“ I was stunned by the perfection of the insects.”

- Pablo Neruda

Entomological fluid dynamics

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Supported by the NSF's Fluids Division (CTS)

Motivation

- to explore the fluid mechanics of the insect world
- to rationalize some of Nature’s microfluidic designs

Bonus

- to inspire and inform biomimetic designs

Outline

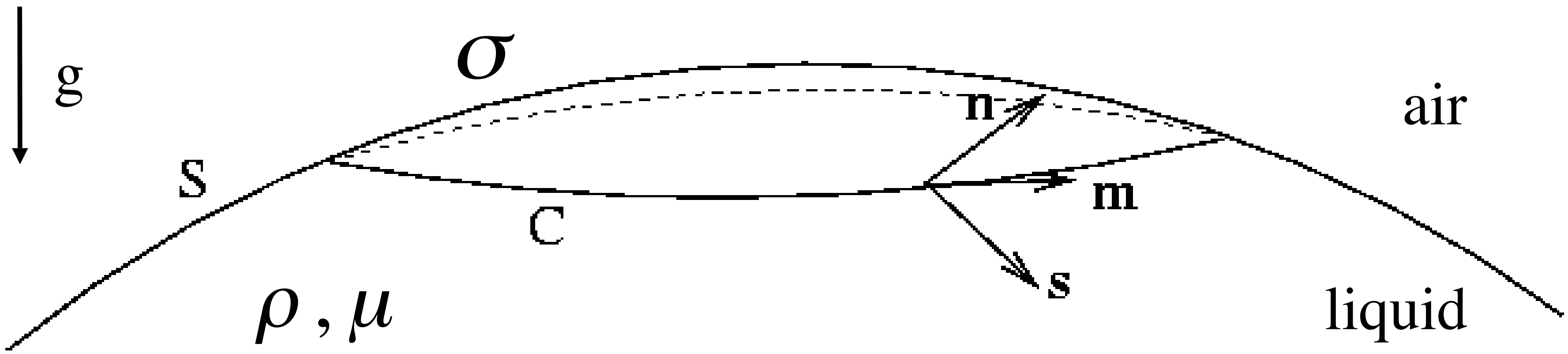
I. Fundamentals
   - surface tension, wetting, water-repellency

II. Walking on water: from the macro to the micro

III. Underwater breathing

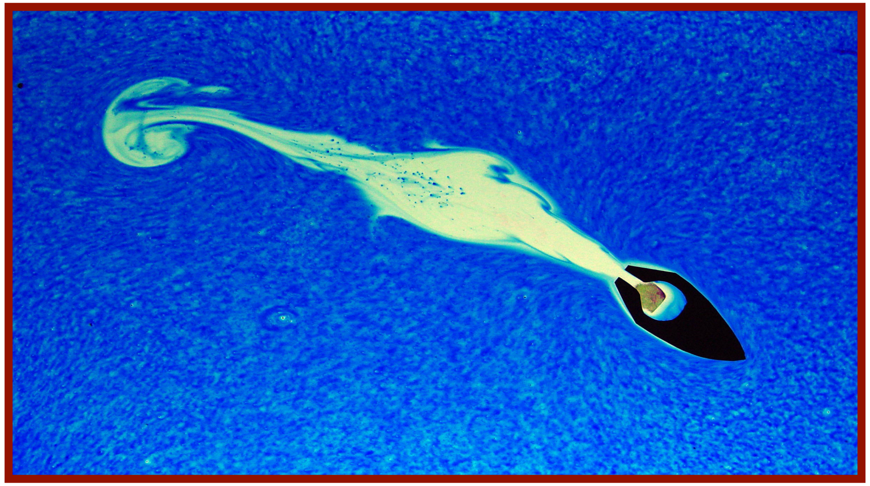
Surface tension σ

Working definition: along a contour C bounding a surface S there is a tensile force σ acting in the **s** direction



Result 1) normal curvature pressure $\sigma \nabla \cdot \mathrm{n}$ resists surface deformation

2) tangential stresses may arise from $\nabla \sigma$



When is surface tension important relative to gravity?

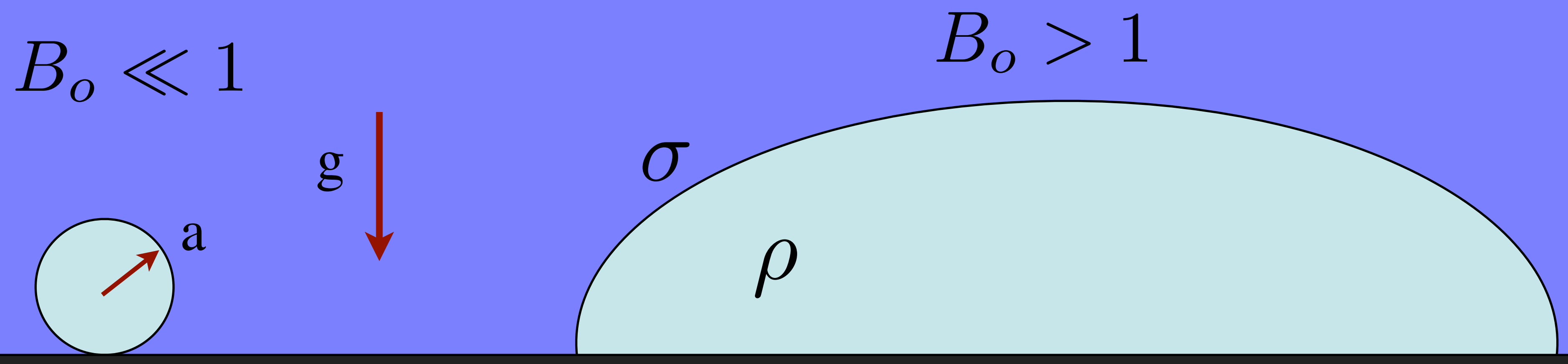
- when curvature pressures large relative to hydrostatic:

Bond number: $$B_o = \frac{\rho g a}{\sigma / a} = \frac{\rho g a^2}{\sigma} < 1$$

i.e. for drops small relative to the capillary length:

$$a < l_c = \left(\frac{\sigma}{\rho g} \right)^{1/2}$$

$\sim$ 2 mm for air-water ($\sigma = 70$ dynes/cm)



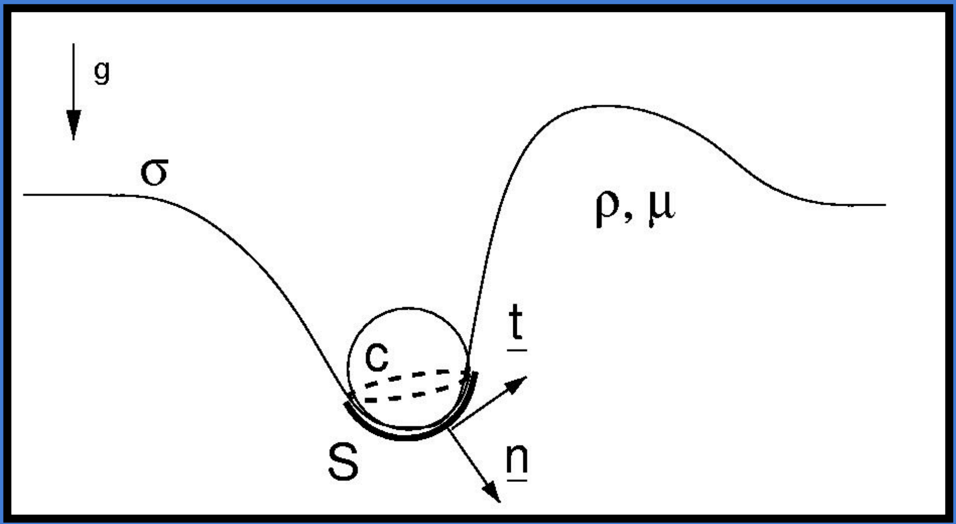
The world of insects is dominated by surface tension.

II. Walking on water

A fluid mechanician’s perspective

with David Hu
(now at Courant Institute, NYU)

Lateral propulsion at the interface



$$\underline{F}_H = \int_S \underline{\underline{T}} \cdot \underline{n}\, dS + \int_C \sigma\, \underline{t}\, dl$$

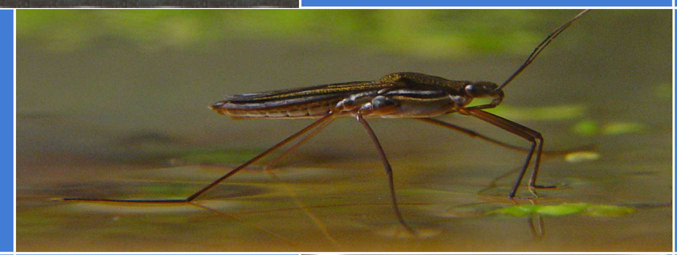
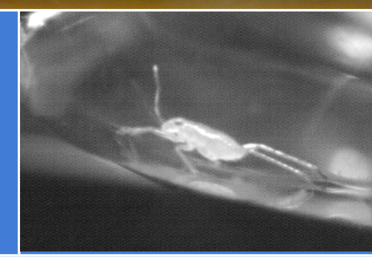
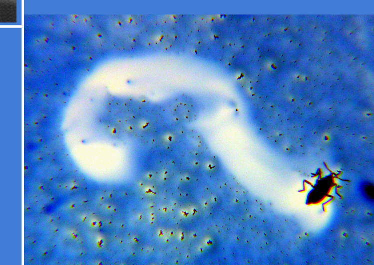
Stress tensor:

$$\underline{\underline{T}} = -p\,\underline{\underline{I}} + \mu\left(\nabla u + (\nabla u)^T\right)$$

Propulsive force

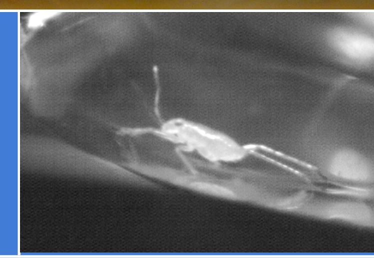
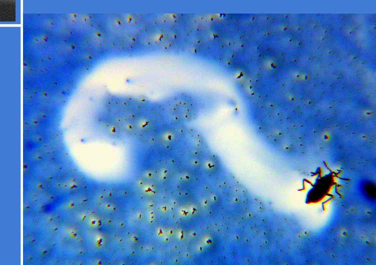
$$F_H \sim \rho g V_s + \rho U^2 A + \rho V \frac{dU}{dt} + \rho \nu U a + \sigma(\underline{\nabla} \cdot \underline{n})A - \underline{\nabla}\sigma A$$

buoyancy	form drag	acceleration reaction	viscous drag	curvature	Marangoni

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

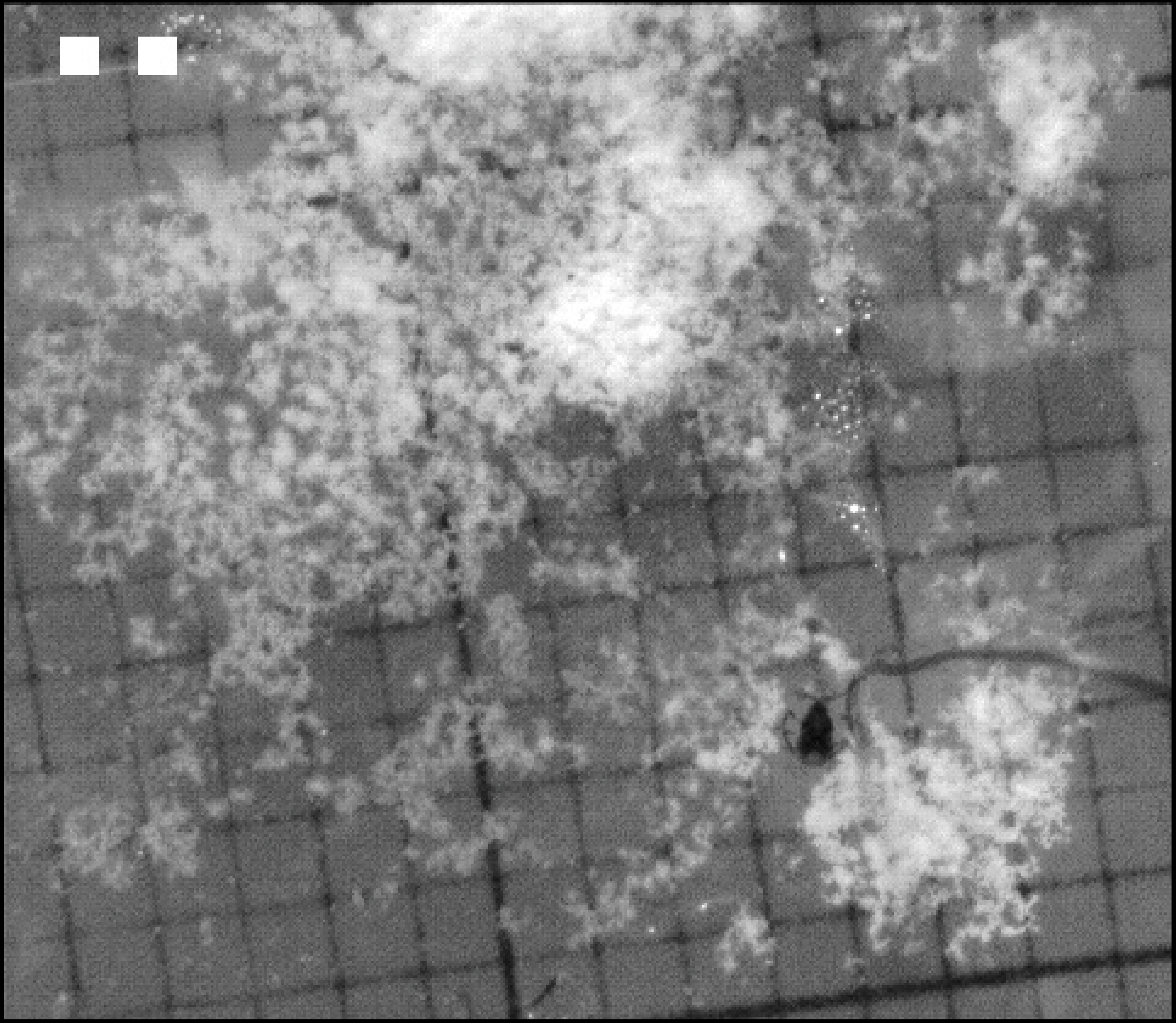
quasi – static propulsion

Clark’s Grebe: clip courtesy of “Winged Migration”

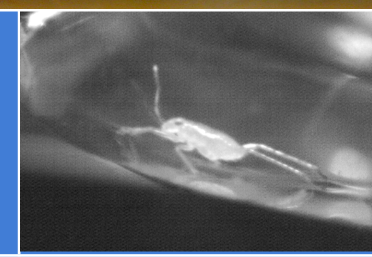
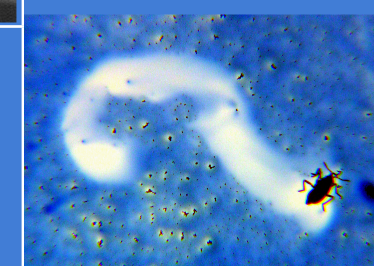
	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

quasi – static propulsion

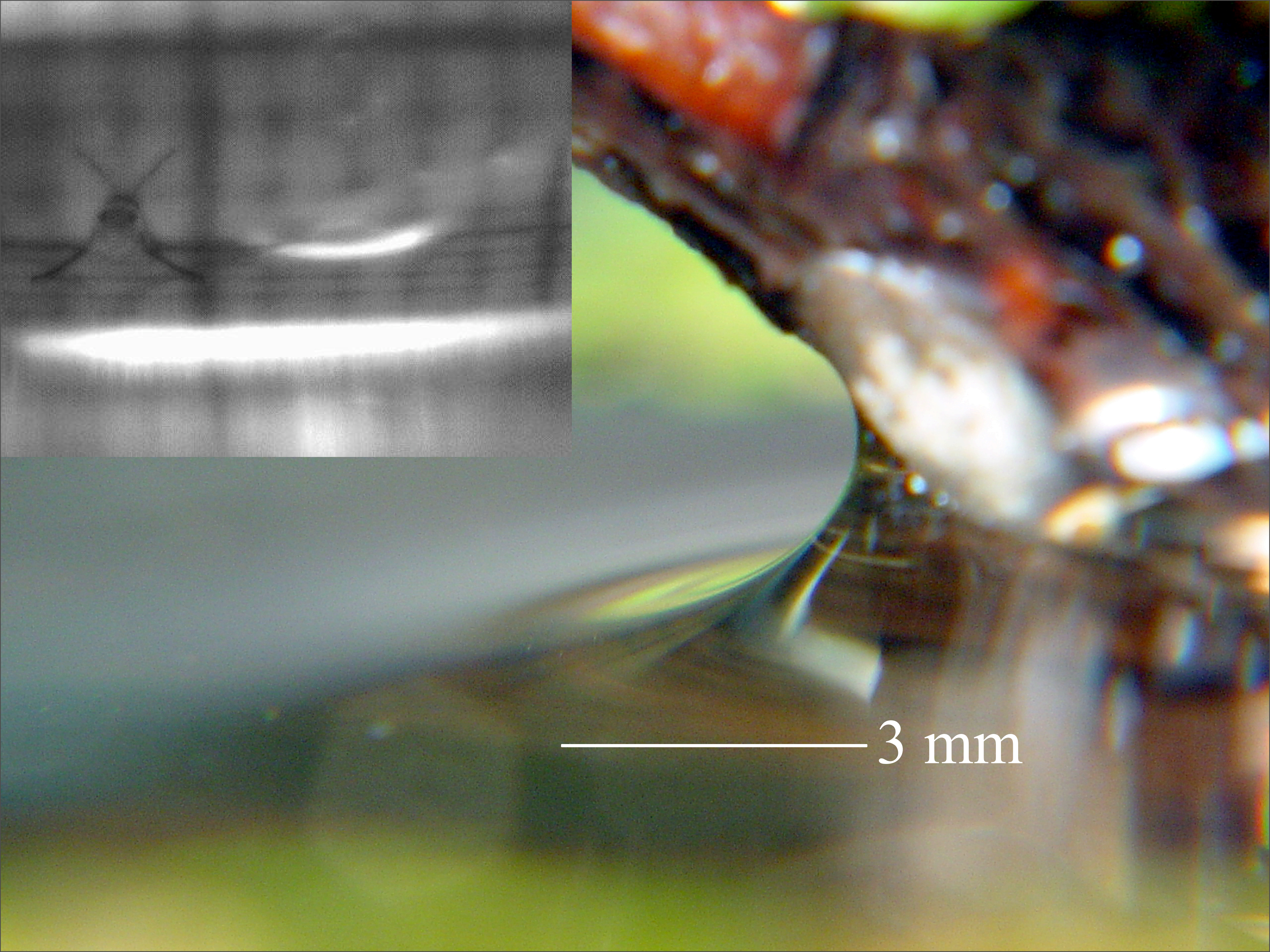
Tangential stress, $\nabla\sigma$, may drive lateral motion.



