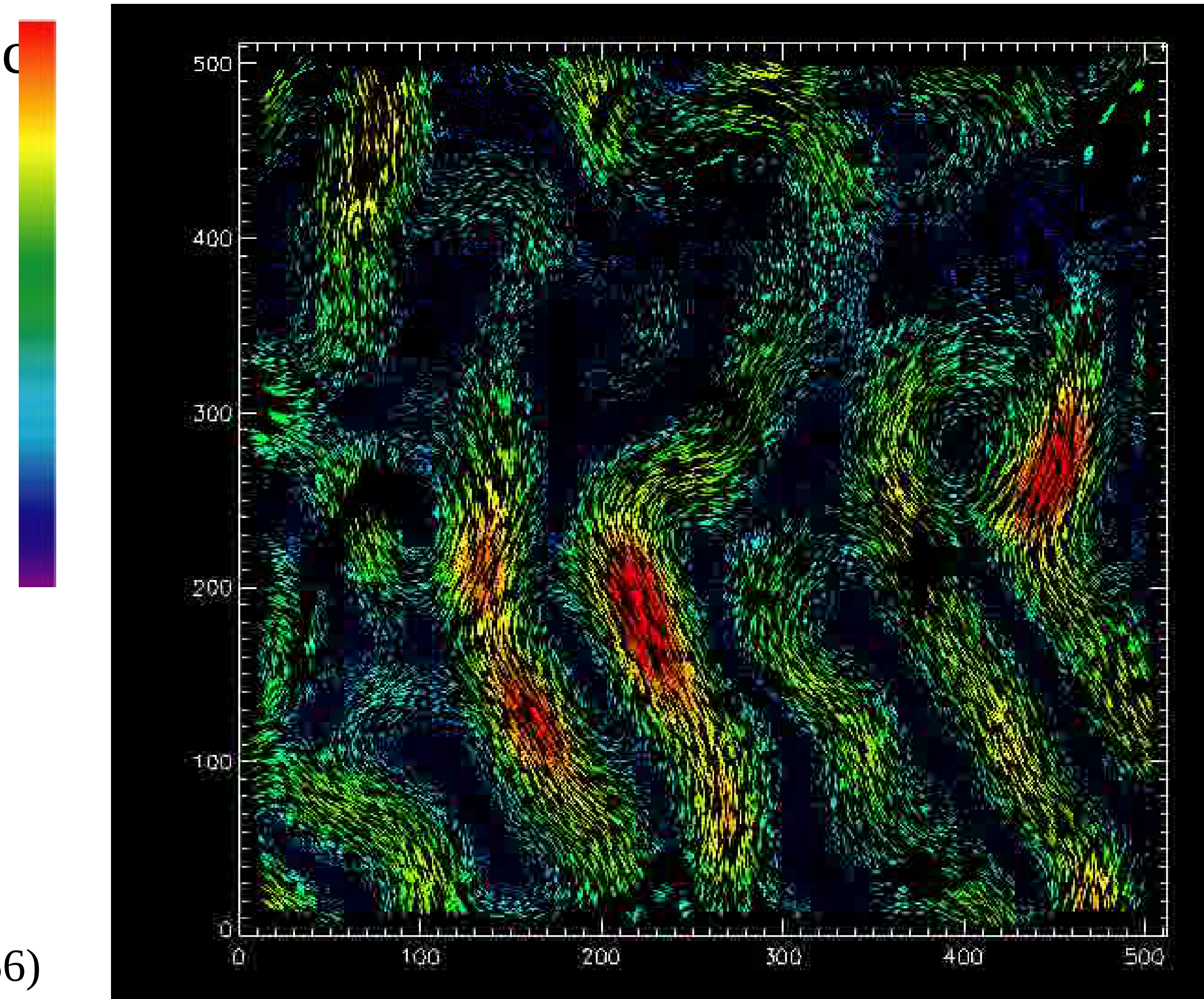
Precise Particle Tracking

- ~ 800 fluorescent particles tracked simultaneously.

Velocity Fields

0.9 cm/sec

0



(p=5, Re=56)

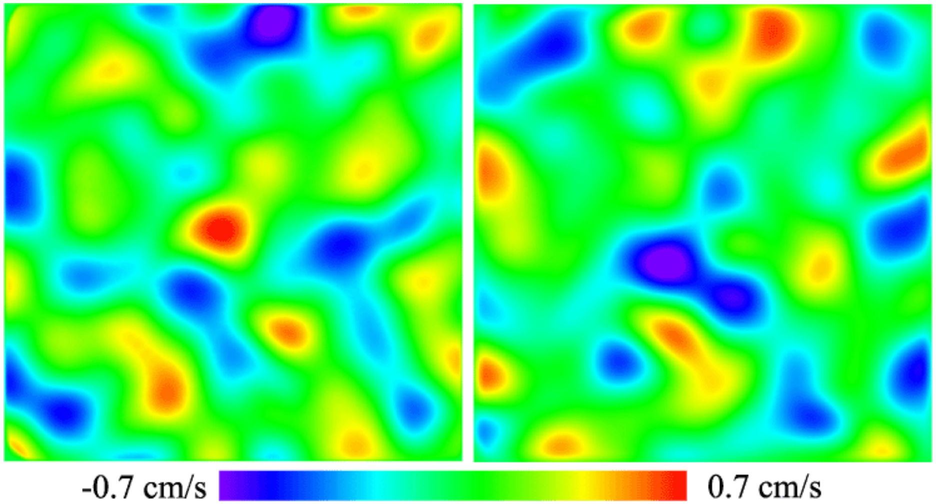
Why Do Periodic Flows Mix? Breaking Time Reversal Symmetry

- To mix, the flow can be time periodic, but must not be time reversible:

$$\mathbf{V}(t) \neq -\mathbf{V}(-t)$$

relative to any starting time.

- **Finite Reynolds number** $Re = VL/\nu$ can break time reversal symmetry:



Velocity at equal intervals before and after the moment of minimum velocity.

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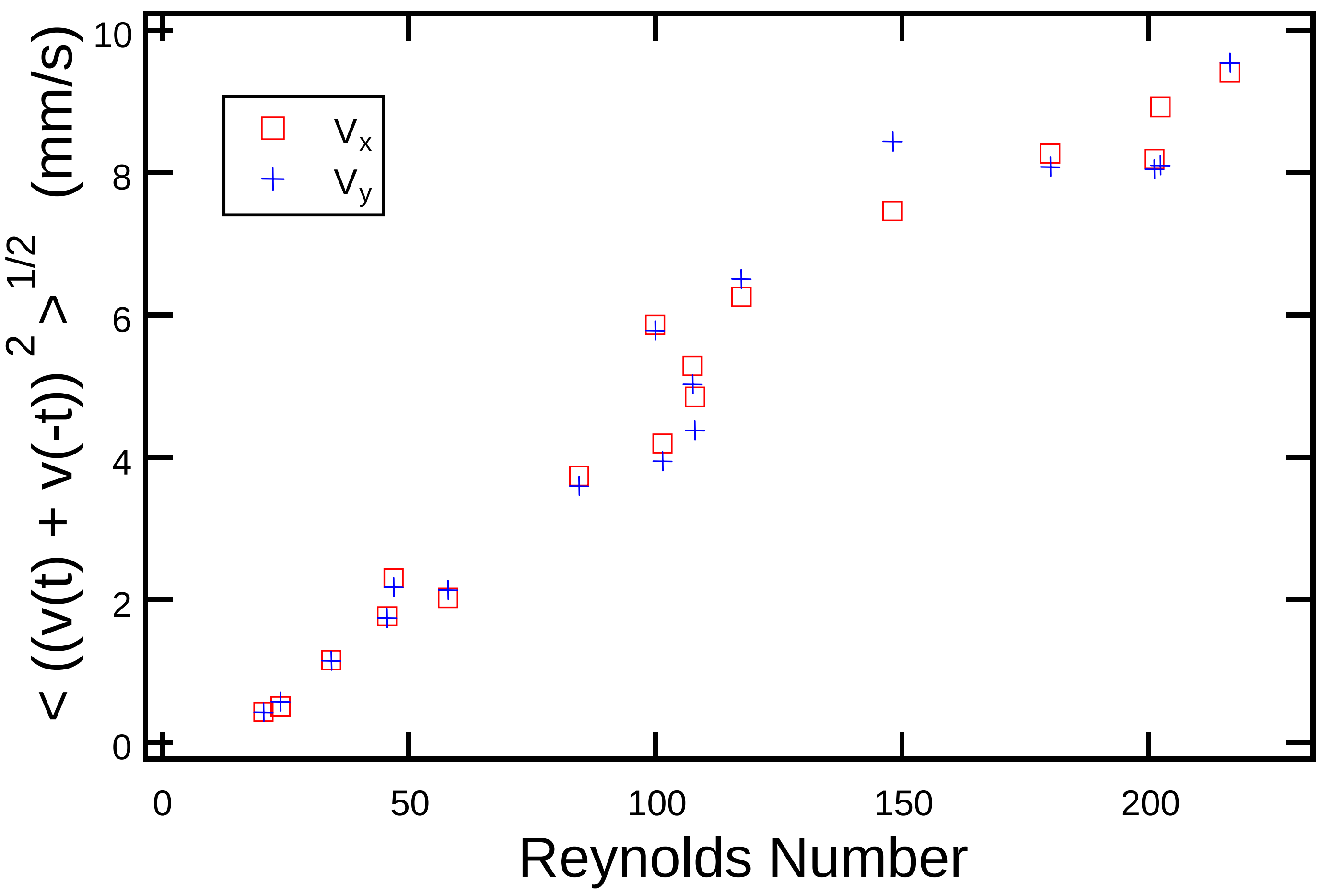
Breaking of Time Reversal Symmetry