Marangoni propulsion: insect uses lipid as fuel.

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
Rowing & walking					
Surface distortion					
Marangoni propulsion					

quasi – static propulsion


3 mm

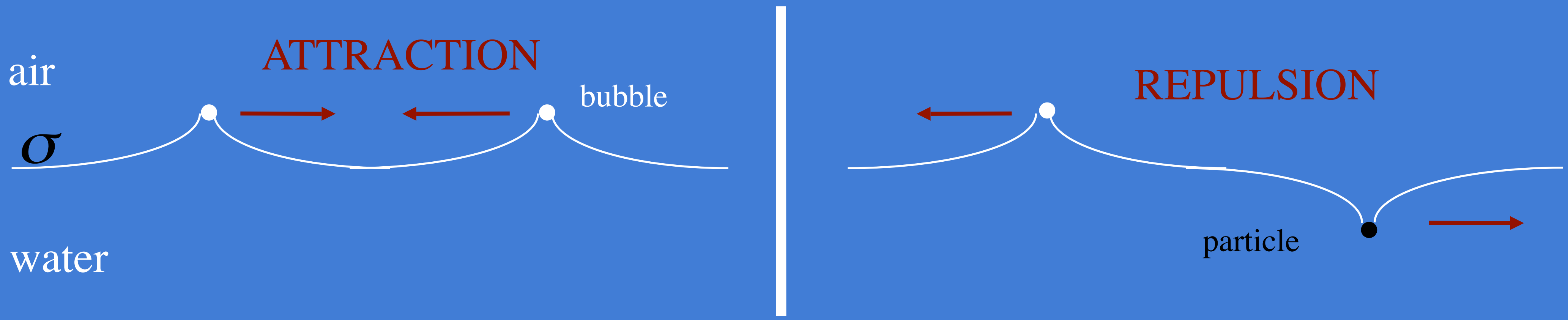
Meniscus-climbing

What if $U < \sqrt{g\ell_c} \sim 35\ cm/s$, the capillary escape velocity?

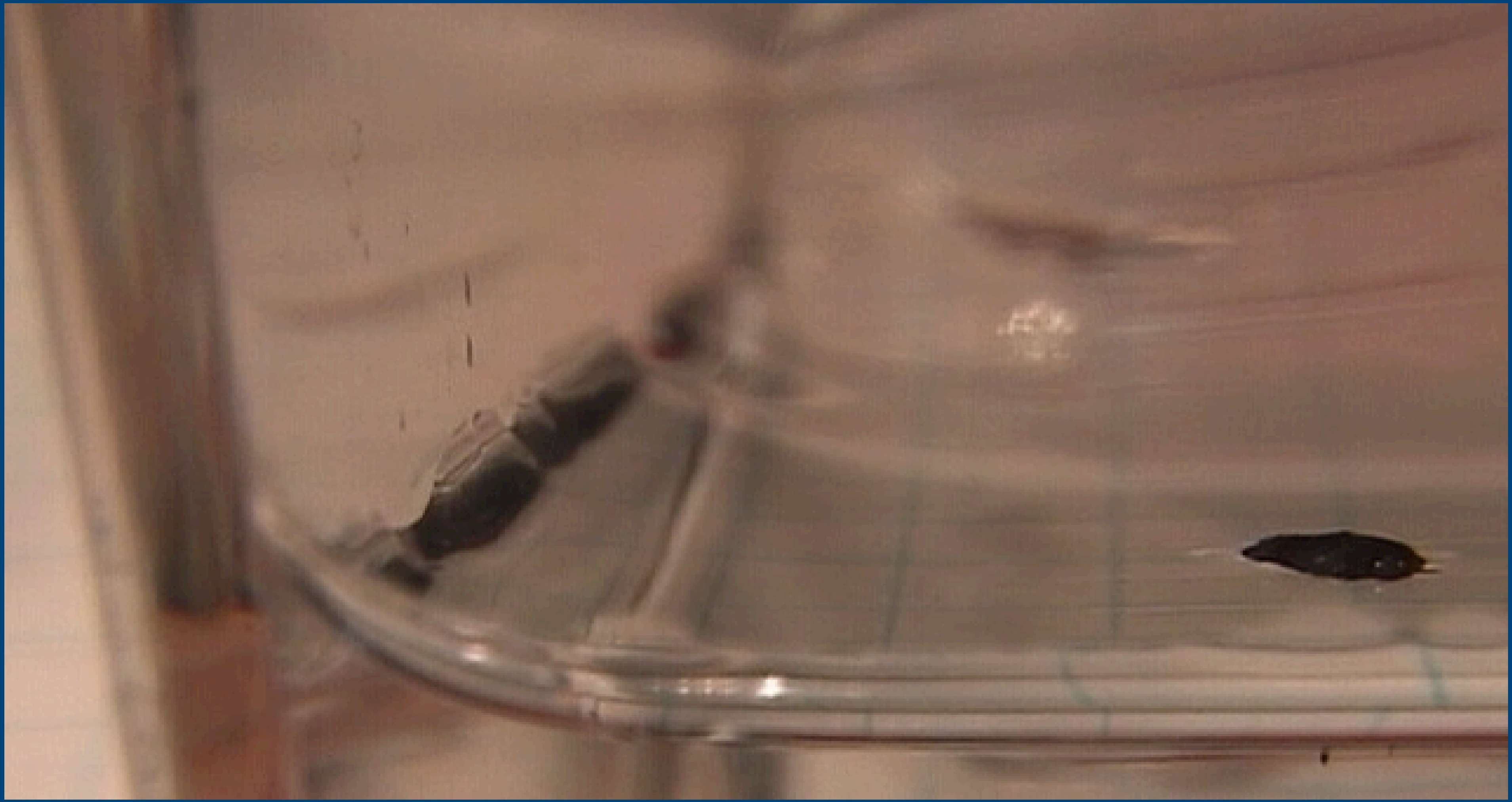
Kralchevsky & Denkov (2001)

Capillary forces

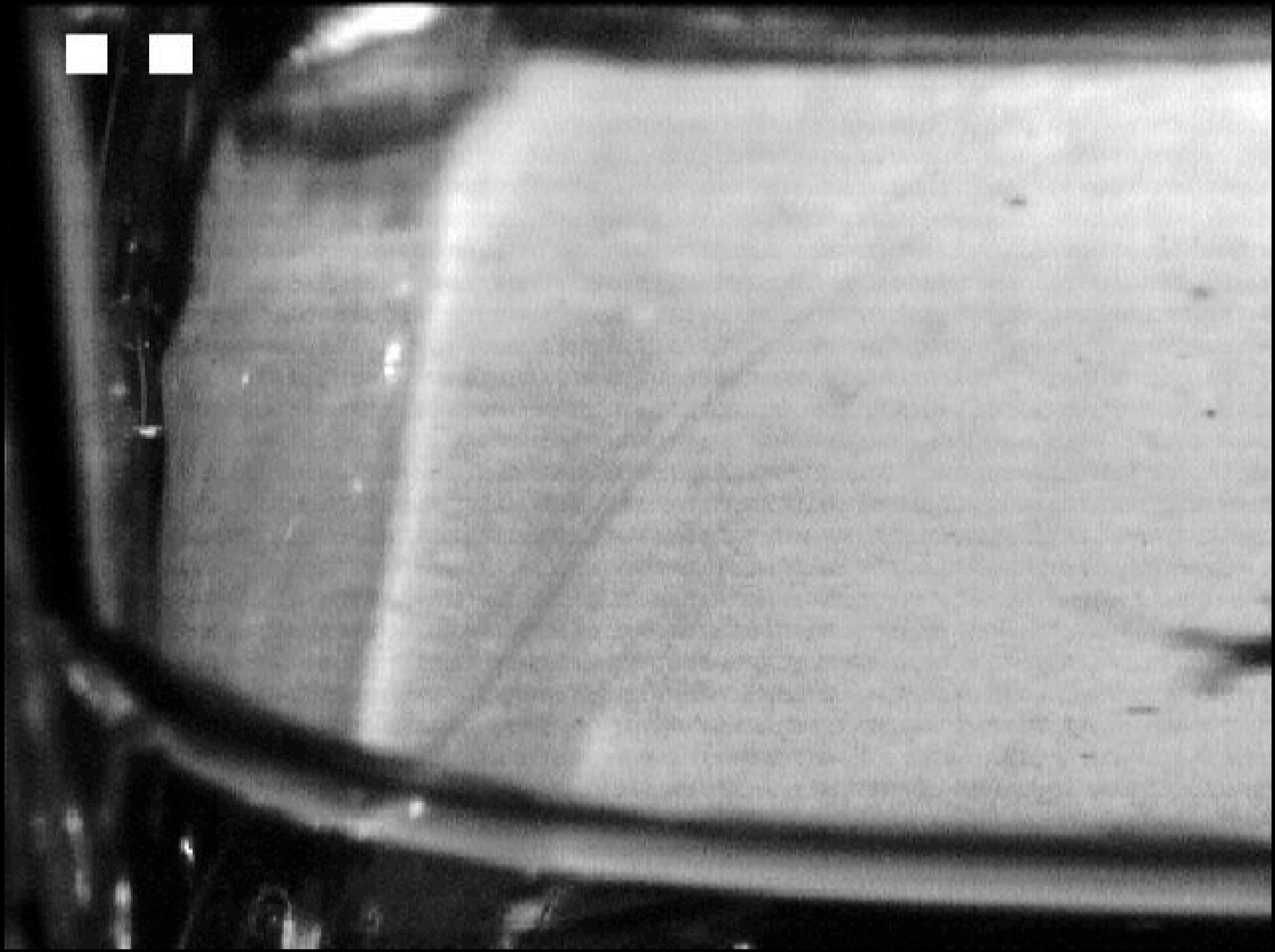
- act between objects floating at a free surface
- attractive/repulsive for menisci of the same/opposite sense



- explains the formation of bubble rafts in champagne
- explains the attraction of Cheerios in a bowl of milk
- used by small insects to move themselves along the free surface



Meniscus-climbing by the beetle larva Pyrrhalta



Meniscus-climbing by *Mesovelia*

	$\rho g z A$	$\rho V dU/dt$	$\rho U^2 A$	$\sigma \nabla \cdot \underline{n} A$	$\underline{\nabla} \sigma A$
Surface slapping					
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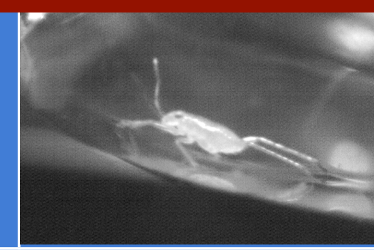
Surface distortion					
Marangoni propulsion					

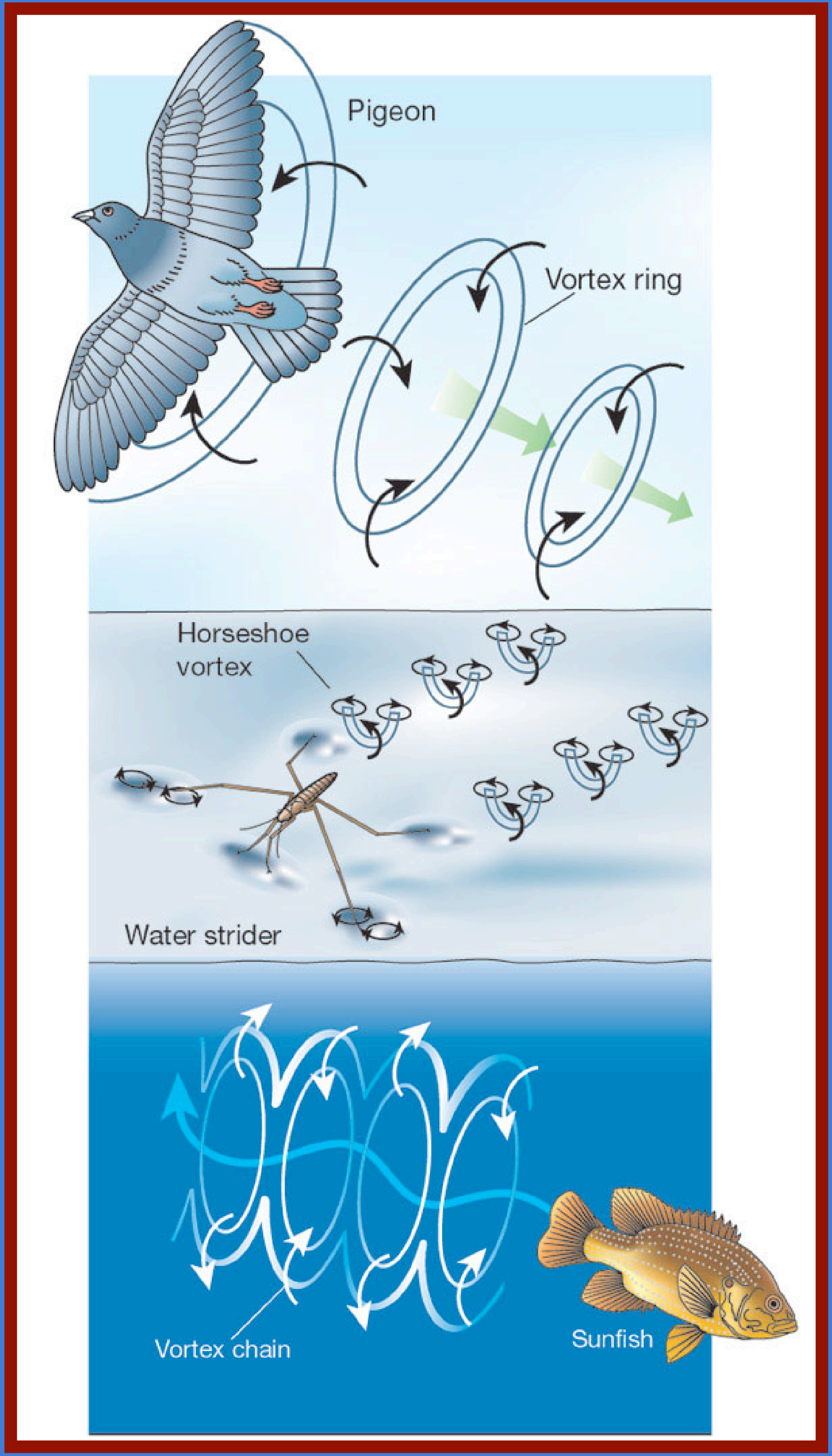
quasi – static propulsion

Vortex generation by the water strider

Flying

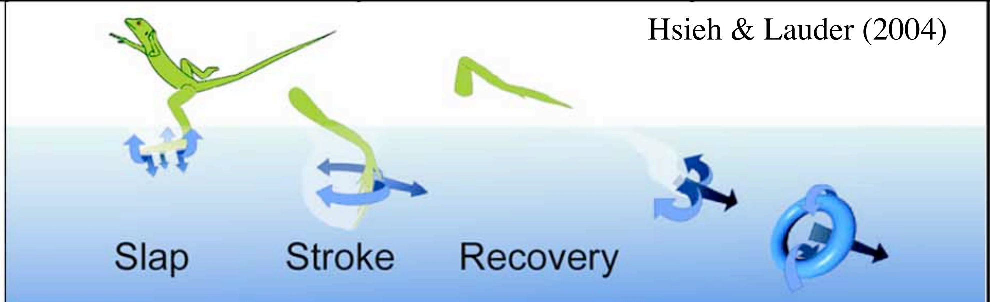
Rowing

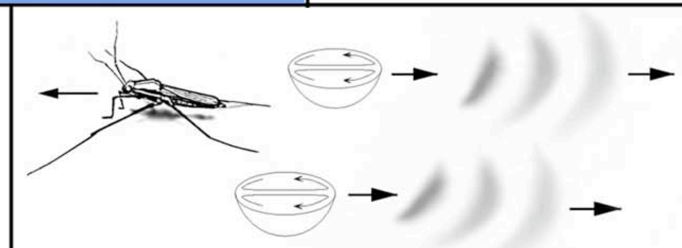
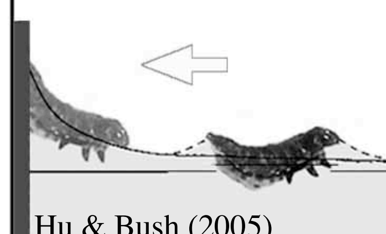
Swimming



Dickinson, 2003

SUMMARY

<table>
<tr><th></th><th>Buoyancy</th><th>Added mass</th><th>Inertia</th><th>Curvature</th><th>Marangoni</th></tr>
<tr><td>Surface slapping</td><td colspan="3">

Hsieh & Lauder (2004)</td><td></td><td></td></tr>
<tr><td>Rowing & walking</td><td></td><td></td><td colspan="2">
Hu, Chan & Bush (2003)</td><td></td></tr>
<tr><td>Meniscus climbing</td><td></td><td></td><td></td><td>
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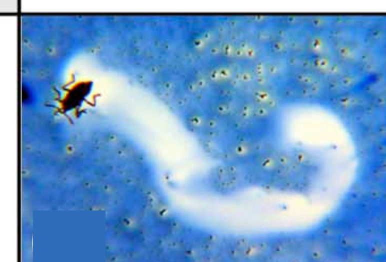
<tr><td>Marangoni propulsion</td><td></td><td></td><td></td><td></td><td></td></tr>
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Bush & Hu, *Ann. Rev. Fluid Dyn.* (2006)

What is happening on the microscale?

What are the precise origins of the propulsive force?

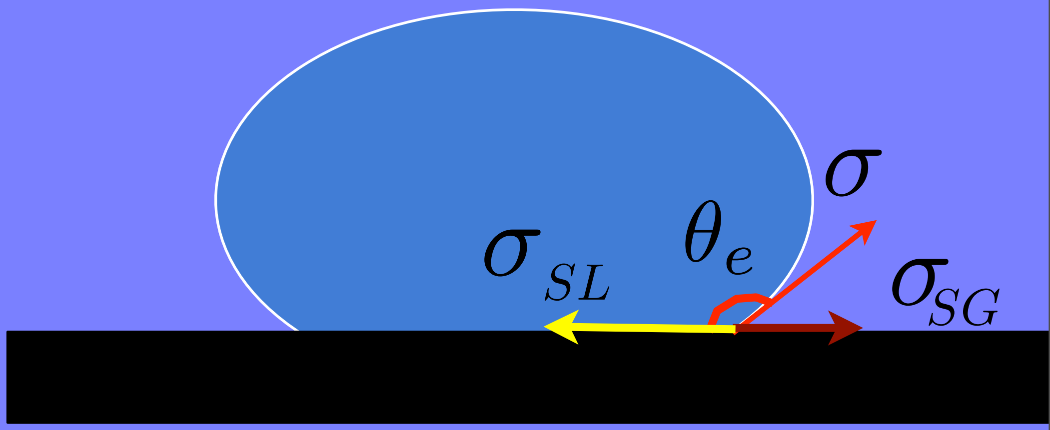
How do these creatures generate vortices?

Walking on water: a closer look

with Manu Prakash

Fluid-Solid Contact: WETTING

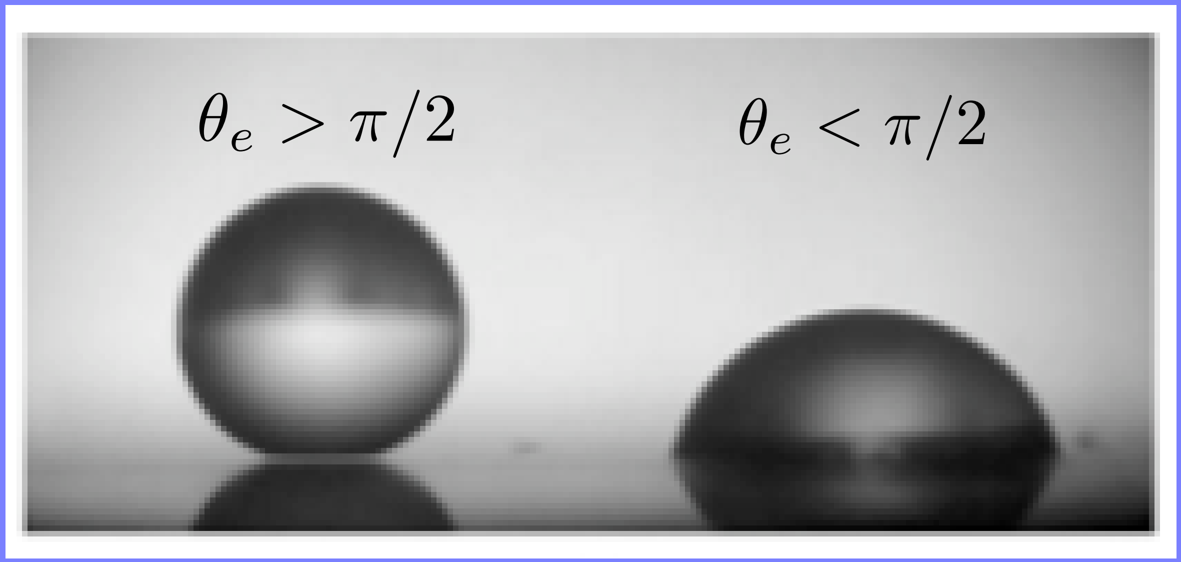
Reference: de Gennes et al. (2004)



Equilibrium contact angle θ_e

Young's relation:

$$\sigma \cos\theta_e = \sigma_{SL} - \sigma_{SG}$$



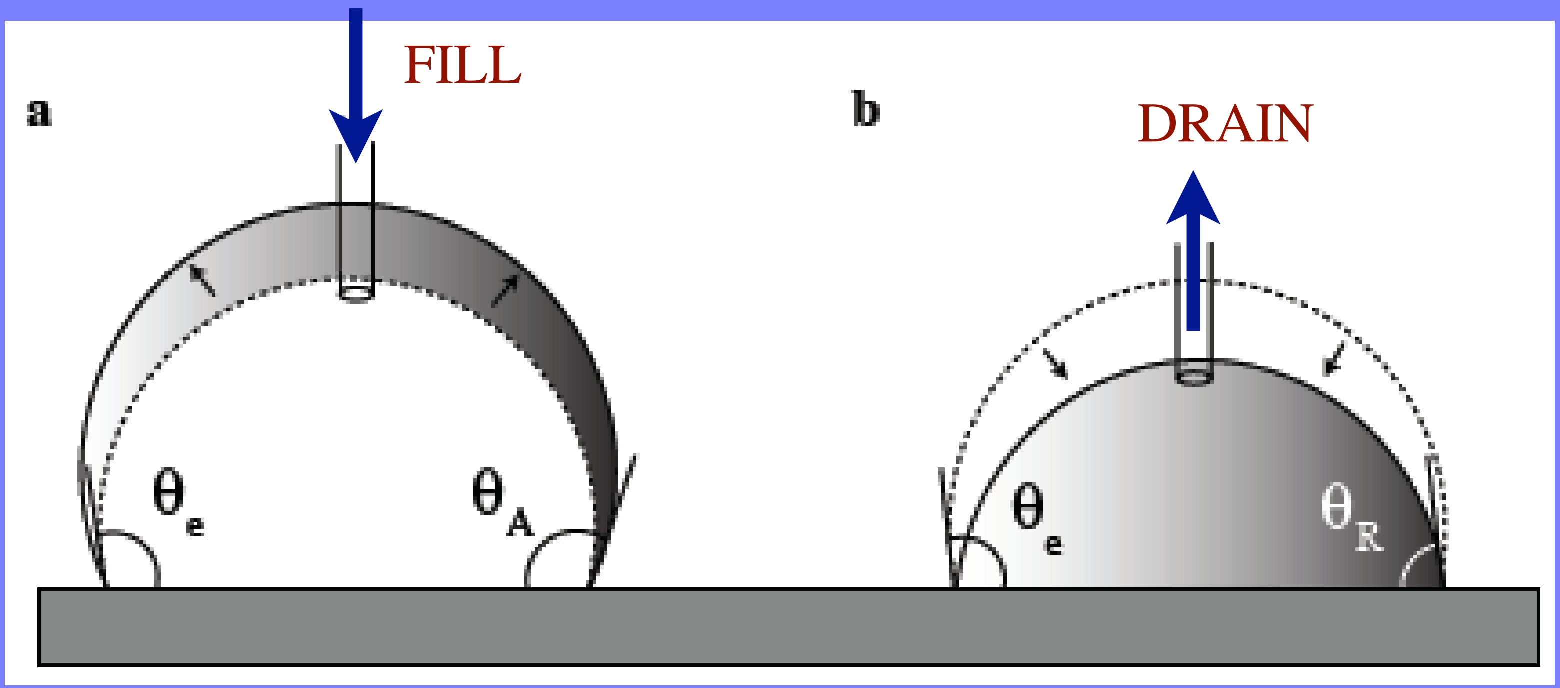
Hydrophobic surface

Hydrophilic surface

Contact angle hysteresis

Static contact angle is not uniquely θ_e

Reality: drop is stable over a range of $\theta_r < \theta < \theta_a$



Origins: advancing contact lines pinned on surface irregularities

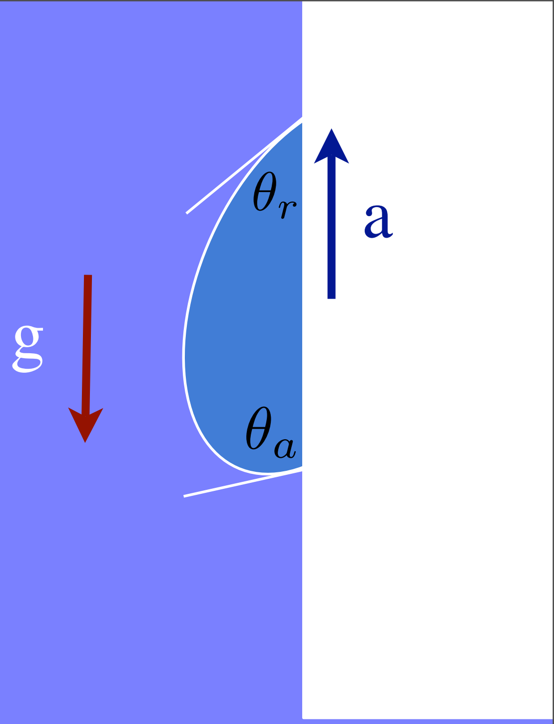
Joanny & de Gennes (1984)

→ FORCE of ADHESION resists drop motion

increases with $\Delta\theta = \theta_a - \theta_r$

The force of adhesion (Dussan & Chow 1983)

Raindrop stuck on a window



- small drops supported by contact line resistance

$$F_c \sim 2\pi a \, \sigma \, (\cos\theta_r - \cos\theta_a)$$

- drops grow by accretion until weight prompts rolling

The force of adhesion (Dussan & Chow 1983)

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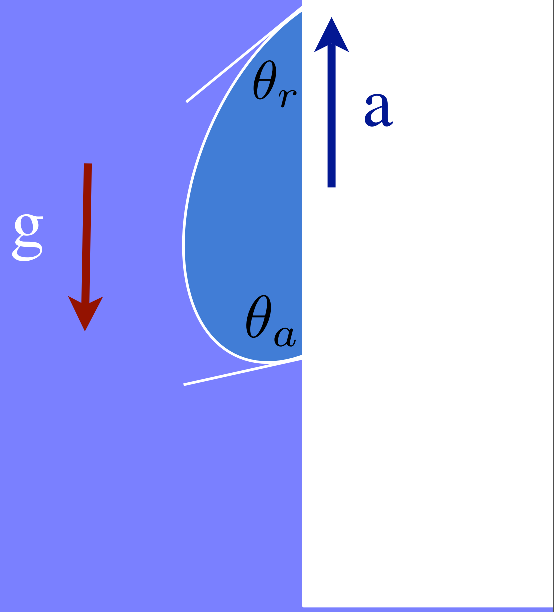

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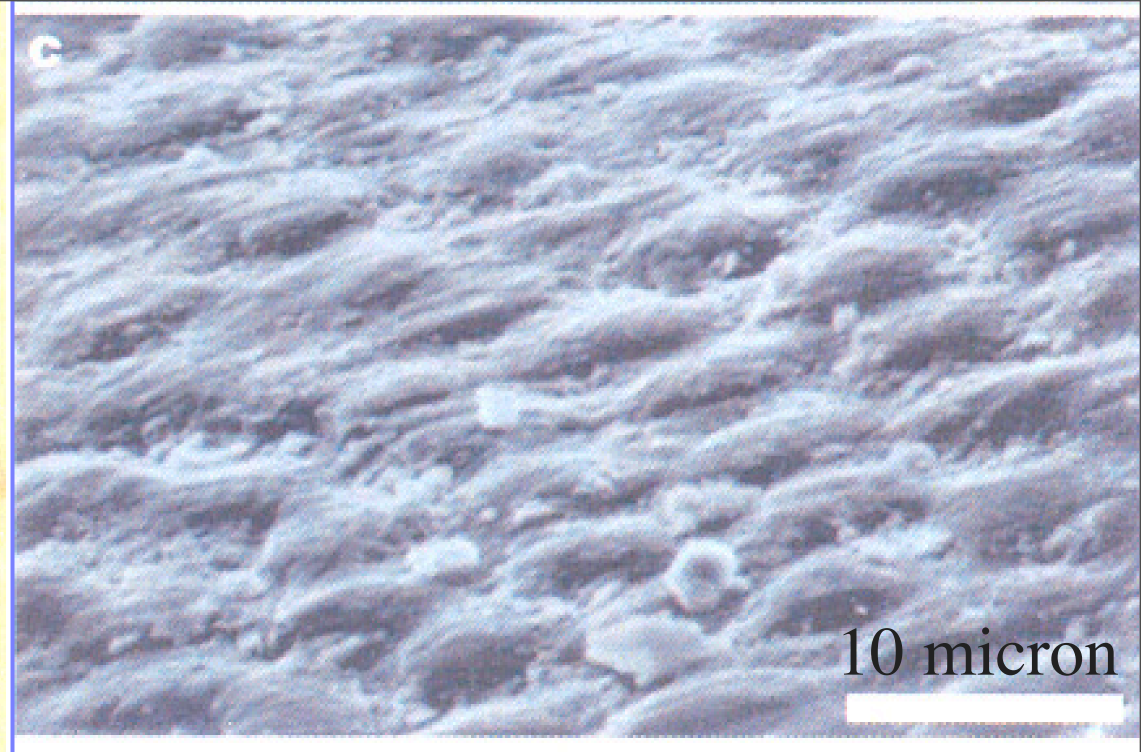
But who cares?

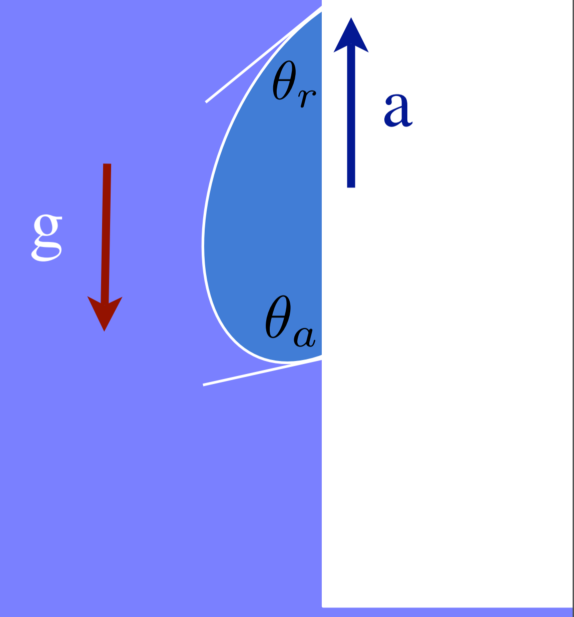


A refrigeration-free condenser

- the Namib desert beetle has hydrophobic bumps to which 5 micron scale fog droplets stick then grow by accretion until rolling onto hydrophobic valleys and into their mouths

Parker & Lawrence (2001)

The force of adhesion (Dussan & Chow 1983)



Raindrop stuck on a window

- small drops supported by contact line resistance

$$F_c \sim 2\pi a\,\sigma\,(\cos\theta_r - \cos\theta_a)$$

- drops grow by accretion until weight prompts rolling

Water-repellency

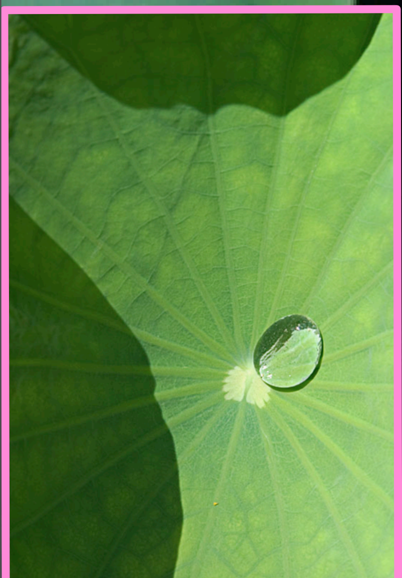
- impinging drops roll off rather than adhering
- requires large θ_e , small $\Delta\theta = \theta_a - \theta_r$

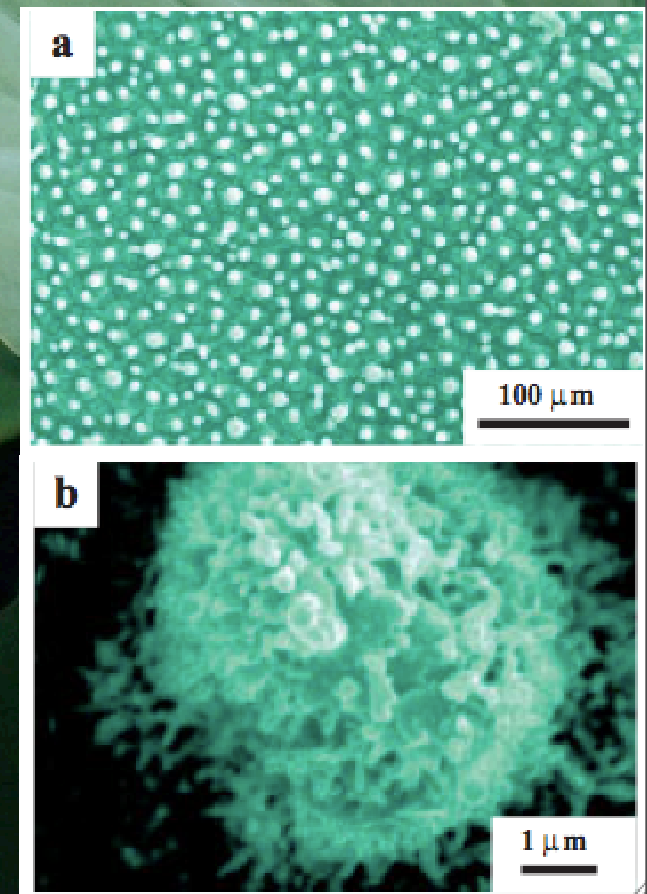
How can we reduce the force of adhesion?