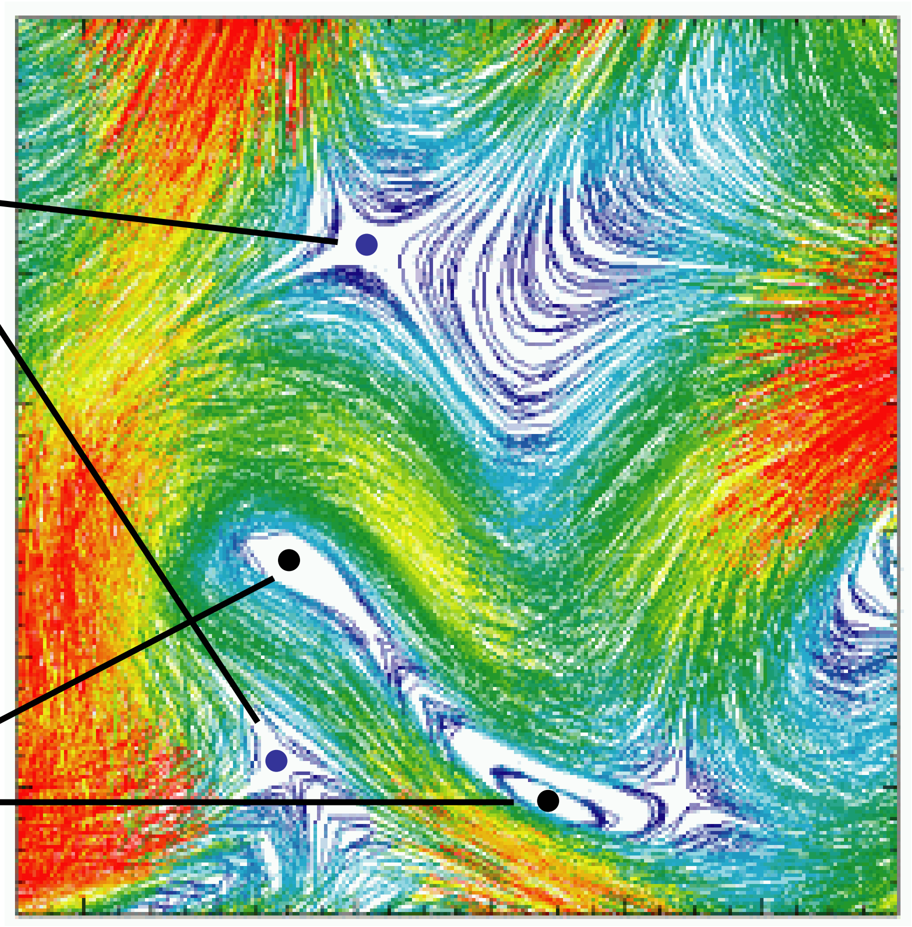
Structures in the Poincaré Map of the Flow

Displacements in one period, color coded.

Hyperbolic Fixed Points

A small part (6%) of the flow.