Water repellency in nature

"One who performs his duty without attachment, surrendering the results unto the Supreme Being, is unaffected by sinful action, as the lotus leaf is untouched by water."

Bhagavad Gita 5.10

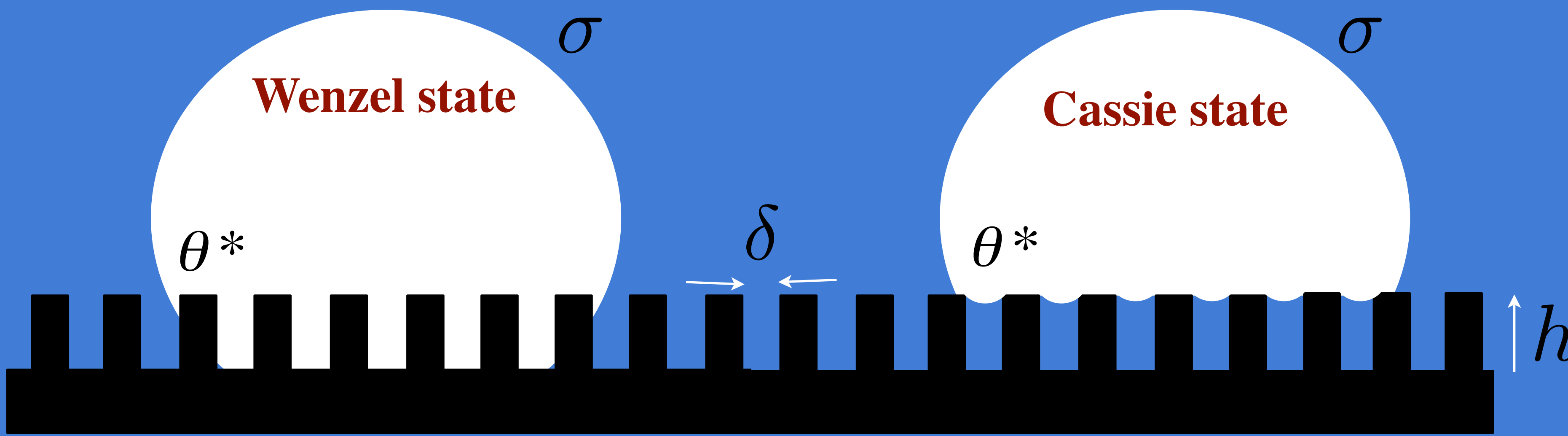




Feng et al. (2004)

- the lotus leaf is superhydrophobic and self-cleaning by virtue of its hierarchical surface roughness (M. Reyssat, 2007)

Wetting of a rough hydrophobic surface: Wenzel vs. Cassie



$$\cos\theta^* = r\ \cos\theta$$

where r is total/planar area

θ^* INCREASES

$\Delta\theta$ INCREASES

$$\cos\theta^* = -1 + f_s + f_s \cos\theta$$

where f_s is exposed/planar area

θ^* INCREASES

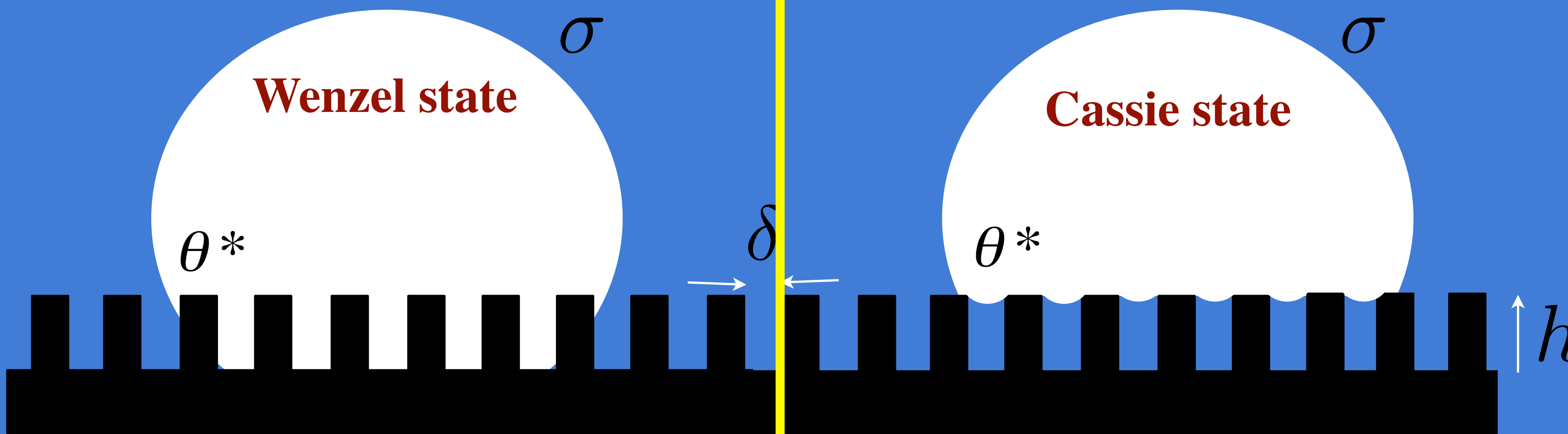
$\Delta\theta$ DECREASES

Water-repellency: requires the maintenance of a Cassie state

$$\longrightarrow \quad P_{applied} < \sigma \left(\frac{1}{\delta}\ ,\ \frac{h}{\delta^2} \right)$$

Bartolo et al. (2006)
Reyssat et al. (2006)

Wetting of a rough hydrophobic surface: Wenzel vs. Cassie



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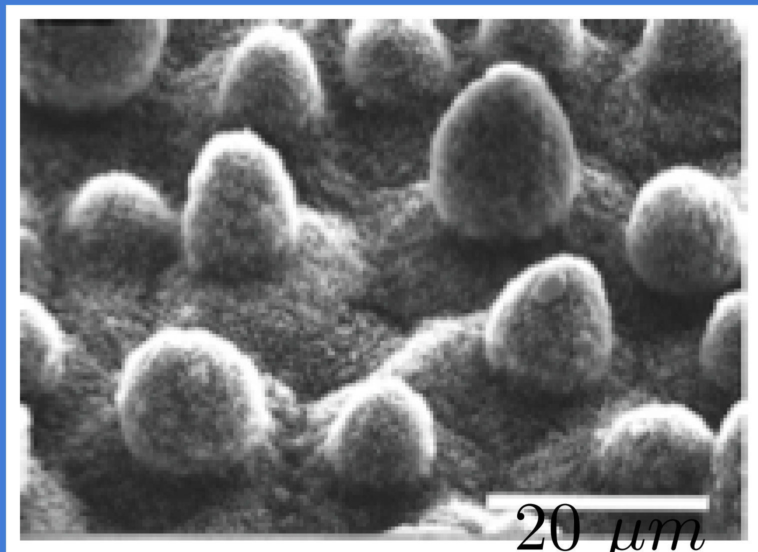
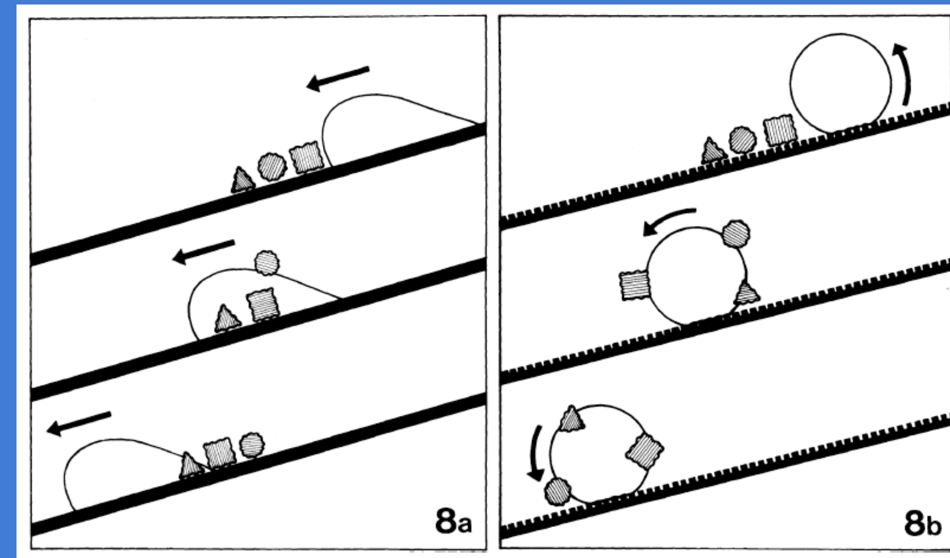
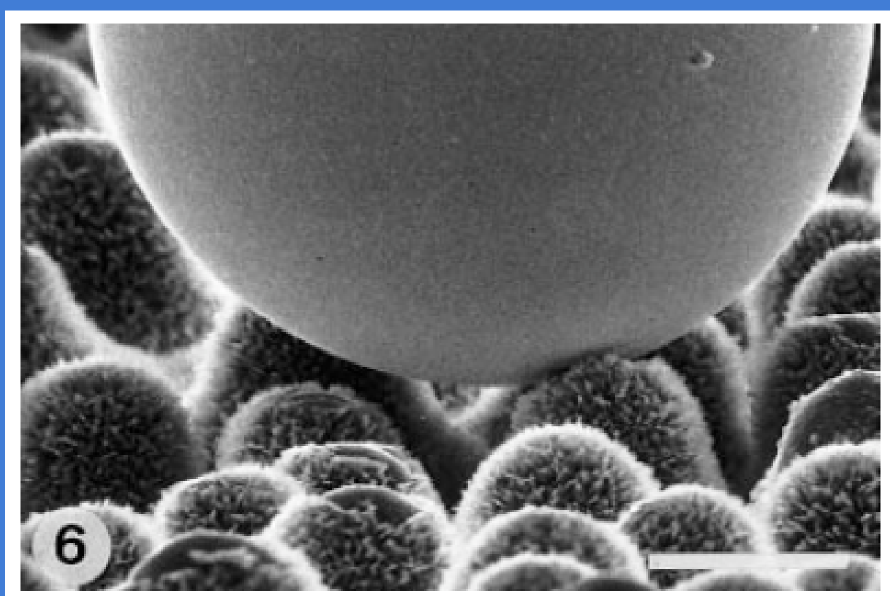
Bartolo et al. (2006)
Reyssat et al. (2006)

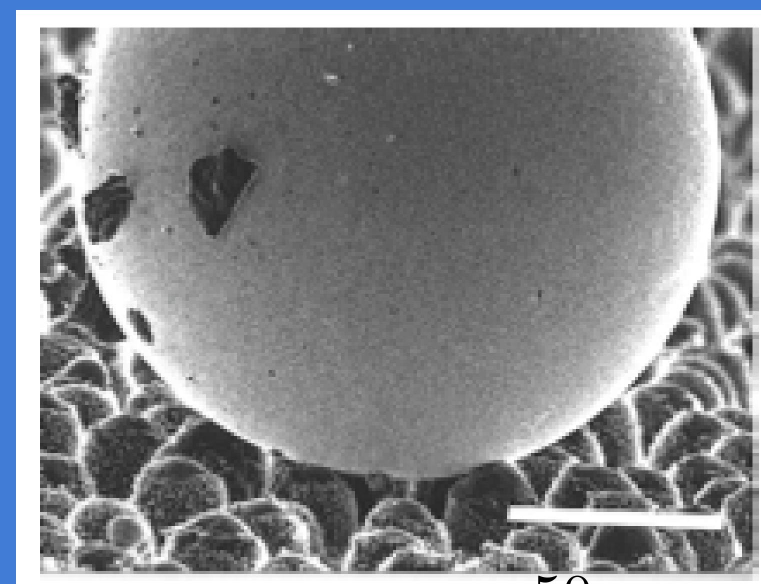
The lotus leaf

Barthlott & Neinhuis (1997)





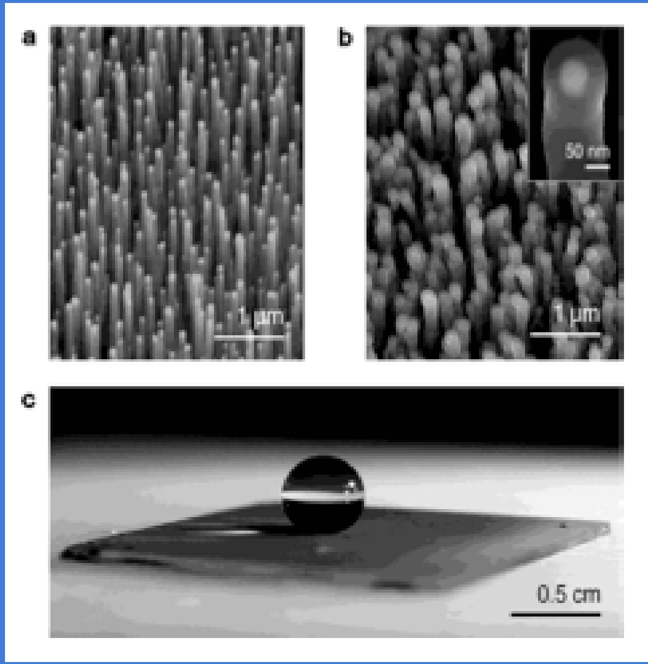




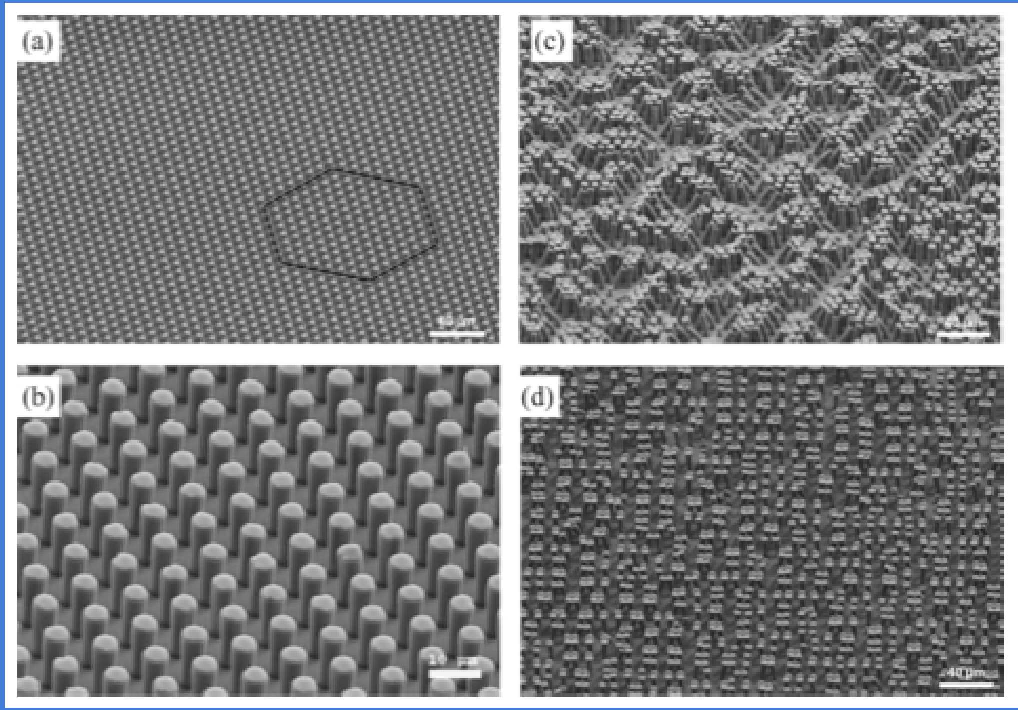
- water-repellent: Cassie state maintained, contact forces minimized
- self-cleaning: surface impurities (e.g. dust) adhere to droplets

Synthetic water-repellent surfaces

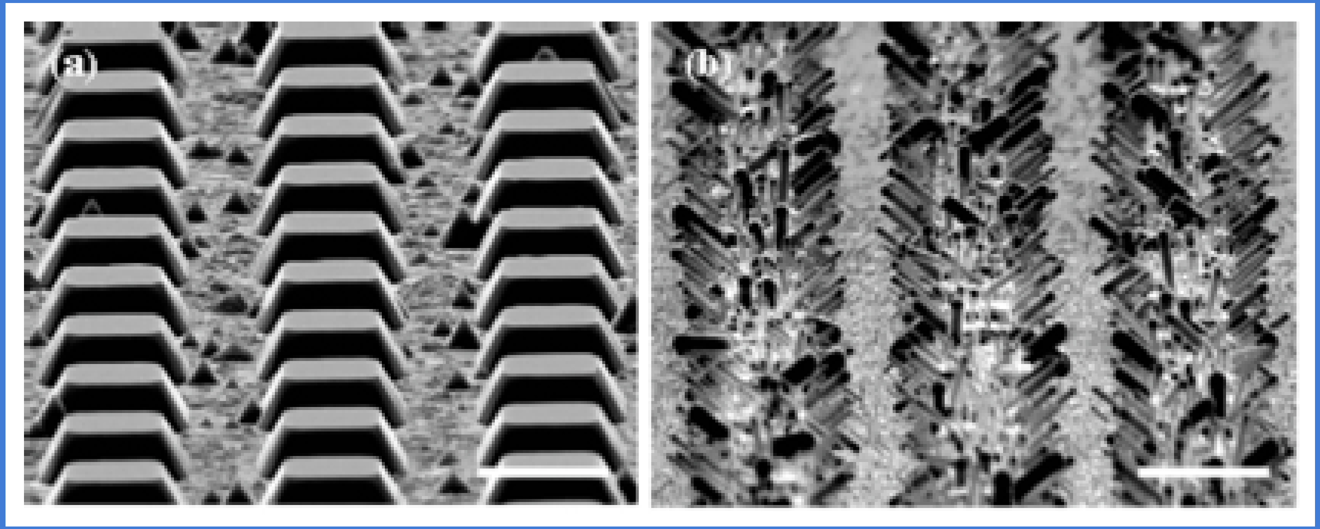
- applications in corrosion protection, rain-proofing and drag-reduction



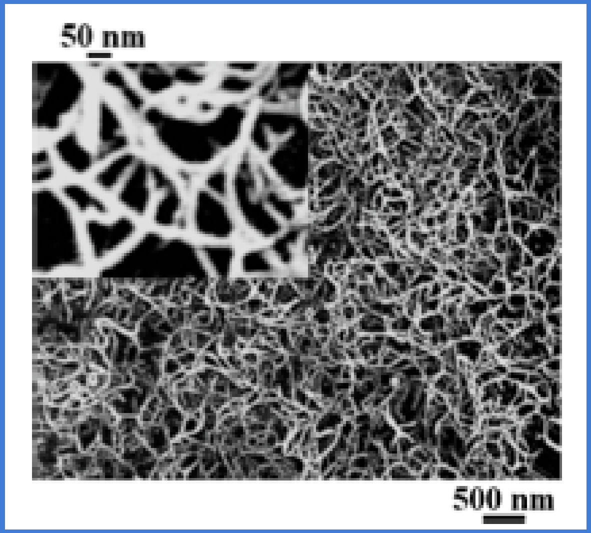
Lau et al. (2003)



Greiner et al. (2007)



Cao et al. (2007)

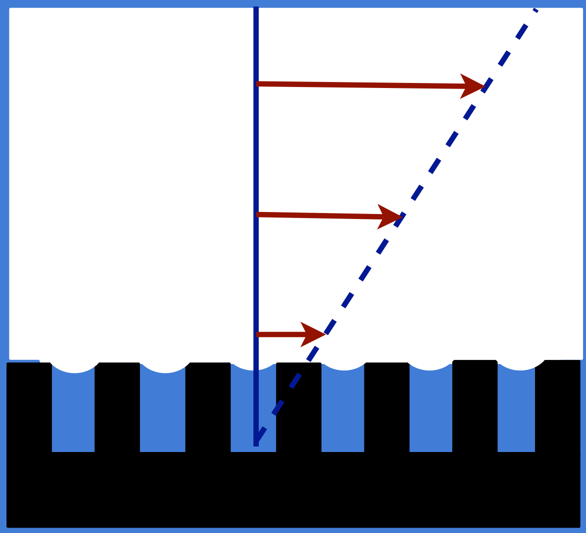


Gao & McCarthy (2006)

Drag reduction and superhydrophobicity

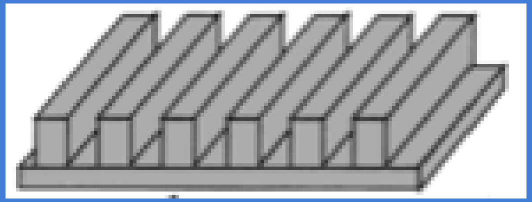
Min & Kim (2006), Joseph et al. (2006)

Drag on a Cassie surface with isotropic roughness



- drag reduced owing to reduced fluid-solid contact

Choi et al. (2006)

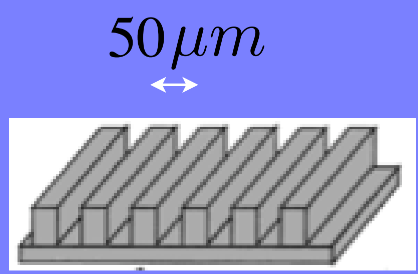


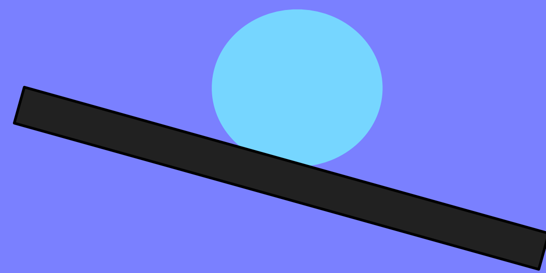
Drag on nanograting in a Cassie state

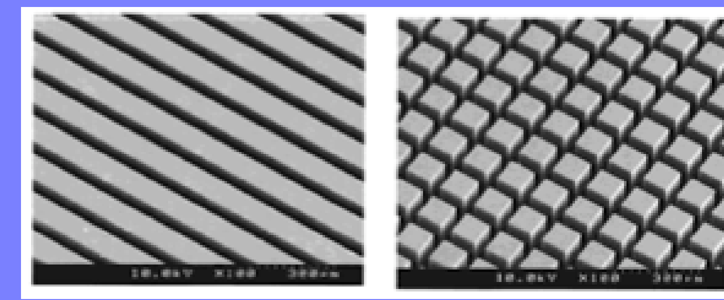
- drag reduced for flow along nanogrooves
- drag increased for flow across nanogrooves

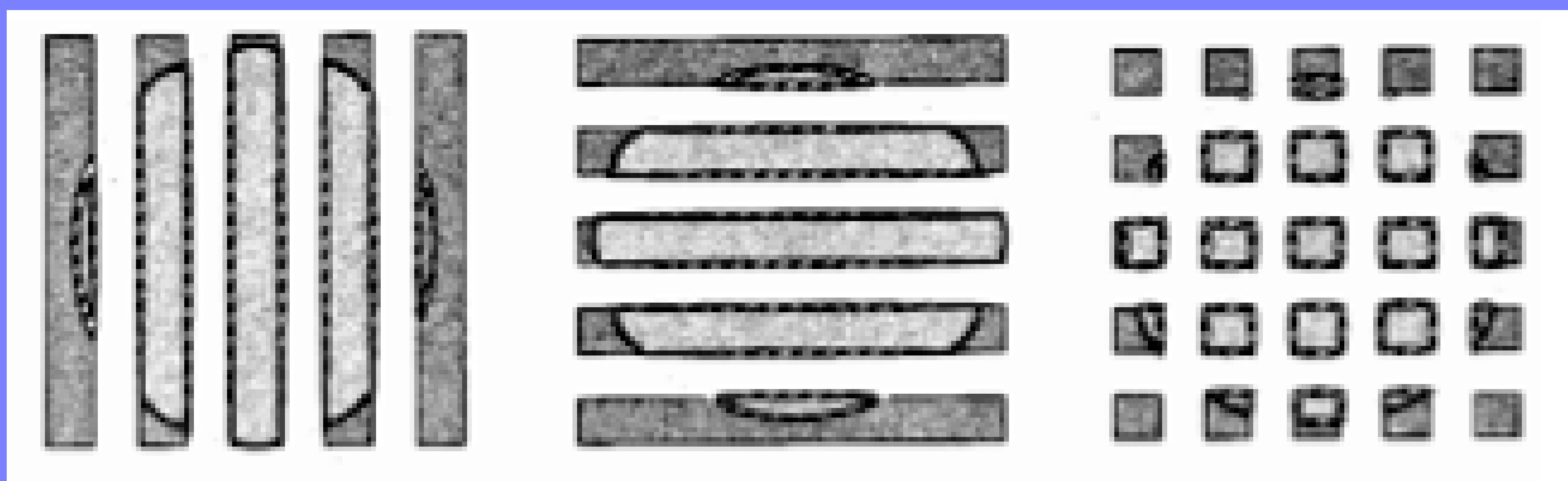
Surface texturing and directional adhesion

Yoshimitsu et al. (2002)









- drops move most easily along nanogrooves
- greatest resistance to motion perpendicular to grooves
- texturing introduces anisotropy in contact line resistance

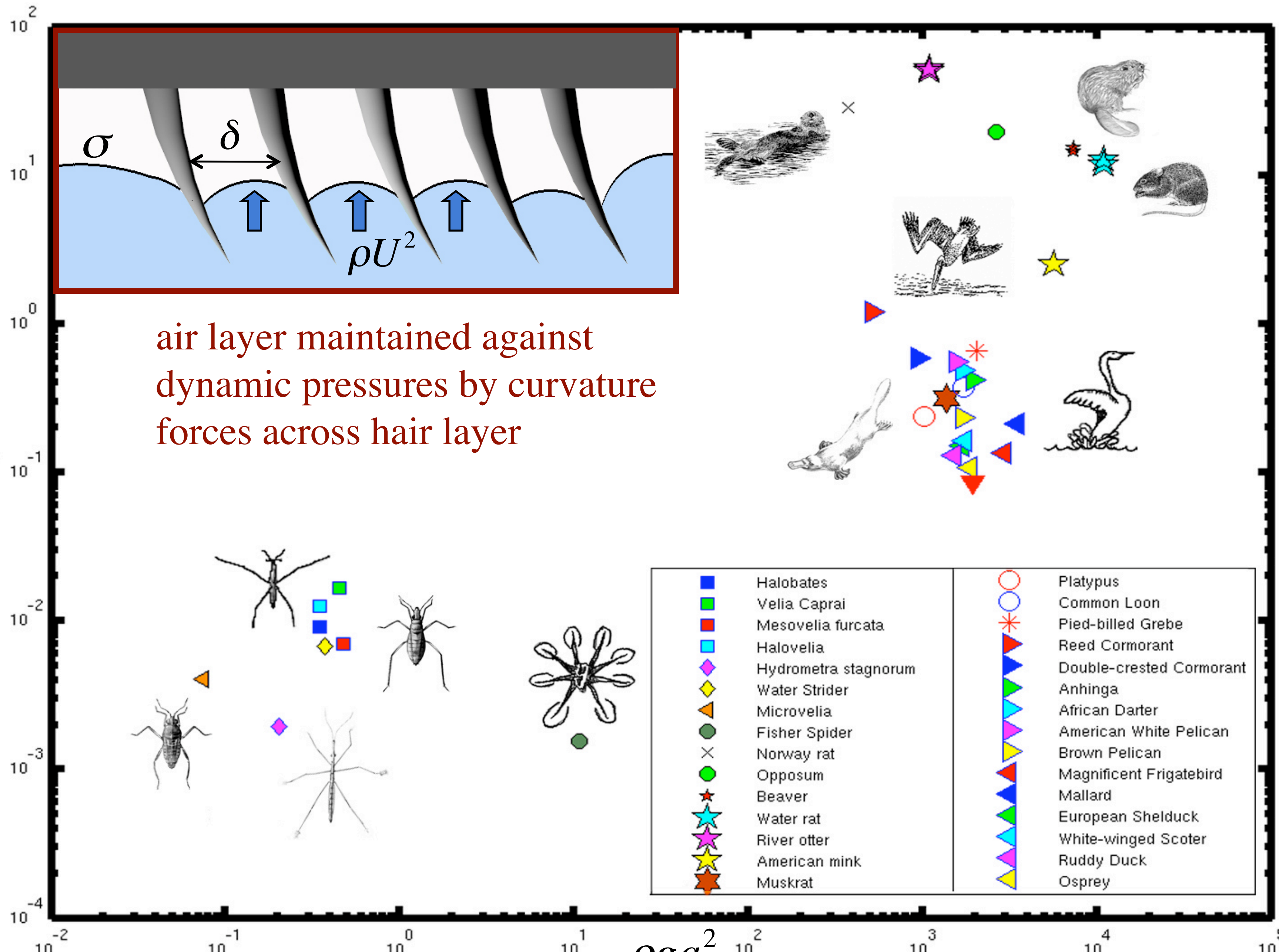
The integument of water-walking insects and spiders

- body and legs covered in dense mat of fine hairs: “the Lotus Effect”

- integument covered in a waxy, hydrophobic surface: $\theta_e = 108^\circ > \pi/2$
- hair layer increases surface area and so energetic cost of wetting
- hair mat renders surface superhydrophobic: $\theta^* \sim 130 - 175^\circ$ (Holdgate 1958)

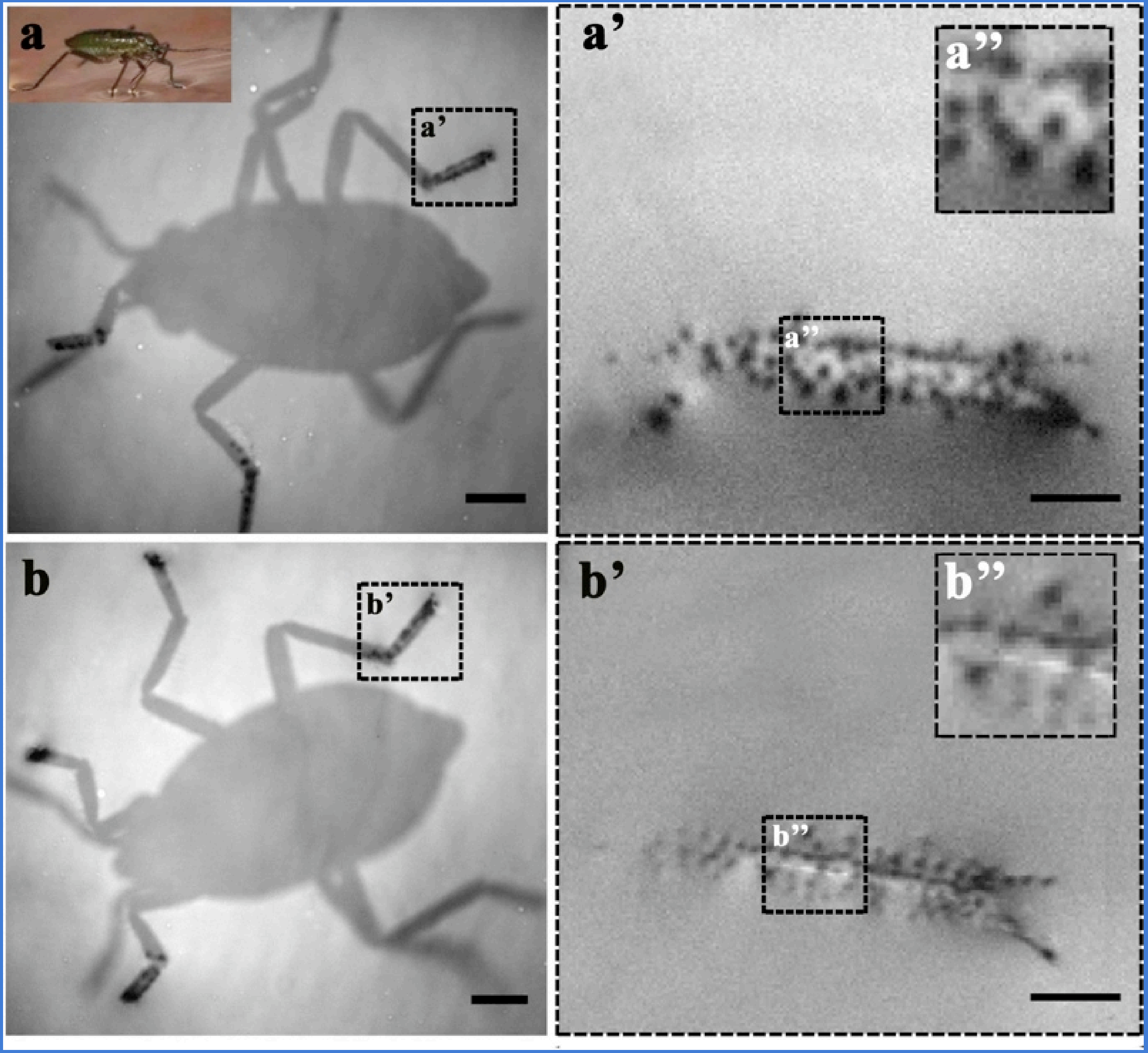
Can water-walking arthropods maintain a Cassie state?