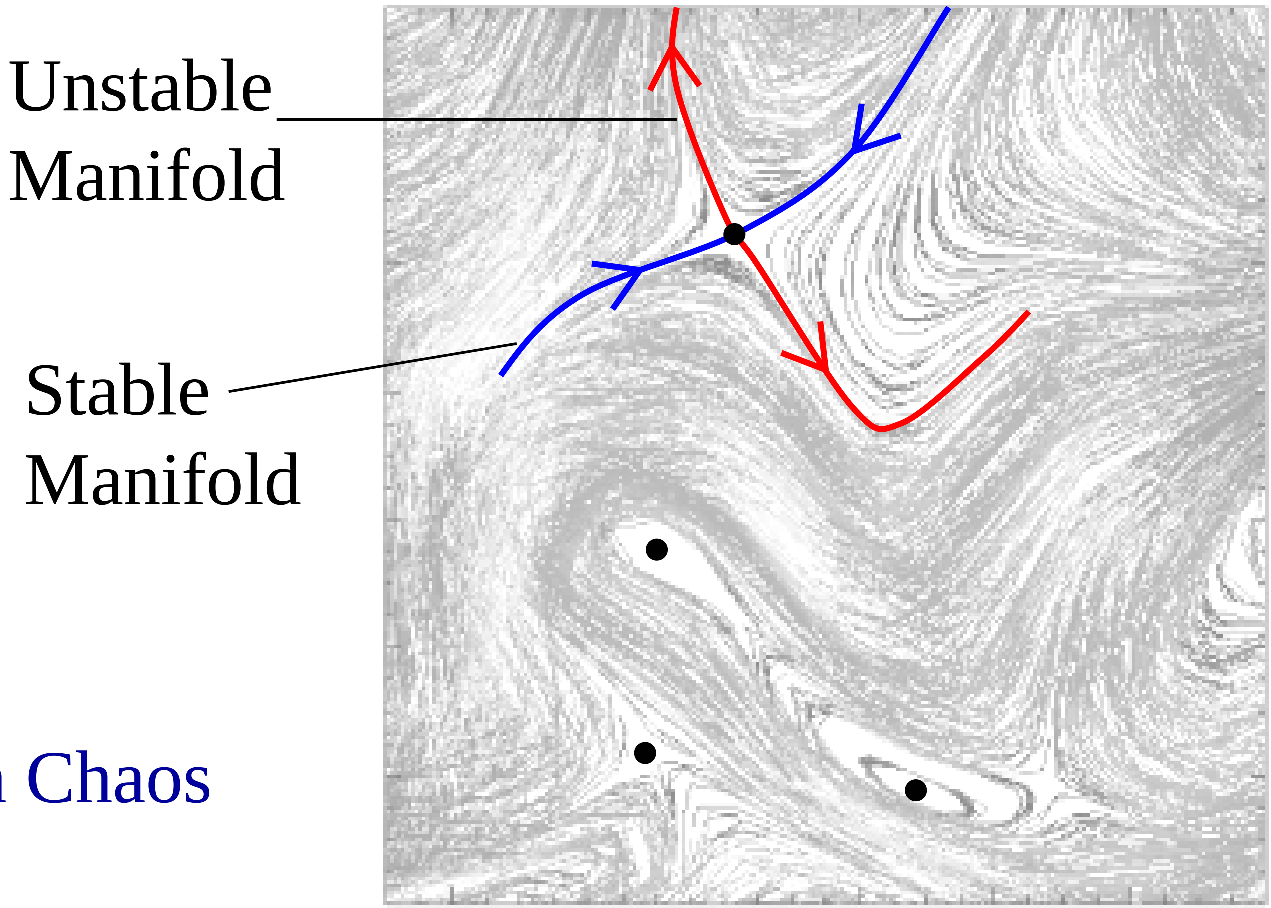
Elliptic Fixed Points



0 cm

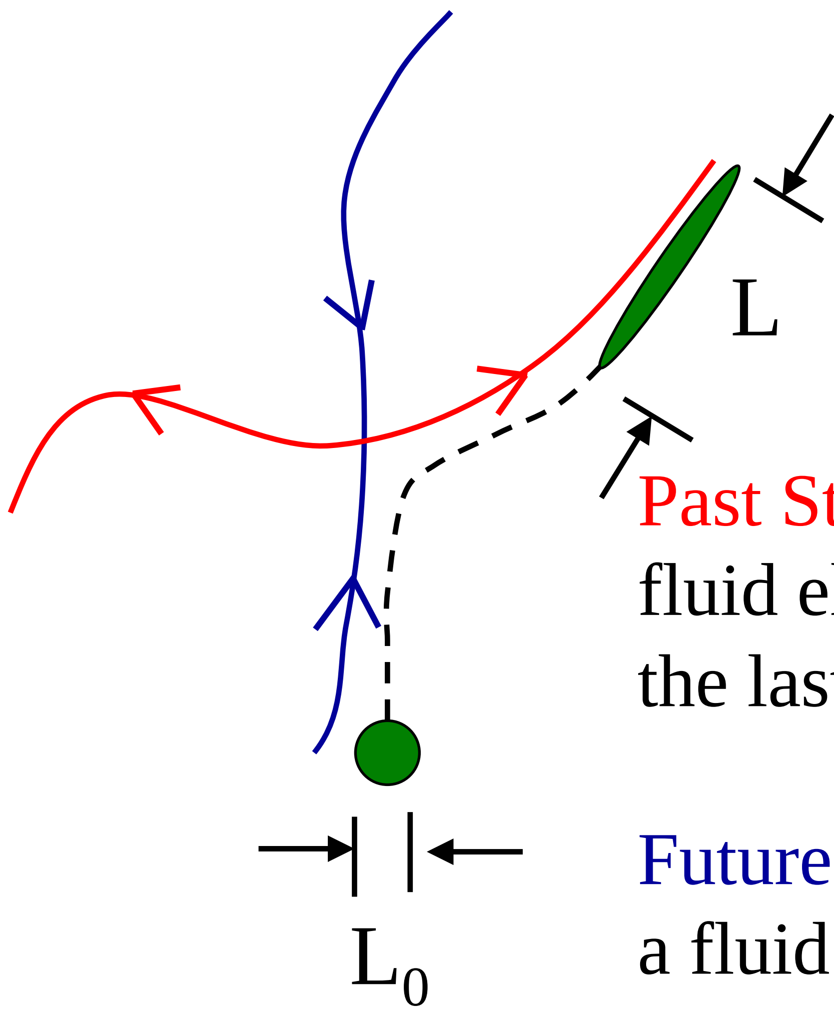
1.7 cm

Manifolds of Hyperbolic Fixed Points



Typical of
Hamiltonian Chaos

Definition of Stretching



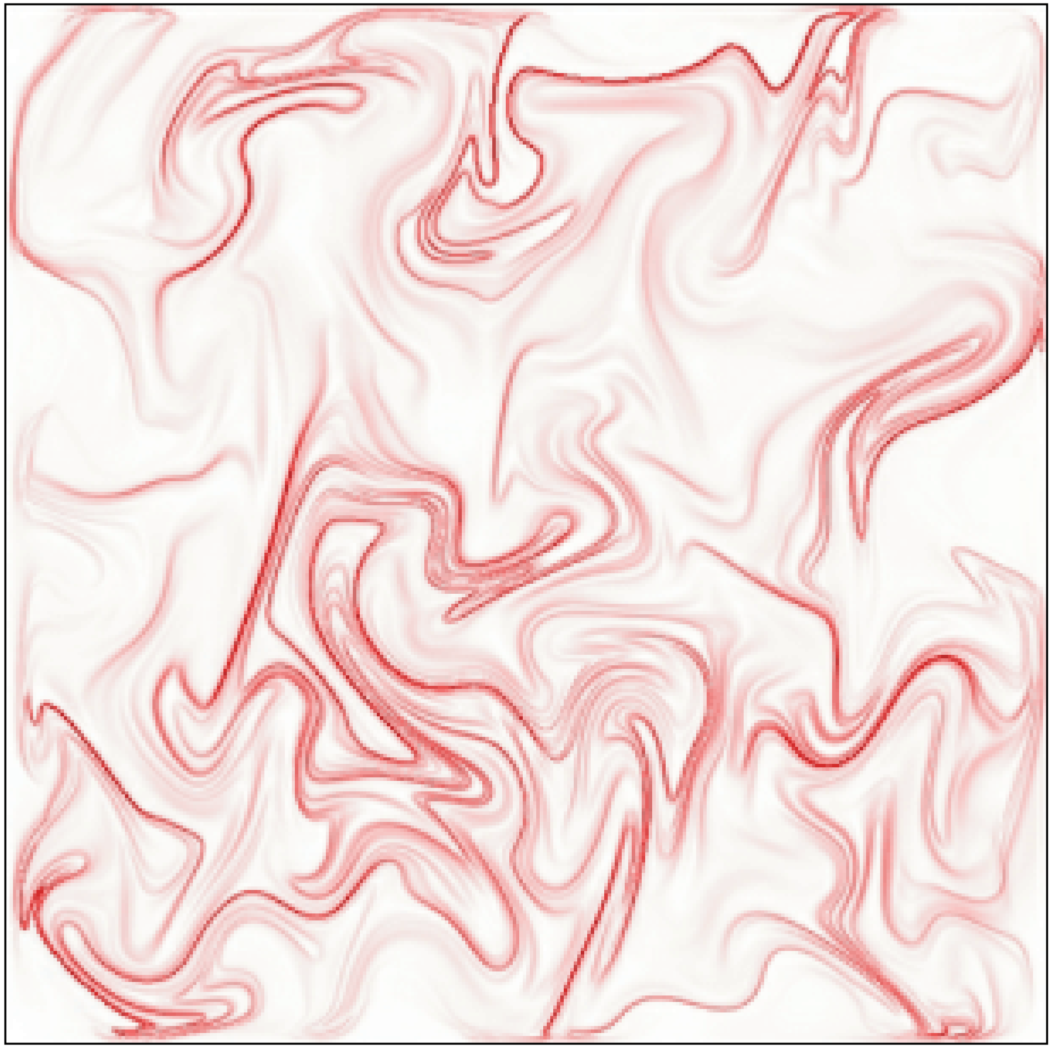
$$\text{Stretching} = \lim_{L_0 \to 0} (L/L_0)$$

Past Stretching Field: Stretching that a fluid element has experienced during the last Δt. (Large near unstable man.)

Future Stretching Field: Stretching that a fluid element will experience in the next Δt. (Large near stable man.)

Past Stretching Field

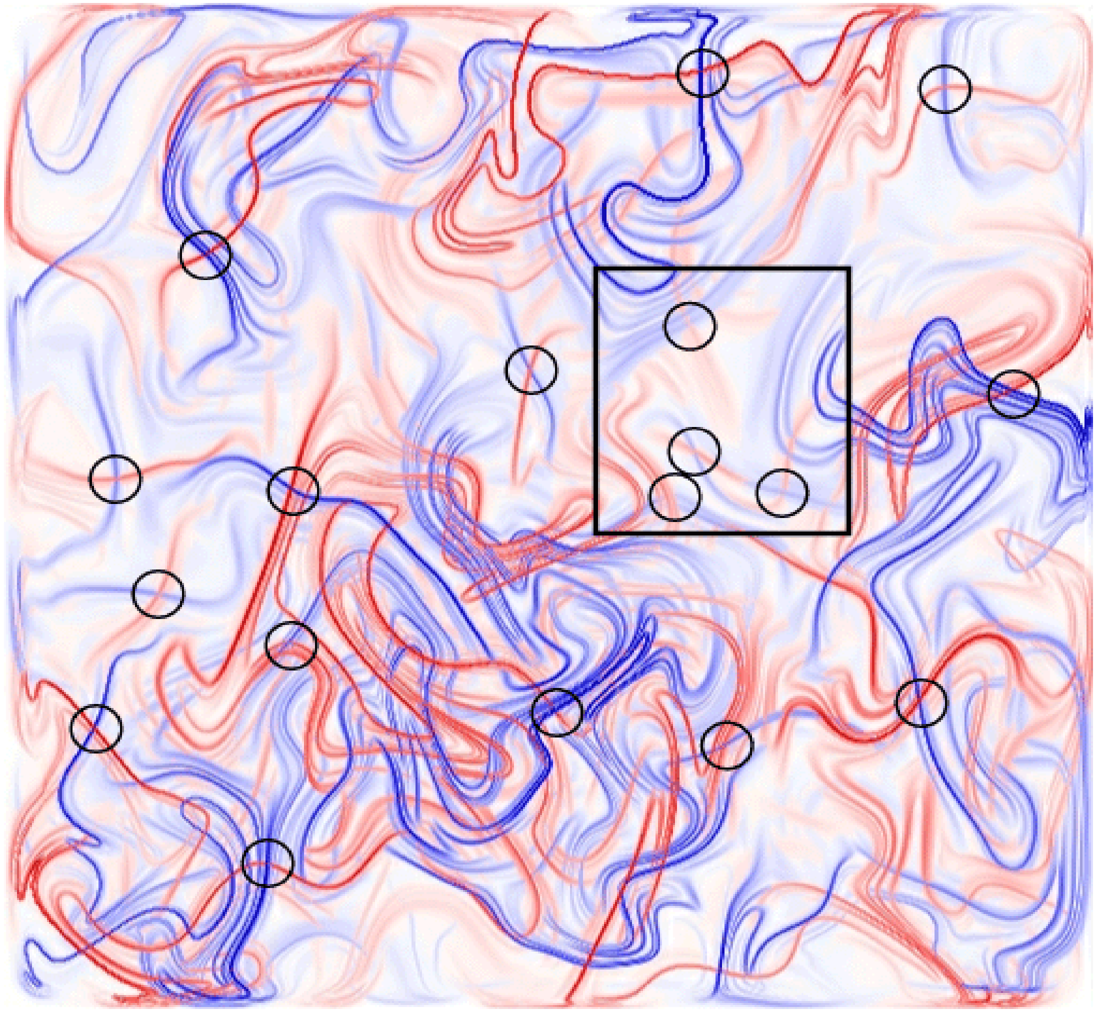
- Stretching is organized in sharp lines.



Re=45, p=1, Δt=3

Future and Past Stretching Fields

- Future Stretching Field (Blue) marks the stable manifold.
- Past Stretching Field (Red) marks the unstable manifold.
- Circles mark hyperbolic points. A "heteroclinic tangle".