$$We_s = \frac{\rho U^2 \delta}{\sigma}$$



$$B_o = \frac{\rho g a^2}{\sigma}$$

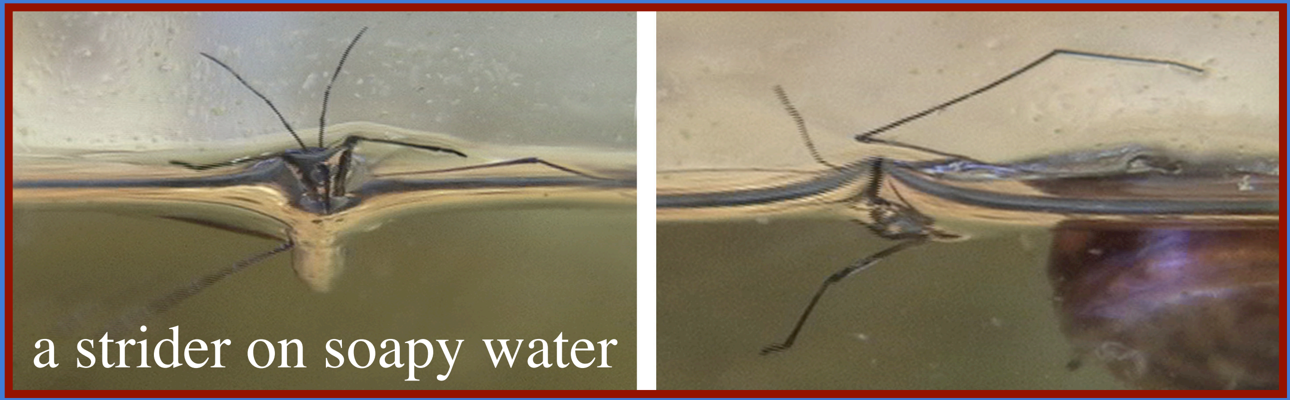
Water-walking arthropods: in a Cassie state



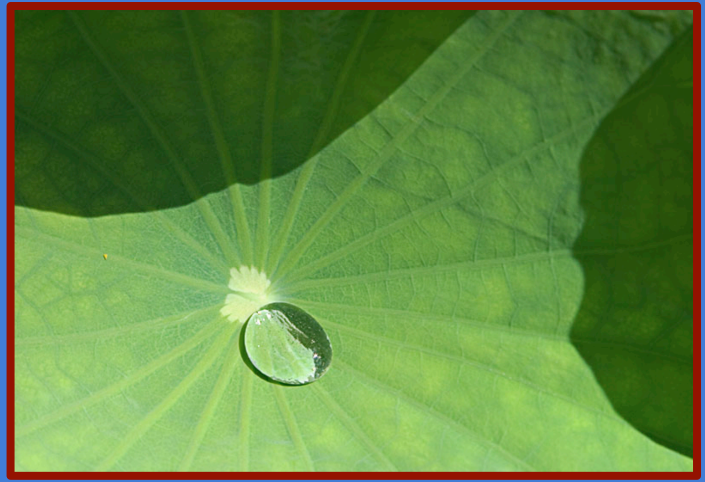
Mesovelia

Conundrum

- in order to avoid falling through the interface, water-walking insects must be water-repellent



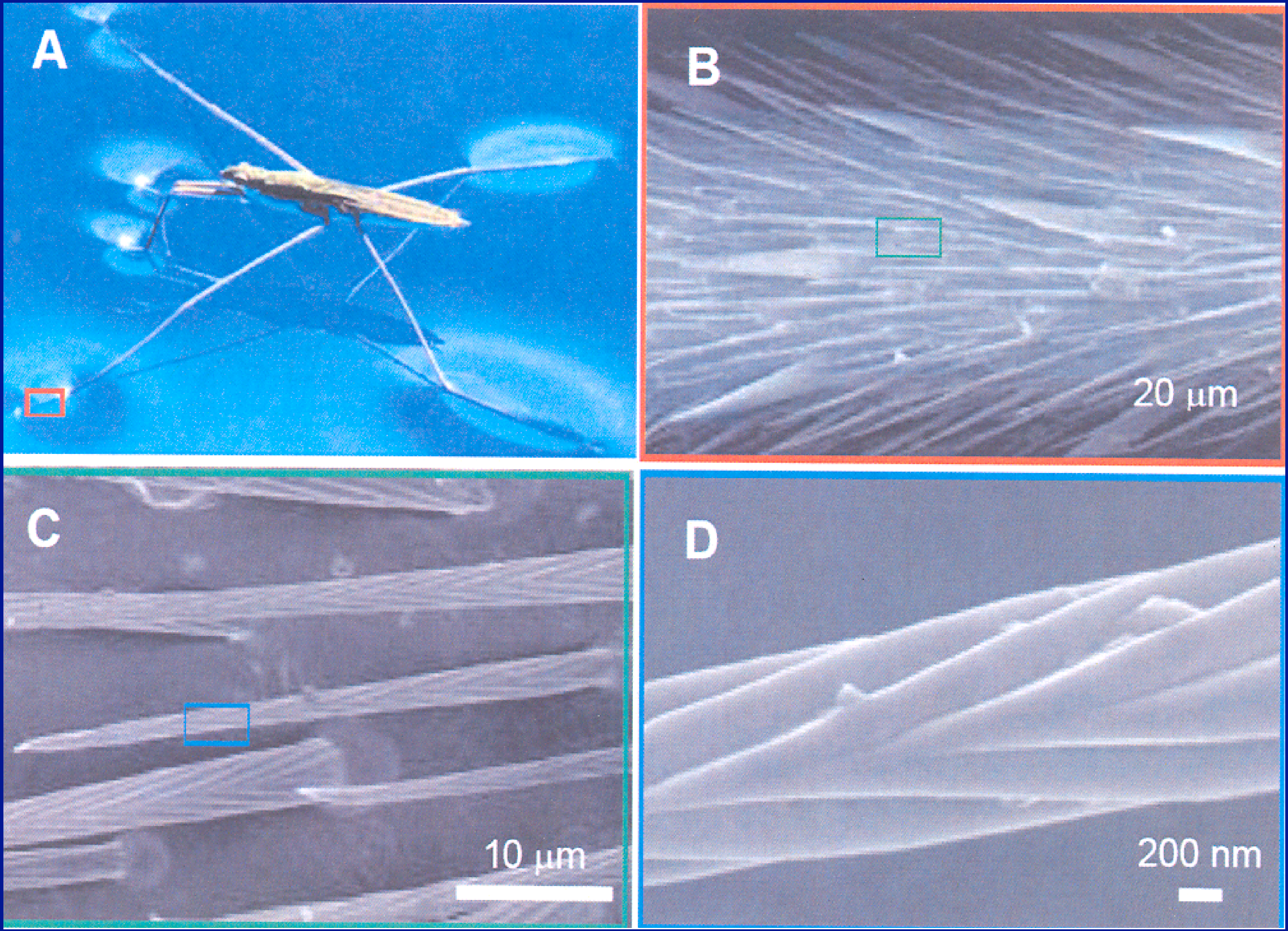
- water-repellent surfaces experience minimal traction on the free surface



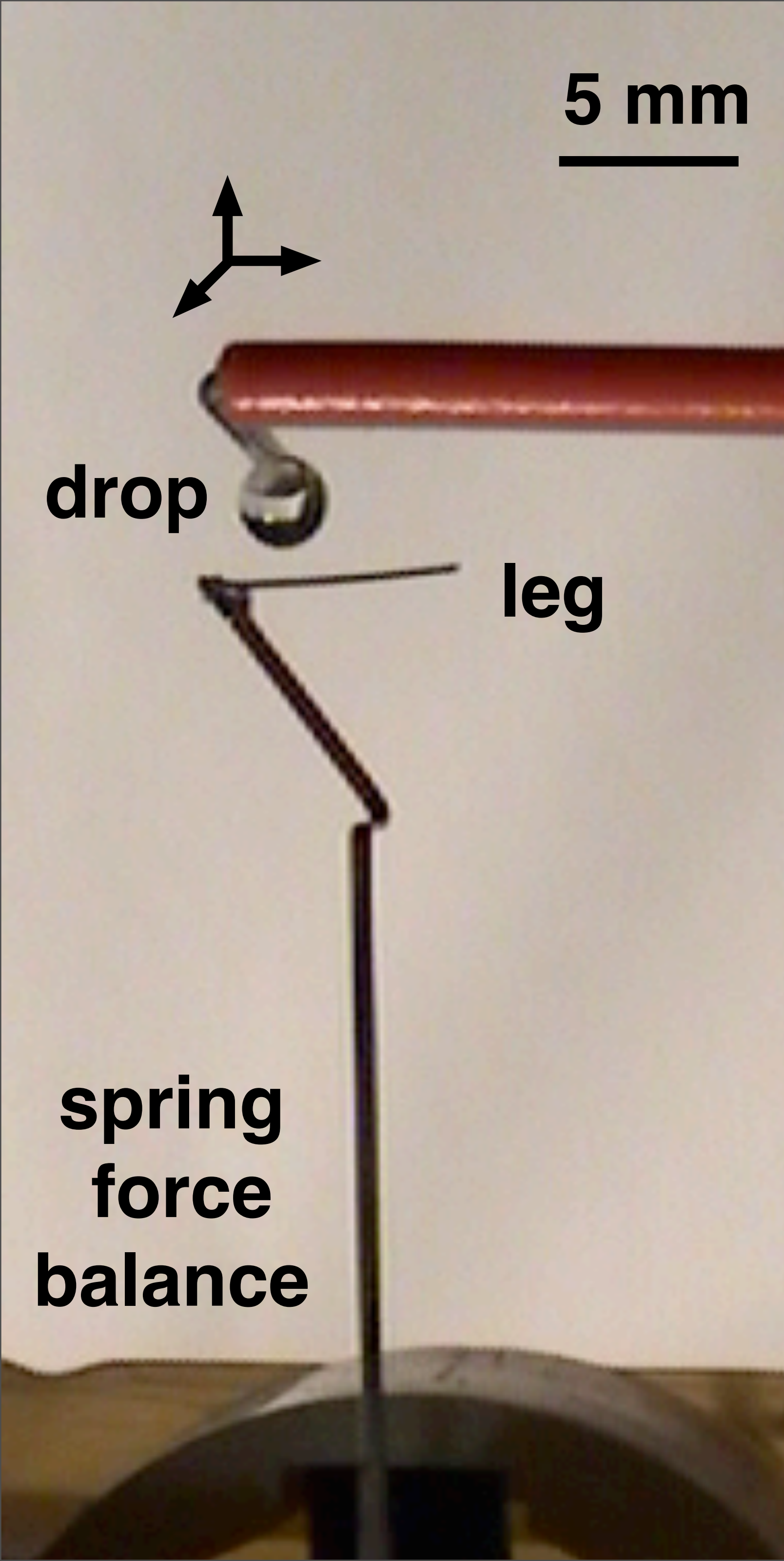
- water-walking insects propel themselves by striking the surface

HOW?

The driving legs of the water strider Gao & Jiang (2004)



- `grating' geometry suggests solution to their conundrum



Contact force measurements

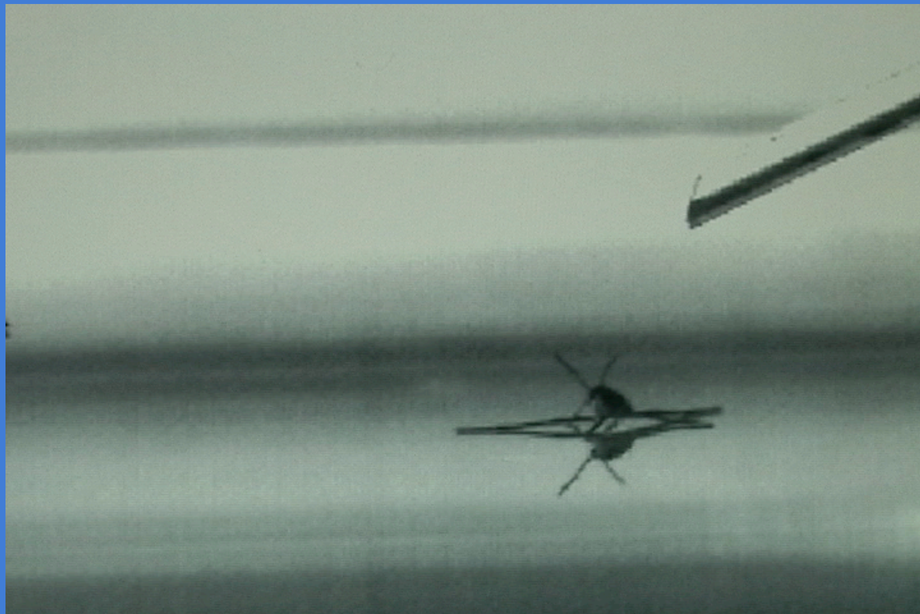
- strider leg mounted on spring force balance
- suspended water droplet brushed past leg in 3 principal directions
- Cassie state maintained
- measurements accurate to 0.1 dynes

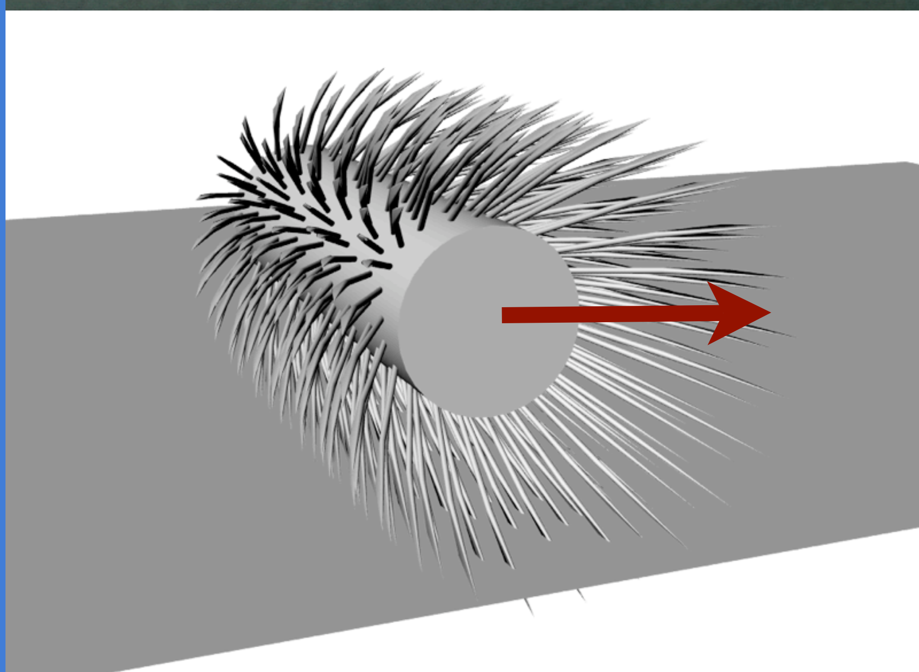
Inferences

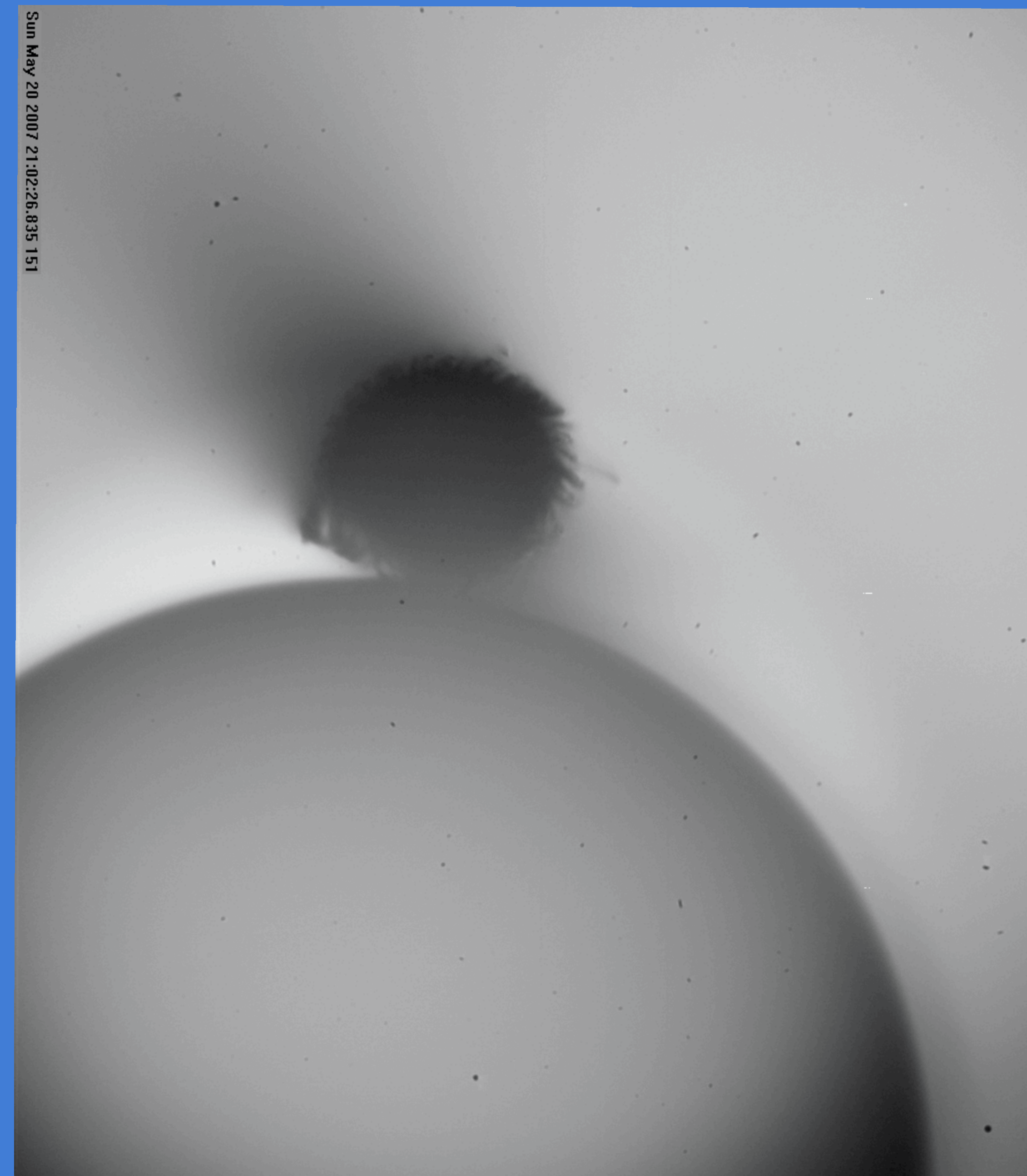
- contact forces depend on penetration depth of hairs, speed
- force/length resisting motion perpendicular, parallel (against the grain) and parallel (with the grain): 4 : 2 : 1