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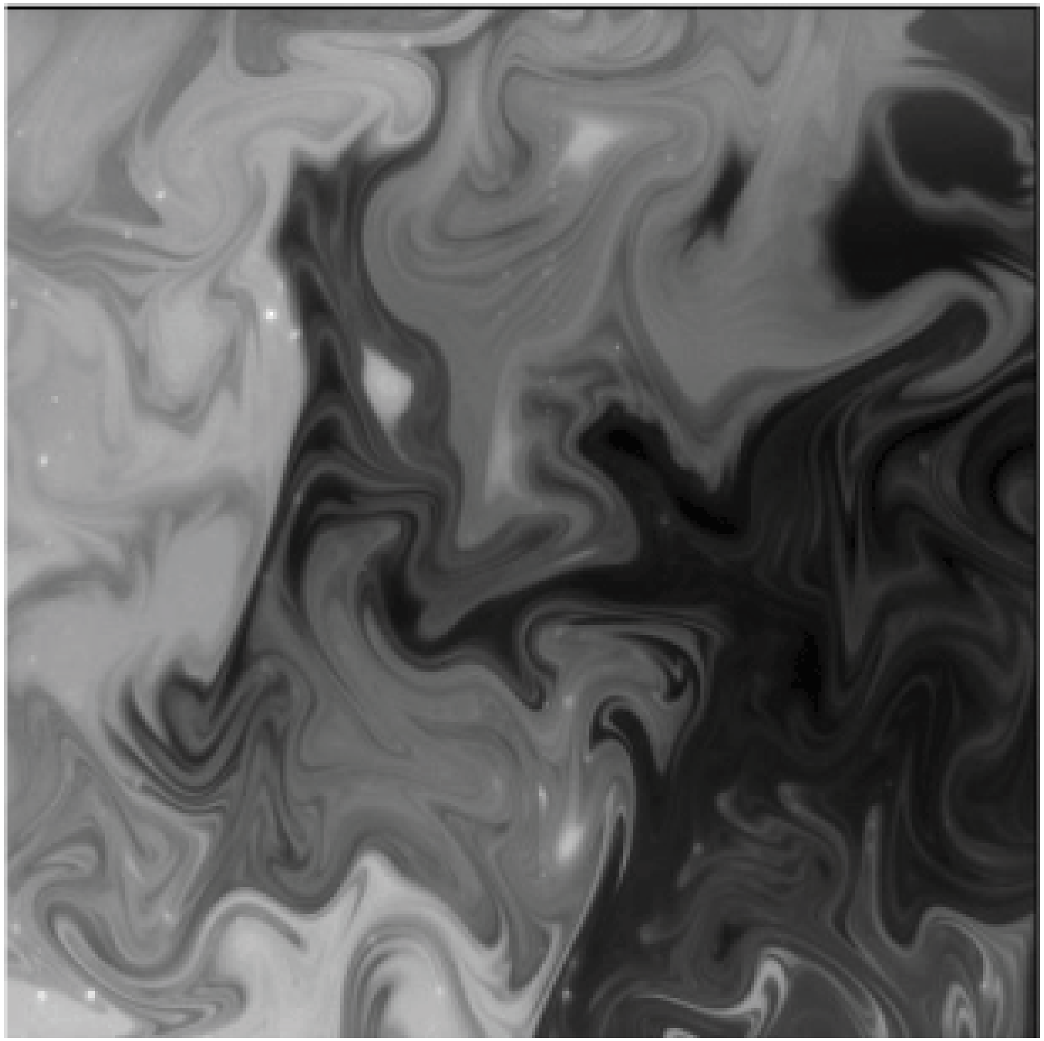
Understanding the Dye Concentration Field



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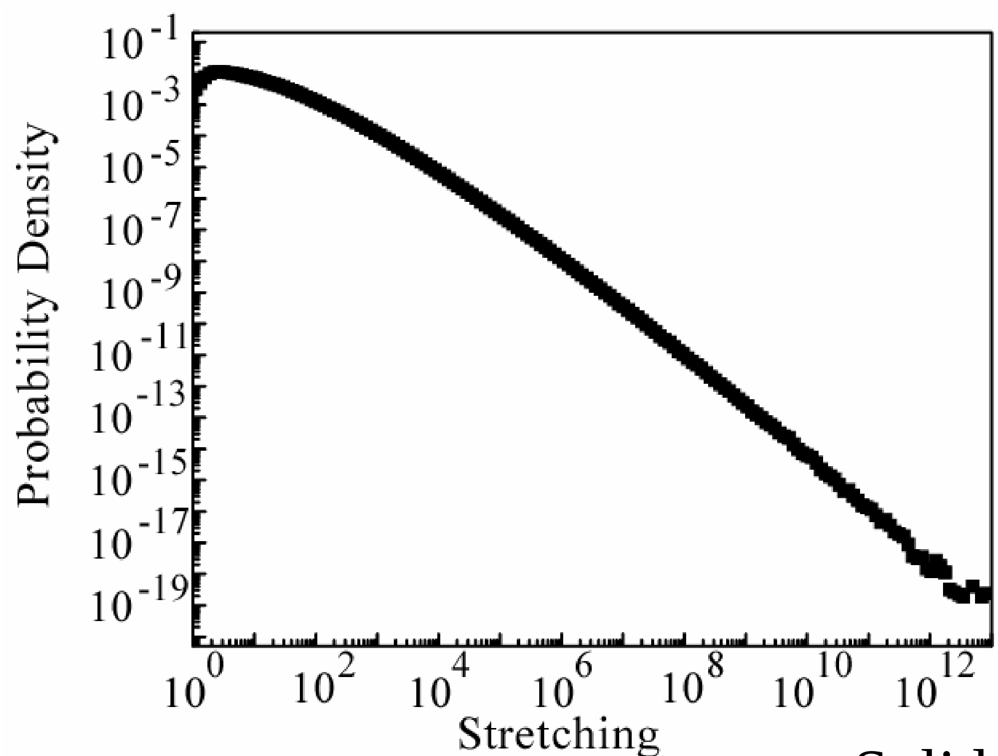
Unstable manifold (past stretching field) and the dye concentration field

- Lines of large past stretching (unstable manifold) are aligned with the contours of the concentration field.
- This is true at every time (phase).

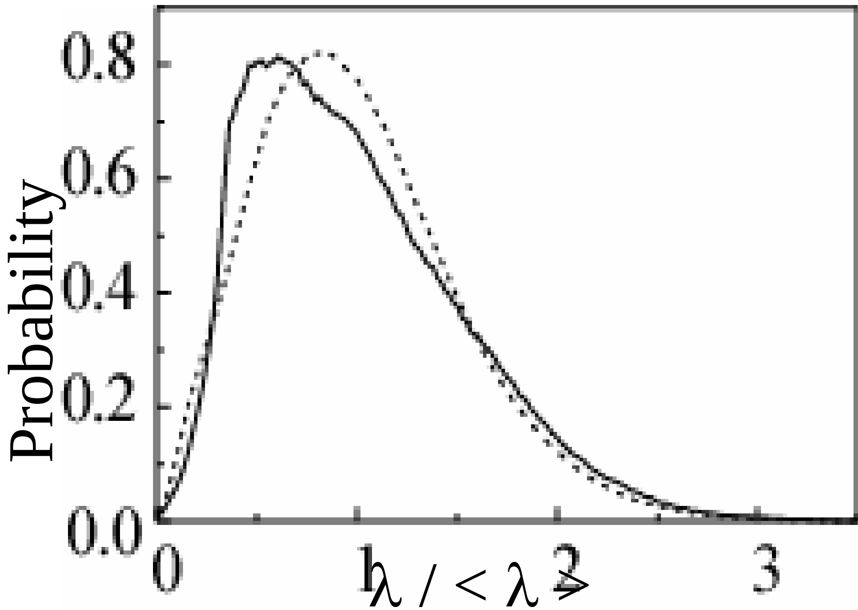
Stretching is Inhomogeneous: PDF

Stretching over one period



(Re=100,p=5)

Log(stretching)
(Finite Time Lyap. Exp.)



Solid: Re=45, p=1, $<\lambda>=1.9$ per^{-1}

Dotted: Re=100, p=5, $<\lambda>=6.4$ per^{-1}

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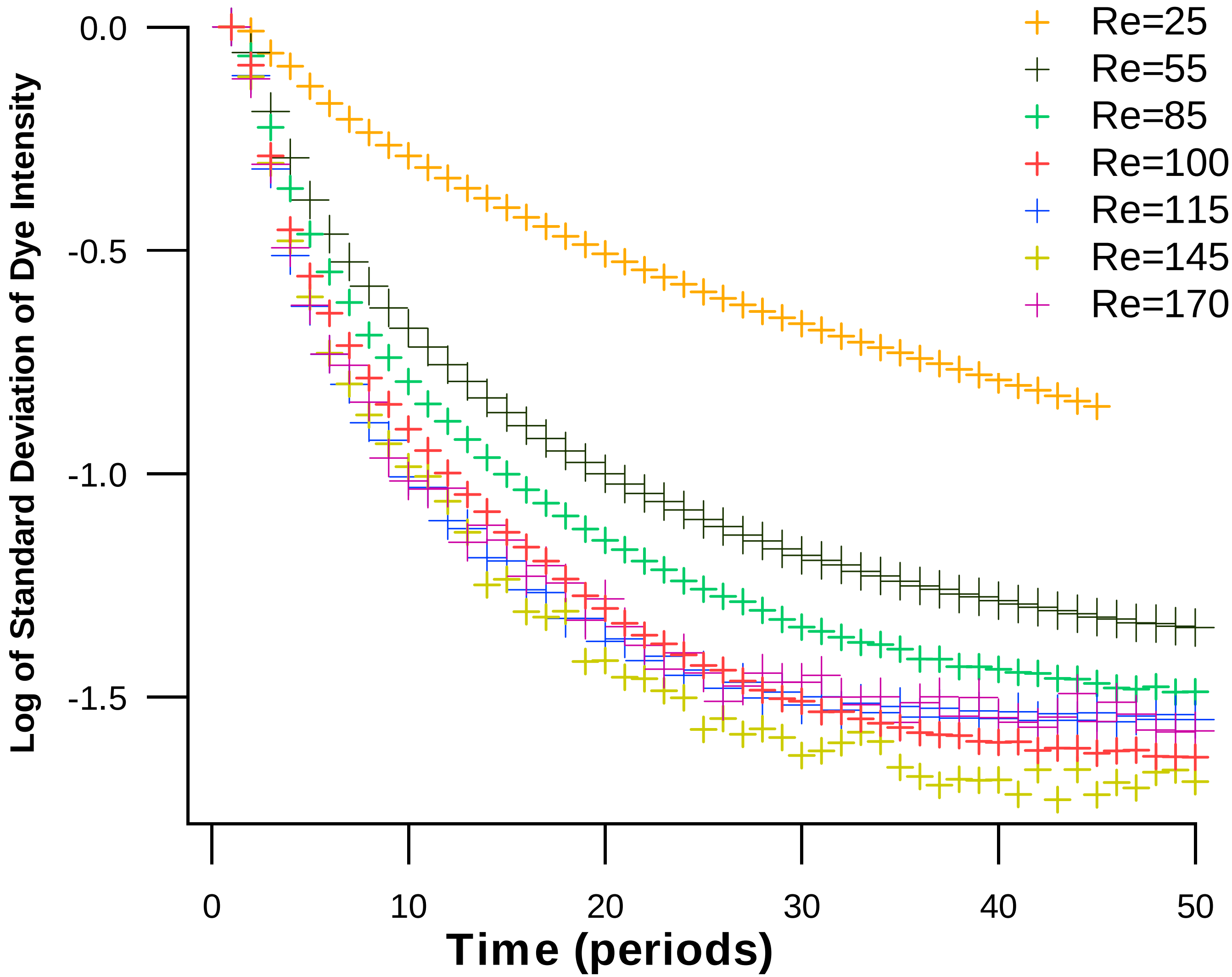
Decay of the Dye Concentration Field

Log

Contrast



Can Mixing Rates Be Predicted?

- Antonsen et al. predict the long time decay of concentration variance based on the prob. dist. P(h,t) of finite time Lyapunov exponents (stretching)
- Using our measured P(h,t) and their theory, we predict mixing (variance decay) rates a factor of 10 larger than is observed.

Predicting Mixing Rates - (cont.)

- Why is mixing so slow? Large system → Transport over long distances is important.
- Enhanced or "eddy" diffusion: By watching particles diffuse, we can succesfully predict the rate of decay of a scalar field.

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Future

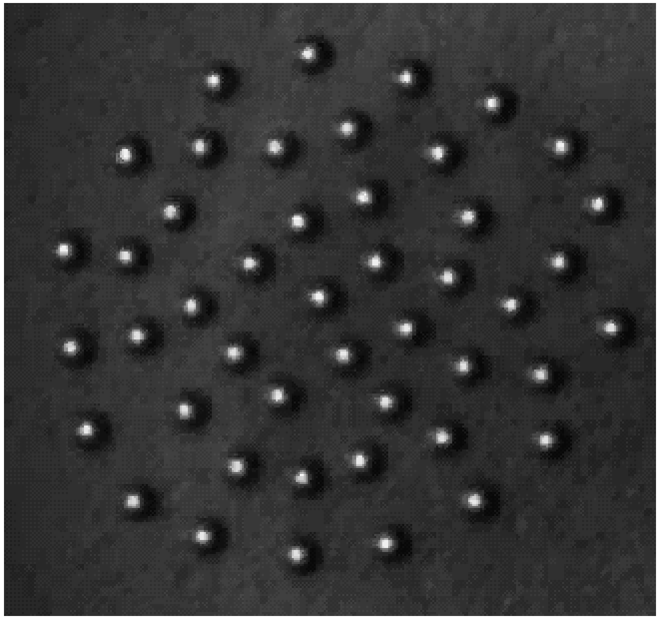
- The onset of non-periodic flow (2D turbulence) does not dramatically change mixing rates, to our surprise.
- Are the main features of non-periodic mixing captured by the periodic case discussed here?
- What will be the topological properties of stretching fields for non-periodic flows?
- Papers: G.A. Voth et al, Phys. Rev. Lett. (2002) and Phys. Fluids (2003)

2. Structure and Dynamics of Hydrodynamically Mediated Particle Clusters

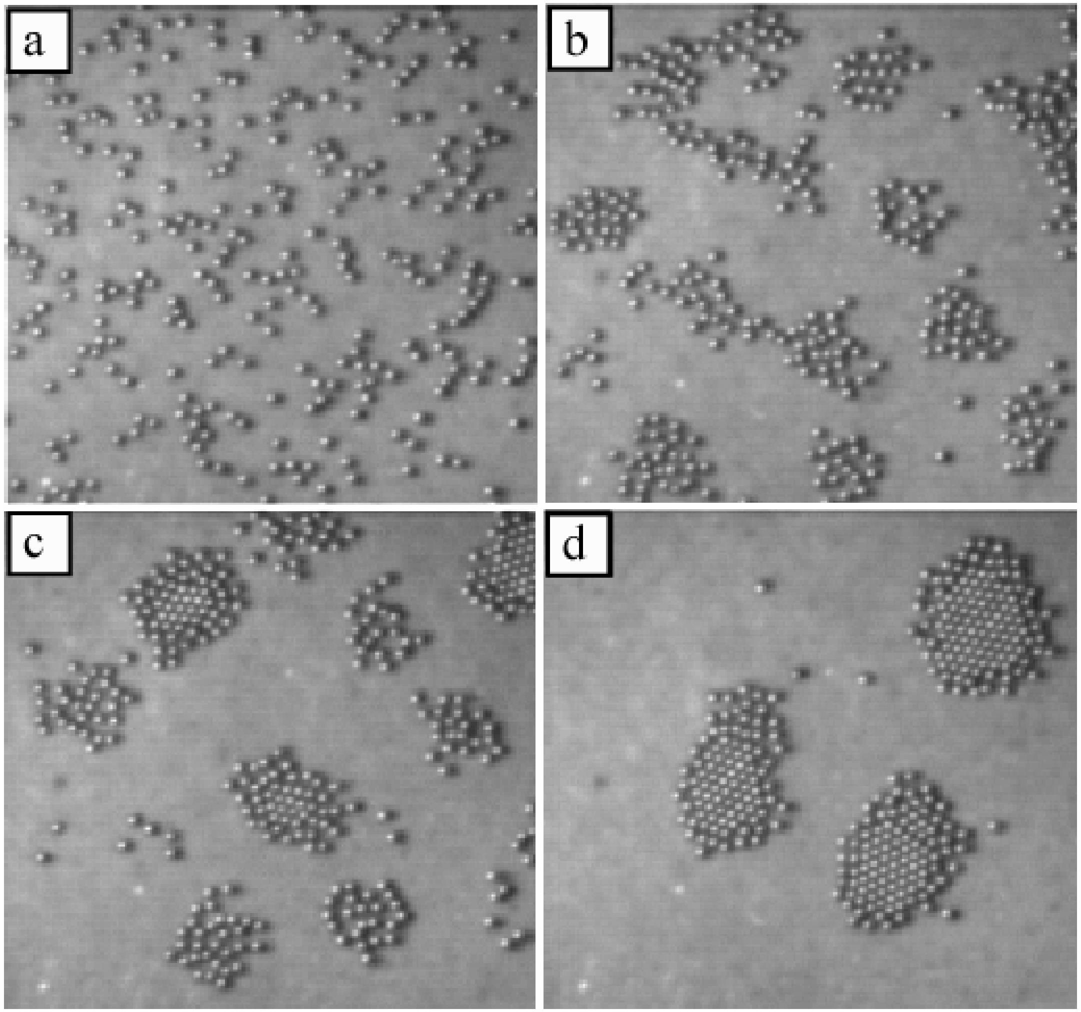
- Fluid mediated interactions between particles lead to forces between them.

- A variety of tunable patterns are produced, including chaotic states.

- **Greg Voth**, Charles Thomas, Ben Bigger, Mark Buckley, Michael Brenner, Howard Stone, and JPG.



Observations of Clustering



f=50 Hz

Γ=4.5

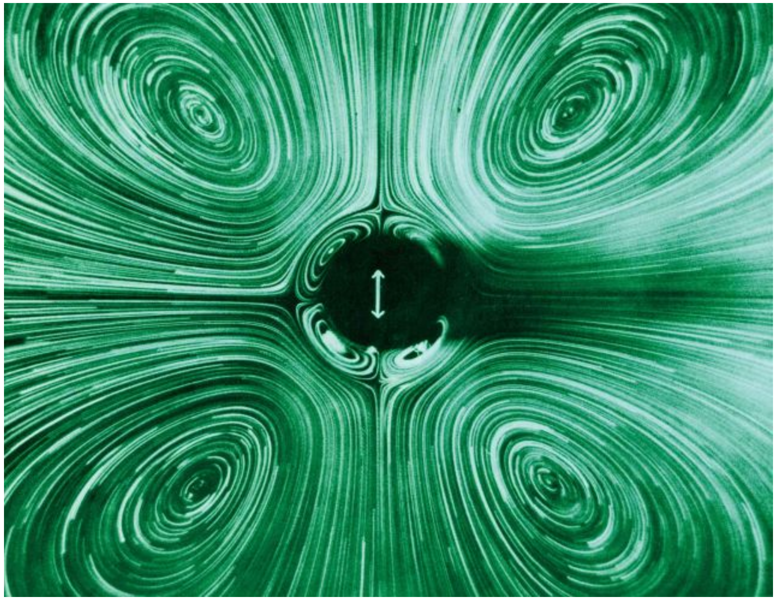
Related Phenomena

- Sedimentation – when particles fall in a fluid, they influence each other.
- Fluidized beds, widely used for catalysis.