Talk tip: Manu Prakash, NF.01, Tues. 11:35

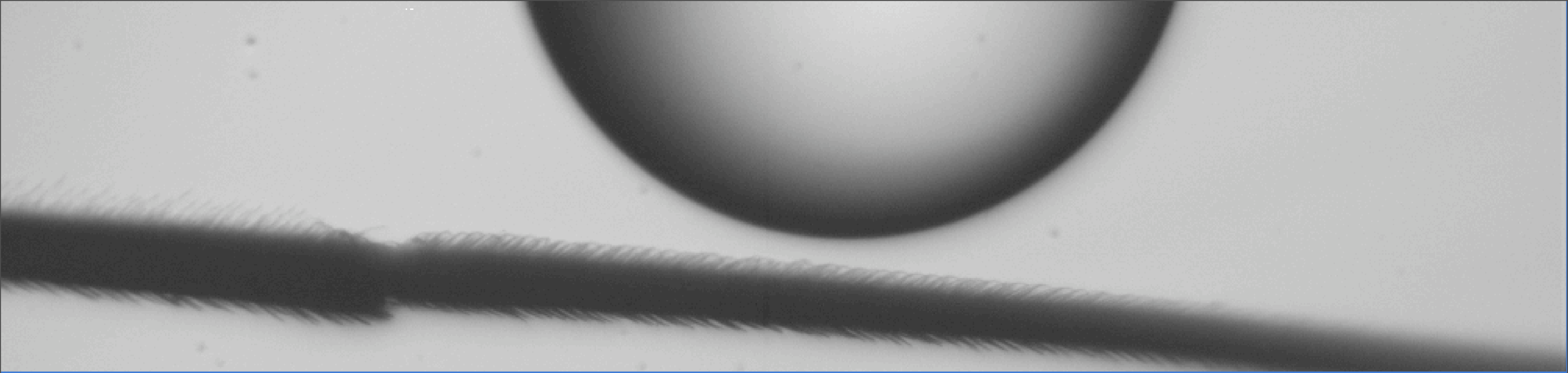
The dynamic interaction between insect cuticle and an interface



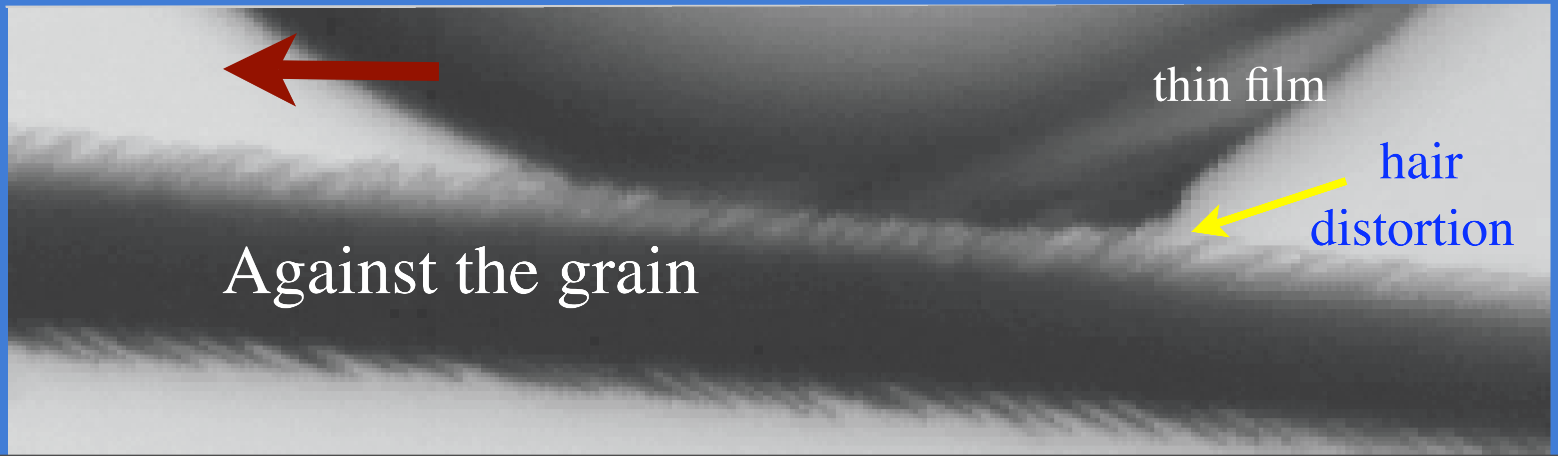




Large contact forces generated by brushing the surface.



Flexible hair generates unidirectional adhesion:



A. By virtue of its tilted, grooved hairs, the strider leg exhibits directional adhesion: drop moves with greatest difficulty perpendicular to leg

B. By virtue of the hair's flexibility, the leg exhibits unidirectional adhesion: drop moves most easily towards leg tip

Yields new insight into the form of their stroke...

Unidirectional adhesion enables:

1) maximum thrust generation by driving stroke

2) minimal drag during the gliding phase

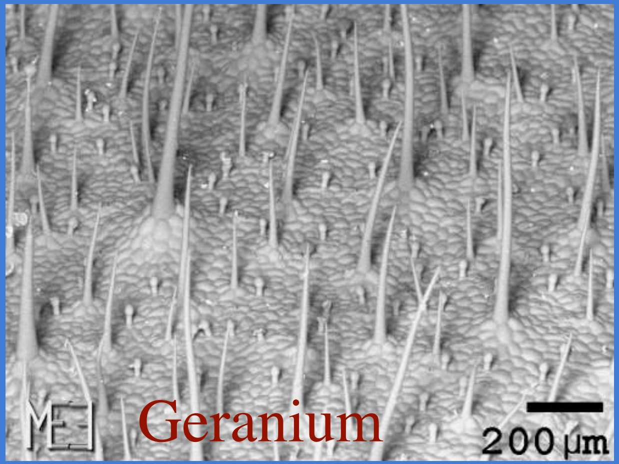
3) minimal adhesion during extraction phase

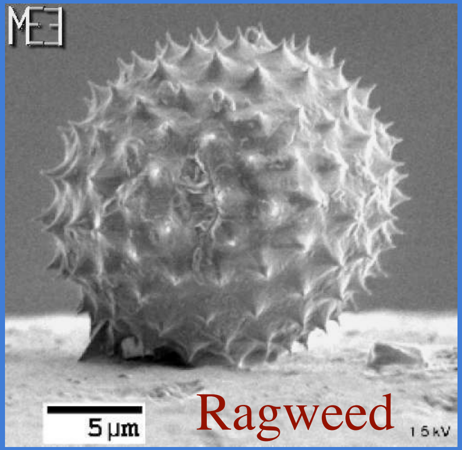
BIG PICTURE

- rationalized anisotropic roughness of water-walking arthropods

Plants are bumpy

- isotropic roughness provides water-repellency





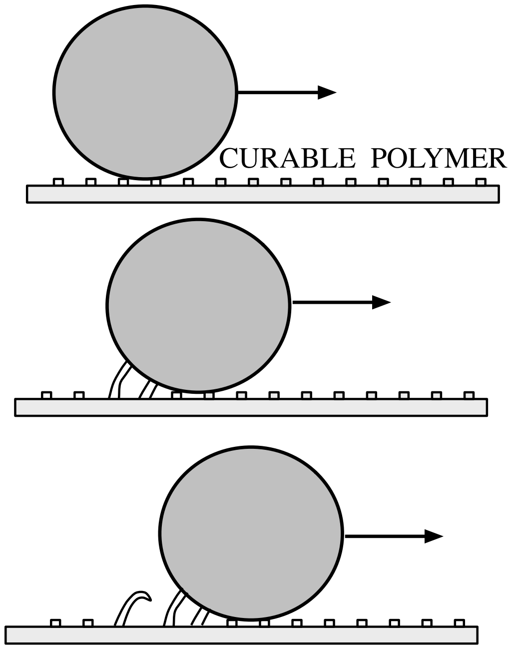
Barthlott & Neinhuis (1997)

Bugs are hairy

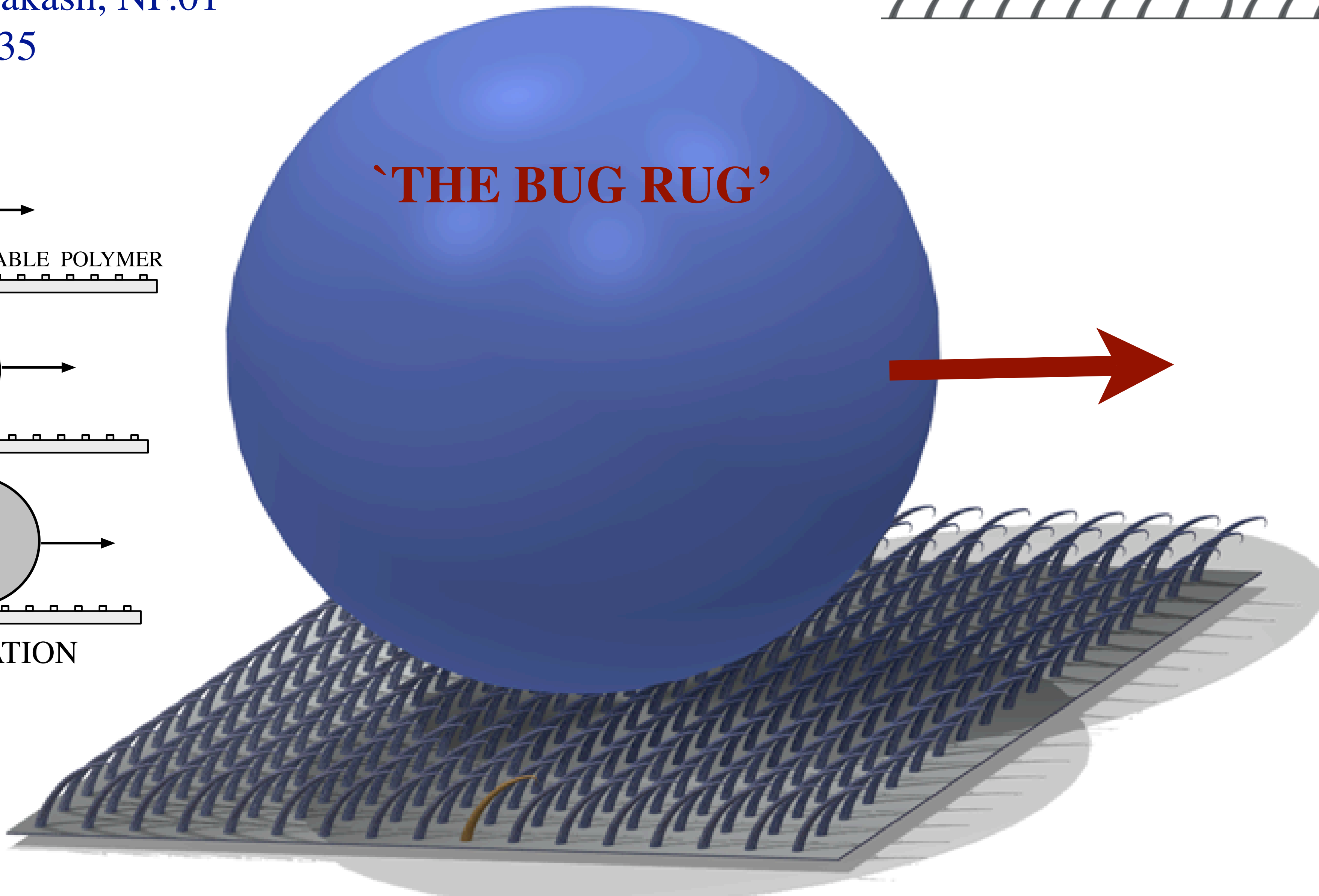
- roughness provides water-repellency
- anisotropic roughness facilitates **propulsion**

Biomimetic unidirectional surface

see Manu Prakash, NF.01
Tues 11:35



FABRICATION



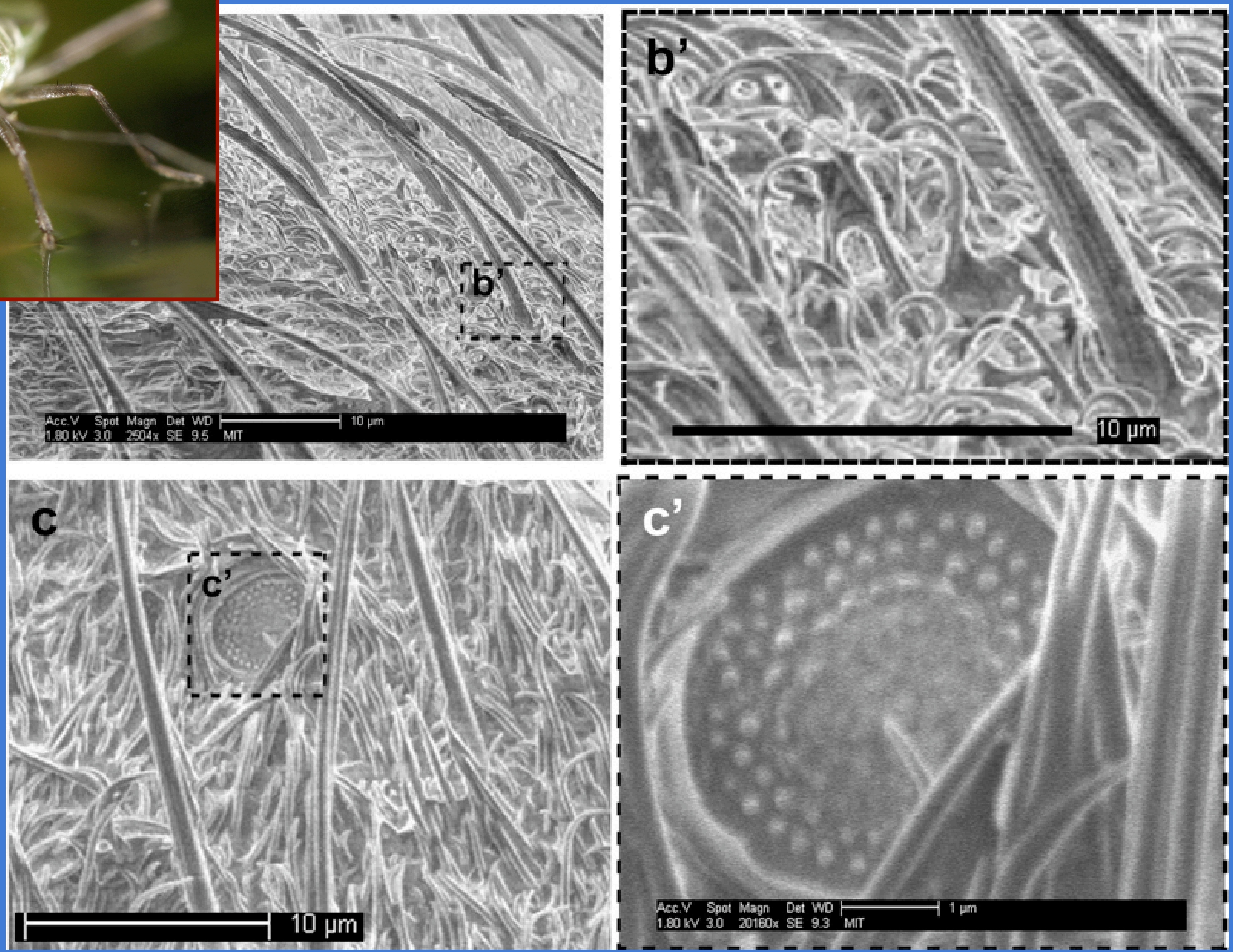
- permits drop motion in only one direction
- applications in directional draining, microfluidics

Another benefit of water-repellency...