Here: An example of physical self-assembly of ordered particulate structures

Chaos in a system of interacting particles

Rayleigh Streaming



Streaming flow for a particle in a fluid - no boundary

From Van Dyke, An Album of Fluid Motion

Origin of Long Range Attraction

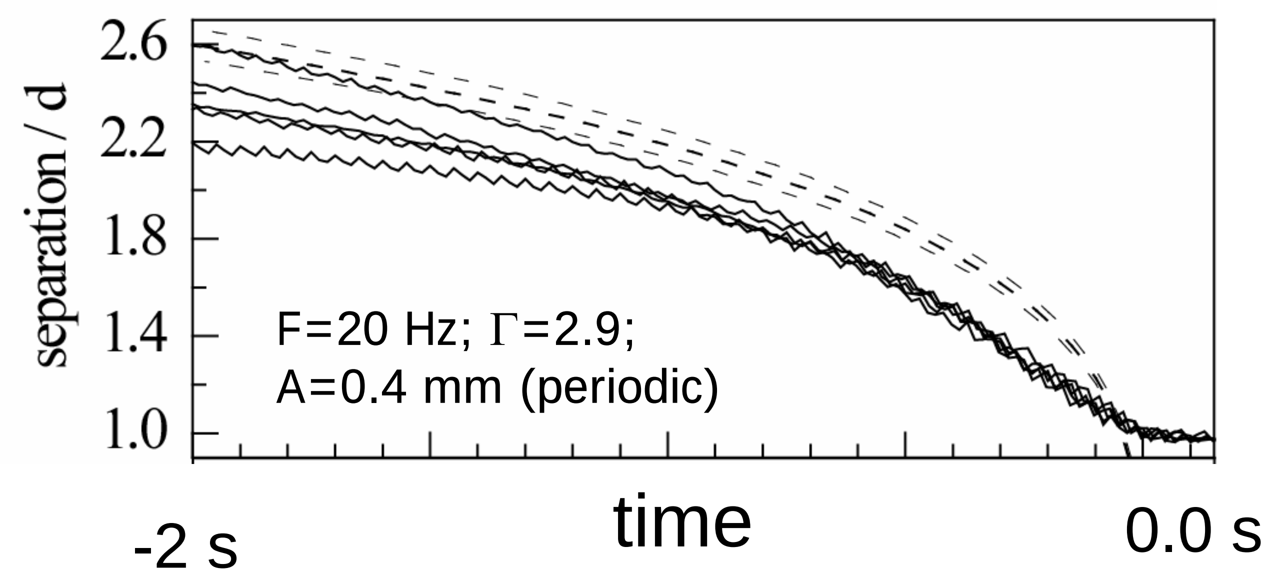
- Rayleigh: steady streaming flow away from poles and toward the equator generated outside the viscous boundary layer (0.2 mm) surrounding the particle.
- Predicted inflow velocity

$$V(r) = -0.53Aa^2\sqrt{\omega\nu}/r^3$$

(via collaborators M. Brenner and H. Stone).

- Particles follow this flow as they approach each other.
- However, see Voth and Otto, KC004, Tuesday 08:39

Approach Curves for 2 Particles



- Dashed: theory;   Solid:experiments;

t = 0 at end of approach.

- At higher excitation amplitude, the particles do not touch. → Repulsive force also.

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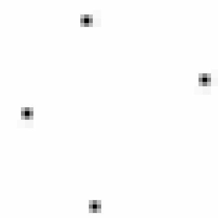
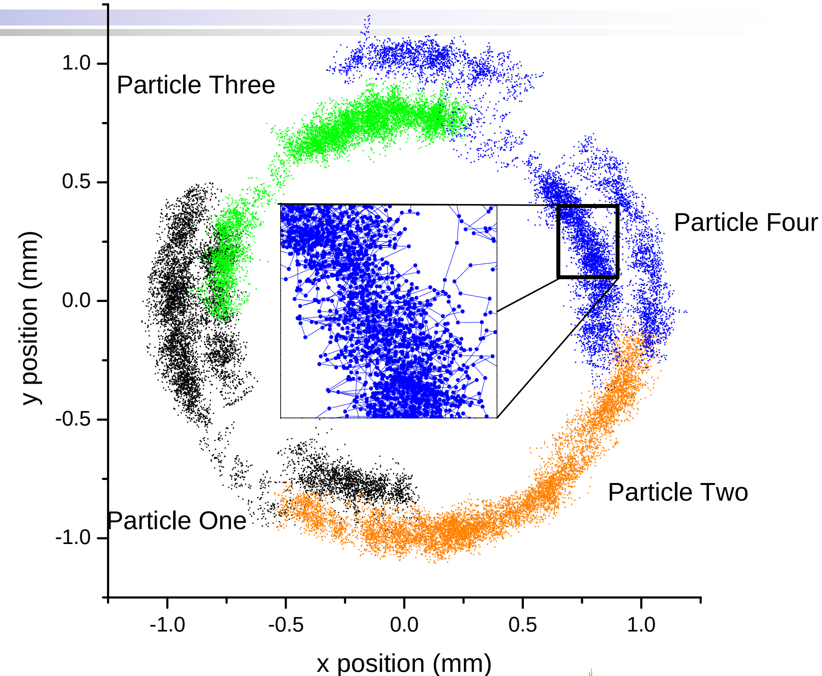
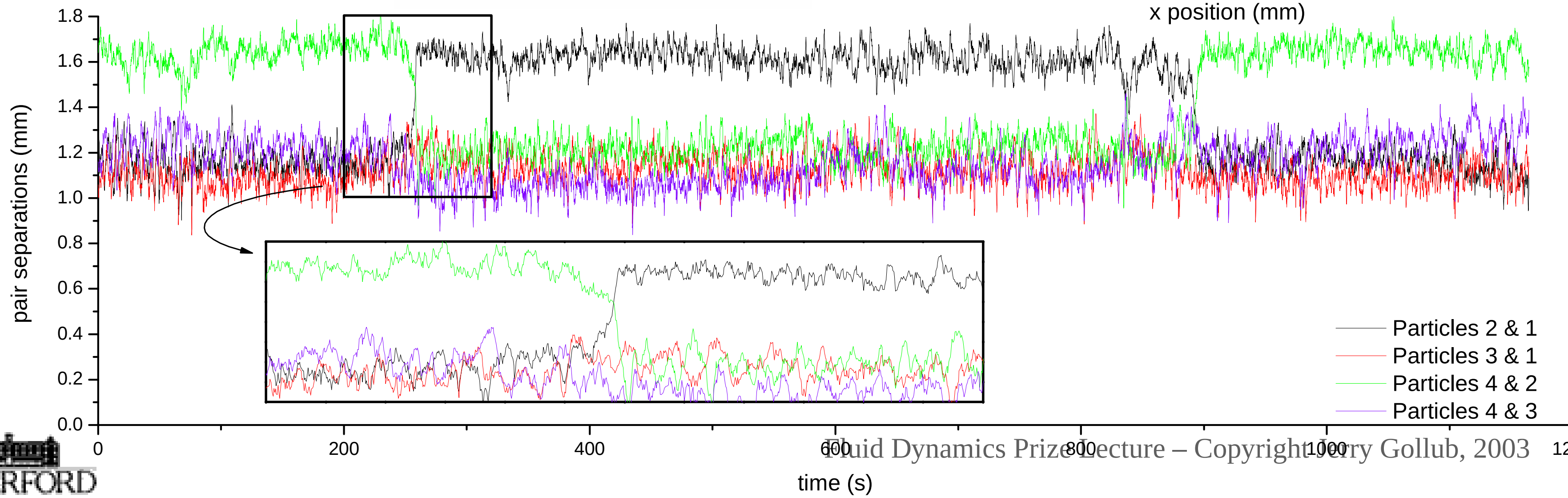
Trapezoidal Cluster - Chaotic Motion

N=4, Γ=2.97.
(a) trajectories;
(b) spacings vs. time.
Note transitions.







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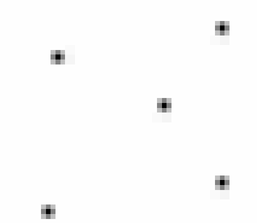
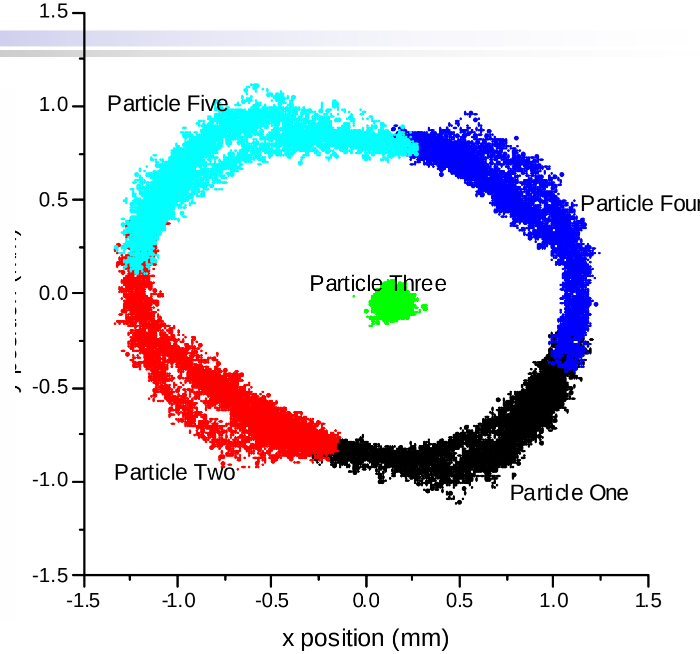
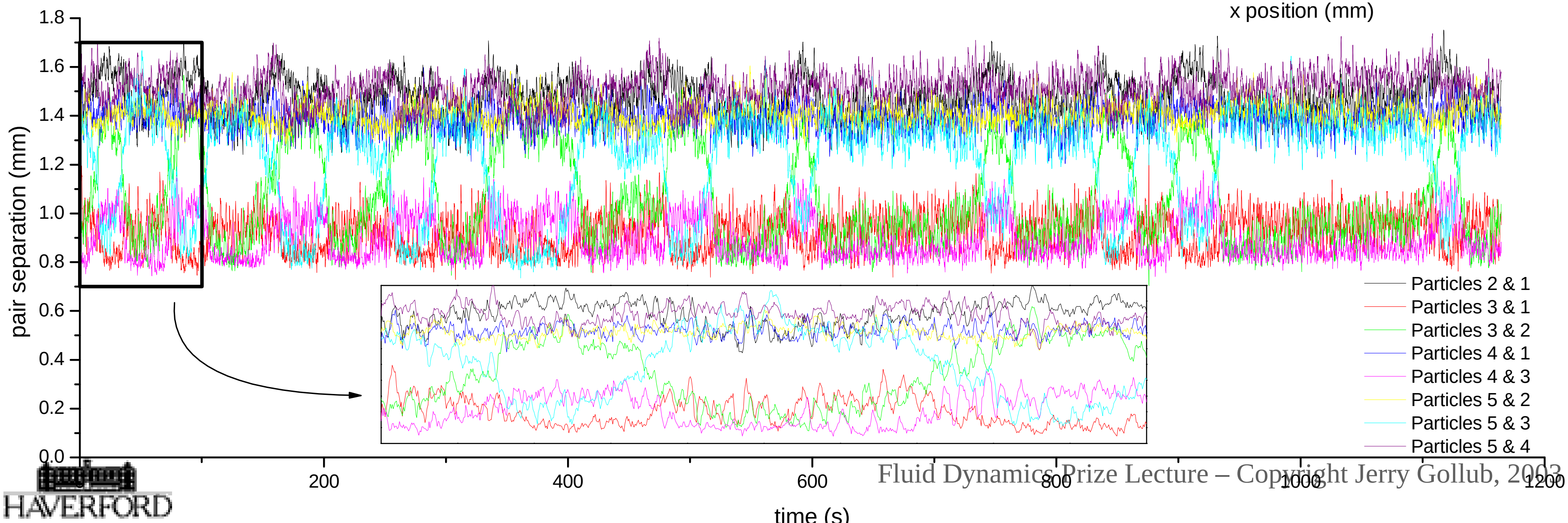
Fluctuating Five Particle Cluster

(a) Trajectories (Γ=2.96)
(b) separations vs. time.
Nonperiodic oscillations.







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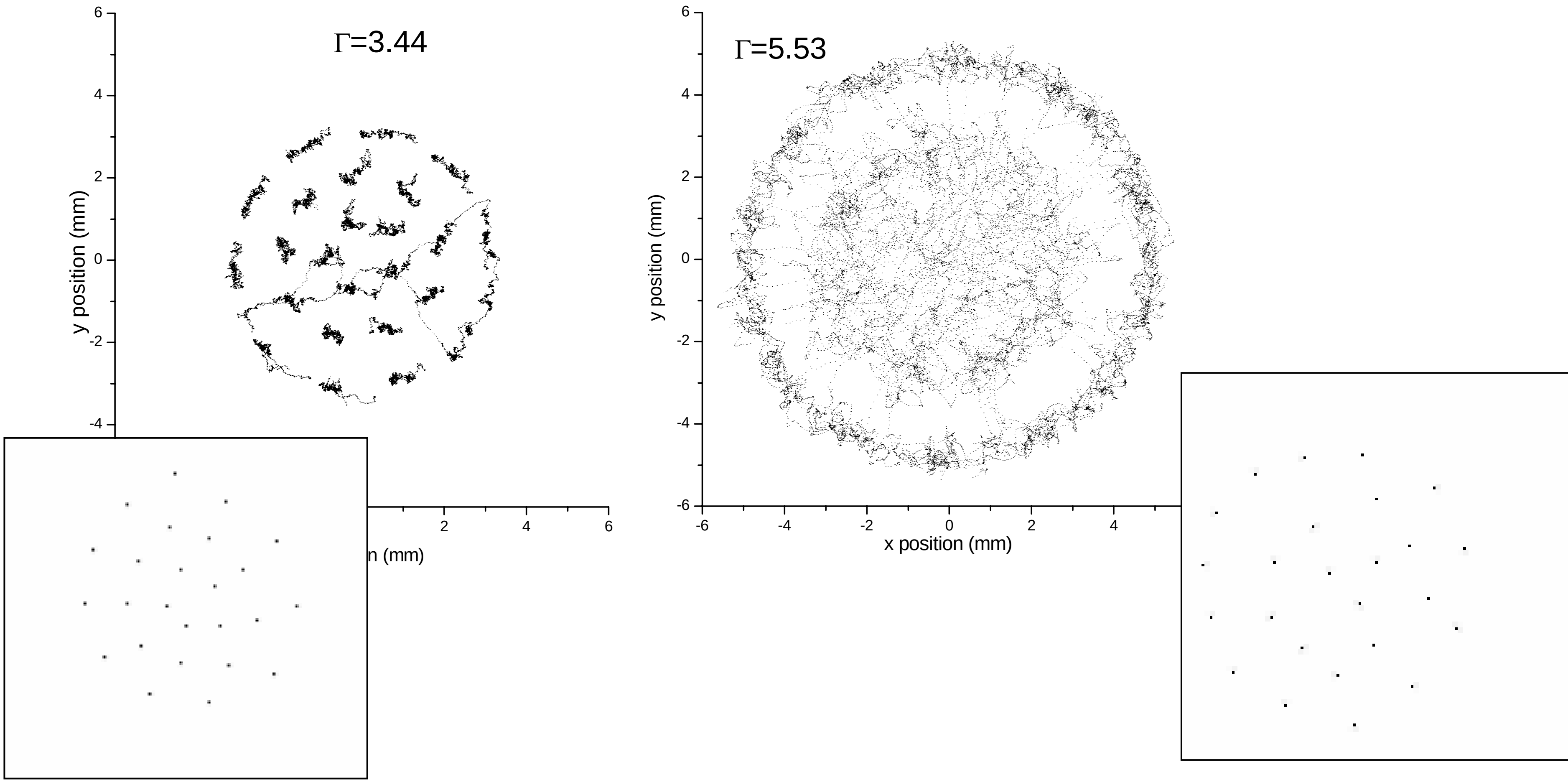
Large Cluster Trajectories

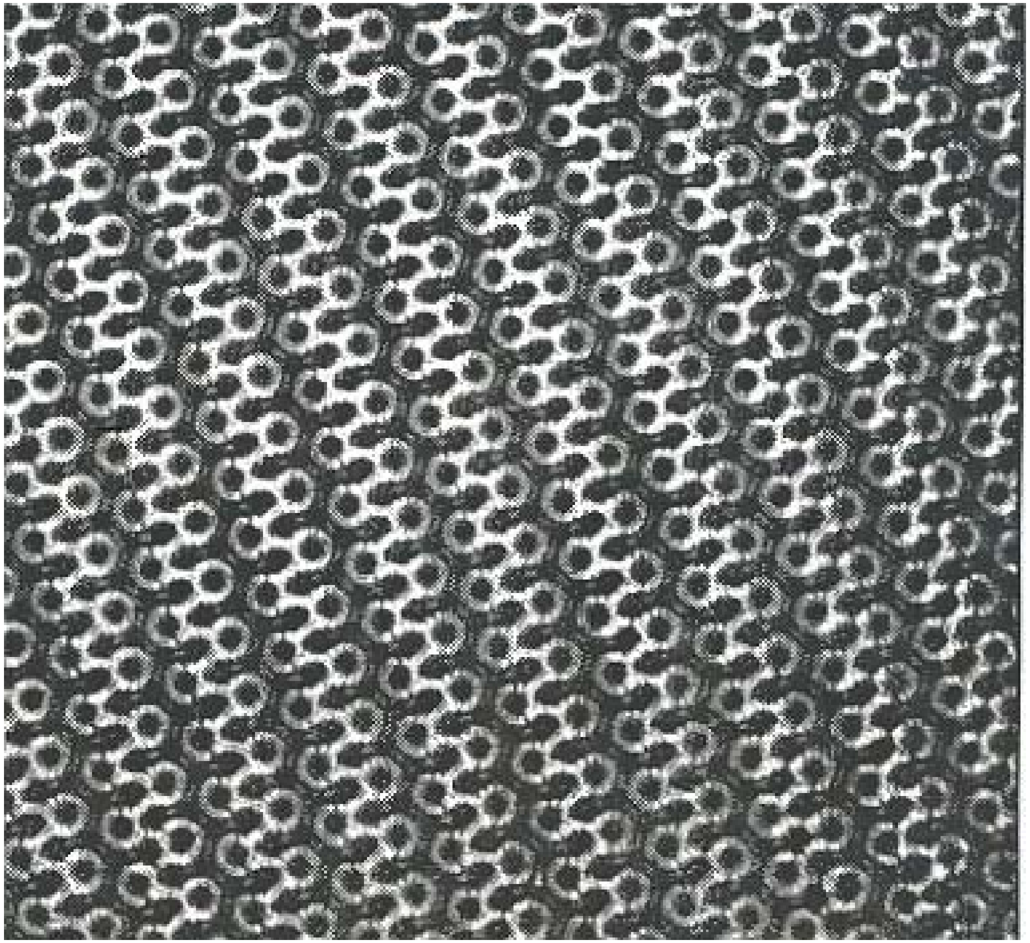


Large cluster trajectories, for several accelerations.

Main Conclusions - Interacting Particles

- Many different structures, ordered, disordered, chaotic
- Observations suggest an effective interaction potential for two particles. But pairwise interactions do not suffice for many particles.
- Large and small clusters behave differently.
- More: Voth et al. PRL 88, 234301 (2002); C.C. Thomas & JPG, in preparation.

Complex wave patterns



Kudrolli, Pier, Edwards, JPG

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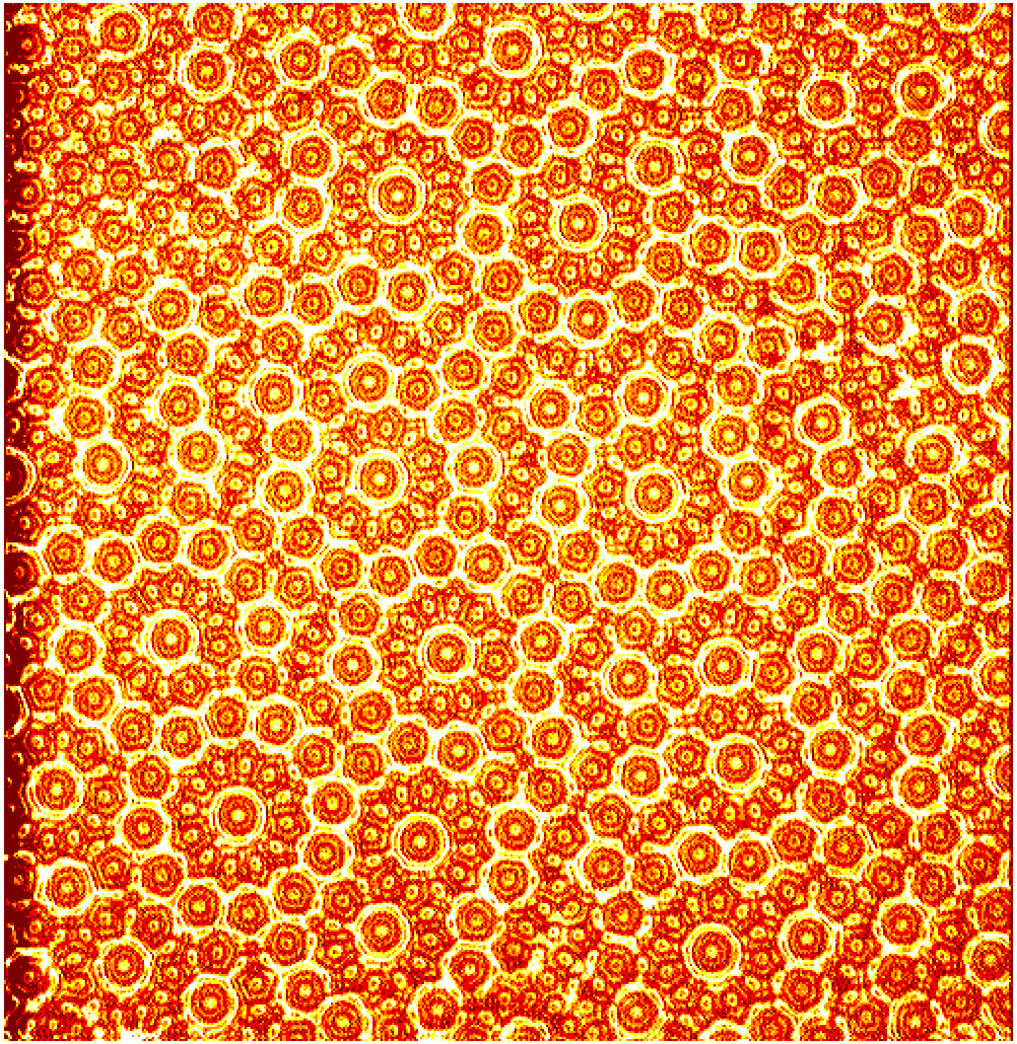
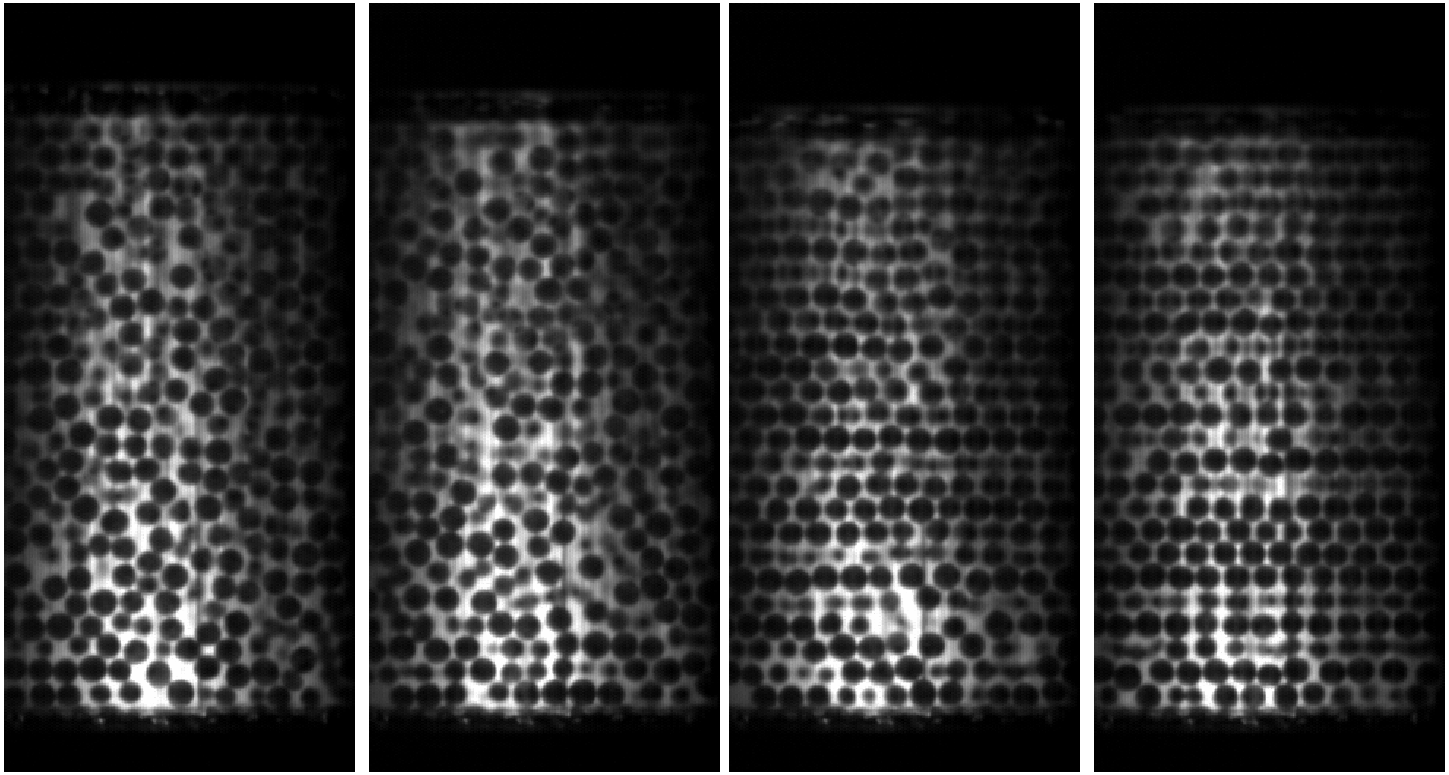
Quasicrystalline wave pattern



Sheared Granular Flow



Shearing induces order; order modifies

Conclusions

- Ideas from nonlinear dynamics contribute to understanding fluid phenomena.
- Equally, fluids illuminate nonlinear dynamics and can be used to teach it.
- Fluids ought to play a larger role in physics teaching. I make a case for this in Physics Today, December 2003: "One of the oddities of contemporary physics education is the nearly complete absence of continuum mechanics…"
- **Movies**: www.haverford.edu/physics-astro/Gollub/lab.html

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END