III. Underwater breathing

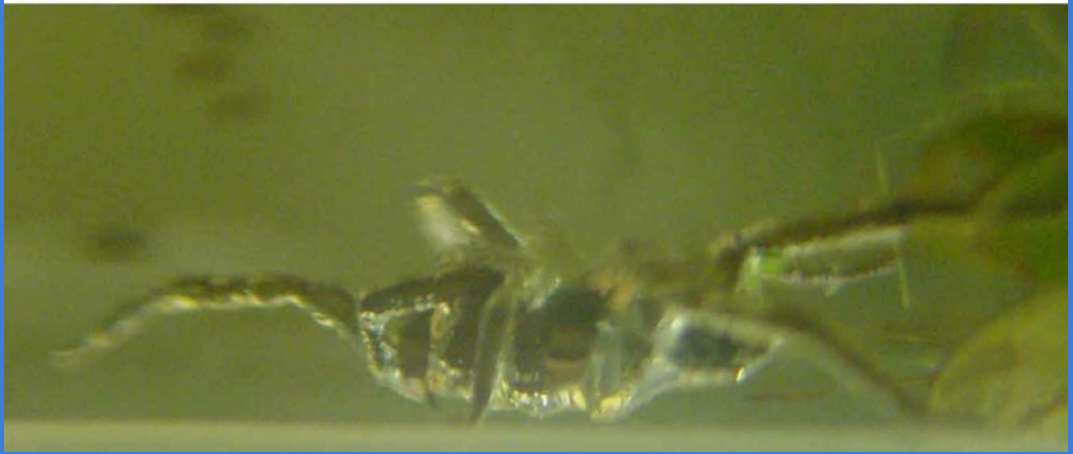
with Morris Flynn

The integument of Mesovelia: breathes through spiracles on thorax

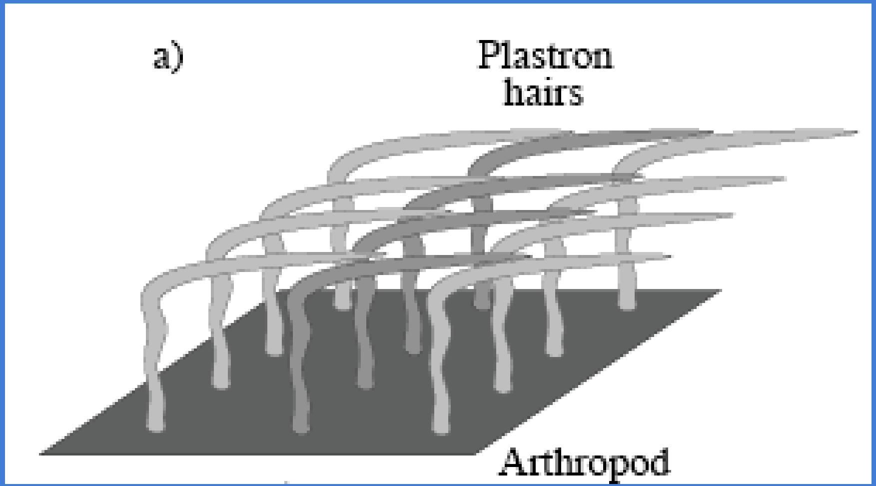


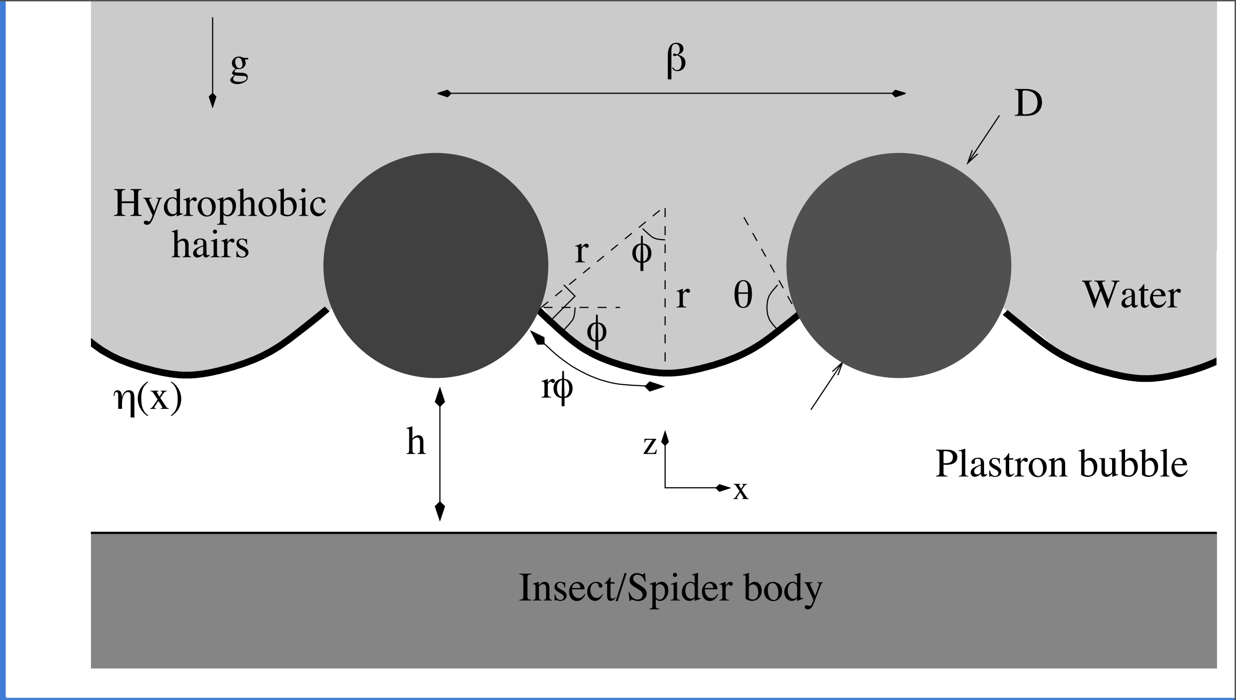
Dense hair layer on body: maintains Cassie state

- thin air layer, termed the `plastron', trapped on body surface



- plastron serves as external gill
- oxygen diffuses into plastron, enabling extended dives
- may sustain bug indefinitely





Intuition

- decreasing hair spacing will mechanically stabilize plastron, but decrease area A through which it breathes

Mechanical stability

$$\frac{\Delta p}{\sigma} = -\nabla \cdot \mathbf{n} = \frac{\eta_{xx}}{(1+\eta_x^2)^{3/2}}$$

BCs: $\eta_x(0) = 0\,, \qquad \eta_x\left(\tfrac{1}{2}[\beta - D\sin(\theta - \phi)]\right) = \tan\phi$

Plastron Chemistry (Thorpe & Crisp 1947; Rahn & Paganelli 1968)

$$\dot{V}_{\mathrm{O}_2} = A\, J_{\mathrm{O}_2}\,(x_{\mathrm{O}_2}\,\mathcal{H}_{\mathrm{O}_2} - p_{\mathrm{O}_2}) - q$$

$$\dot{V}_{\mathrm{N}_2} = A\, J_{\mathrm{N}_2}\,(x_{\mathrm{N}_2}\,\mathcal{H}_{\mathrm{N}_2} - p_{\mathrm{N}_2})$$

$$\dot{V}_{\mathrm{CO}_2} = A\, J_{\mathrm{CO}_2}\,(x_{\mathrm{CO}_2}\,\mathcal{H}_{\mathrm{CO}_2} - p_{\mathrm{CO}_2}) + q$$

Bubble partial volume, pressures: $V_j\ ,\ p_j$

Concentration of dissolved gas in water: x_j

Henry's Law constants: $\mathcal{H}_j$

Rate of O_2 consumption, CO_2 production: q

Invasion coefficients: $J_j \equiv \dfrac{\alpha_j\,\widehat{D}_j}{\delta}$

where $\alpha_j\ ,\ \hat{D}_j\ ,\ \delta$ are the solubility, diffusivity and boundary layer thickness

Bubble pressure

- determined by bubble chemistry

$$p_{bub} = \sum_j p_j \simeq \sum_j x_j \mathcal{H}_j - \left(\frac{\dot{V}_{O_2} + q}{A \mathcal{J}_{O_2}} \right) - \frac{\dot{V}_{N_2}}{A \mathcal{J}_{N_2}}$$

since $\quad \mathcal{J}_{CO_2} \gg \mathcal{J}_{O_2}, \mathcal{J}_{N_2}$

Steady state

$$p_{bub} \simeq \sum_j x_j \mathcal{H}_j - \frac{q}{A \mathcal{J}_{O_2}}$$

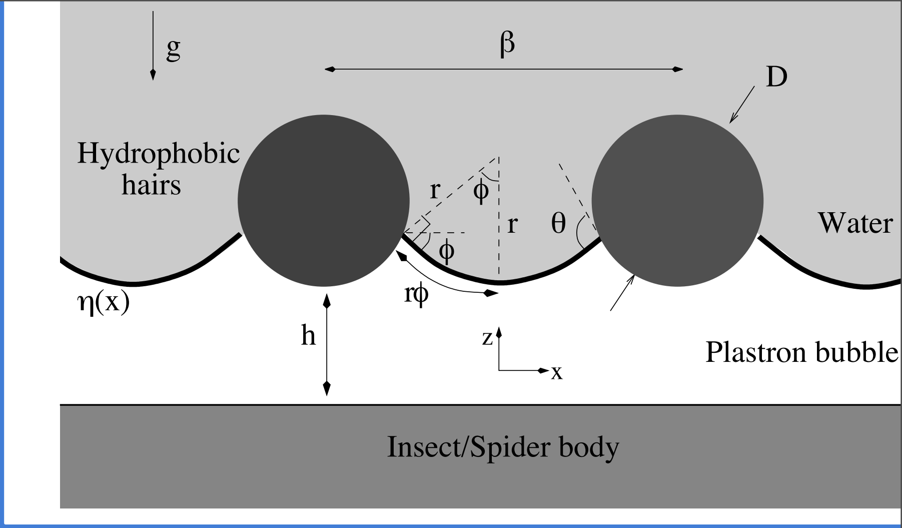
Survival requires conservation of oxygen in plastron:

$$x_{O_2} \mathcal{H}_{O_2} \quad > \quad \frac{q}{A \mathcal{J}_{O_2}}$$

influx from water respiration

Plastron stability

- couple chemistry and mechanics



Normal stress balance across plastron:

$$p_{atm} + \rho\, g\, H - \sum_j x_j\, \mathcal{H}_j + \frac{q}{A\, \mathcal{J}_{O_2}} = \frac{\sigma}{r}$$

Nondimensionalizing:

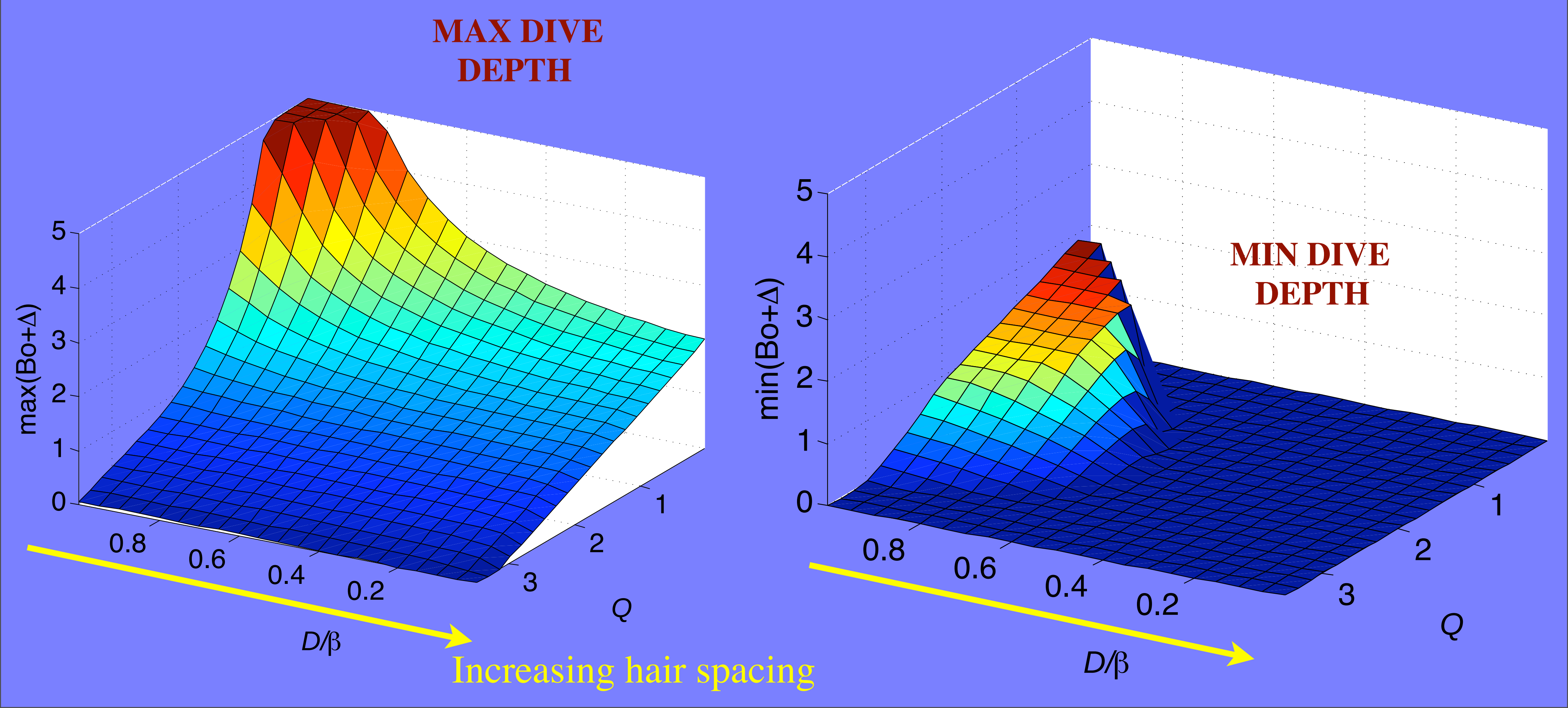
$$\mathrm{Bo} + \Delta = \frac{1}{r/\beta}\left(1 - \frac{\mathcal{Q}}{2\phi}\right)$$

where $\mathrm{Bo} = \dfrac{\rho\, g\, H}{\sigma/\beta}$, $\quad \mathcal{Q} = \dfrac{q}{A\, \mathcal{J}_{O_2}\, (\sigma/\beta)}$, $\quad \Delta = \dfrac{p_{atm} - \sum_j x_j\, \mathcal{H}_j}{\sigma/\beta}$

- seek solutions for a prescribed $\mathrm{Bo},\ \mathcal{Q},\ \Delta$
- assess the mechanical and respiratory stability of the plastron

Range of viable dive depths

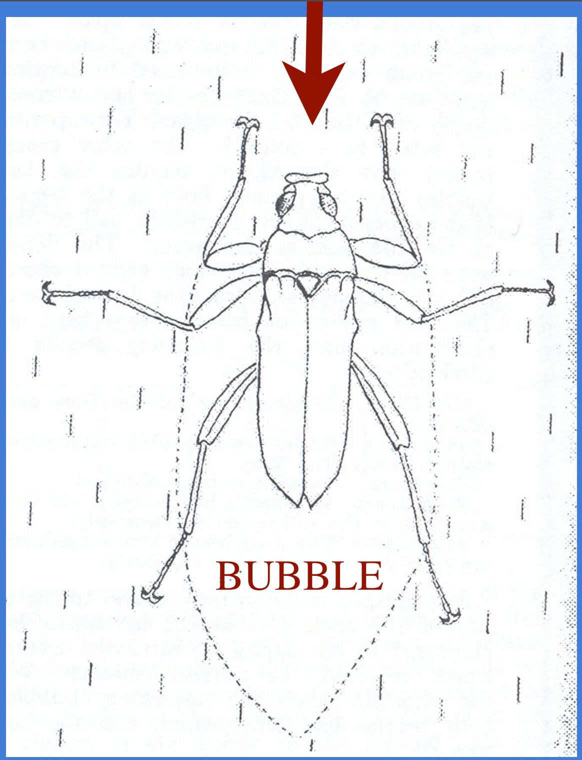
- respiration plays a critical role in limiting range of dive depths
- maximum dive depth prescribed by mechanical stability of plastron: decreases with increasing respiration rate and hair spacing
- minimum depth for large D/β , moderate Q required to increase A



- provides rationale for behaviour of various plastron breathers, e.g. dive depth and duration

- analysis can be extended to dynamic setting by considering influence of dynamic pressures and enhanced invasion coefficients

  e.g. *P. Tuberosis* can survive only in flowing streams (Stride 1954)



Biomimetics: underwater fuel cells (Shirtcliffe et al. 2006)

- submerged cavities covered with superhydrophobic foam function as plastrons, supply oxygen to fuel cells
- could potentially power small underwater vehicles

Human respiration: $q \sim 1.6 \times 10^{-3}\, m^3/s$

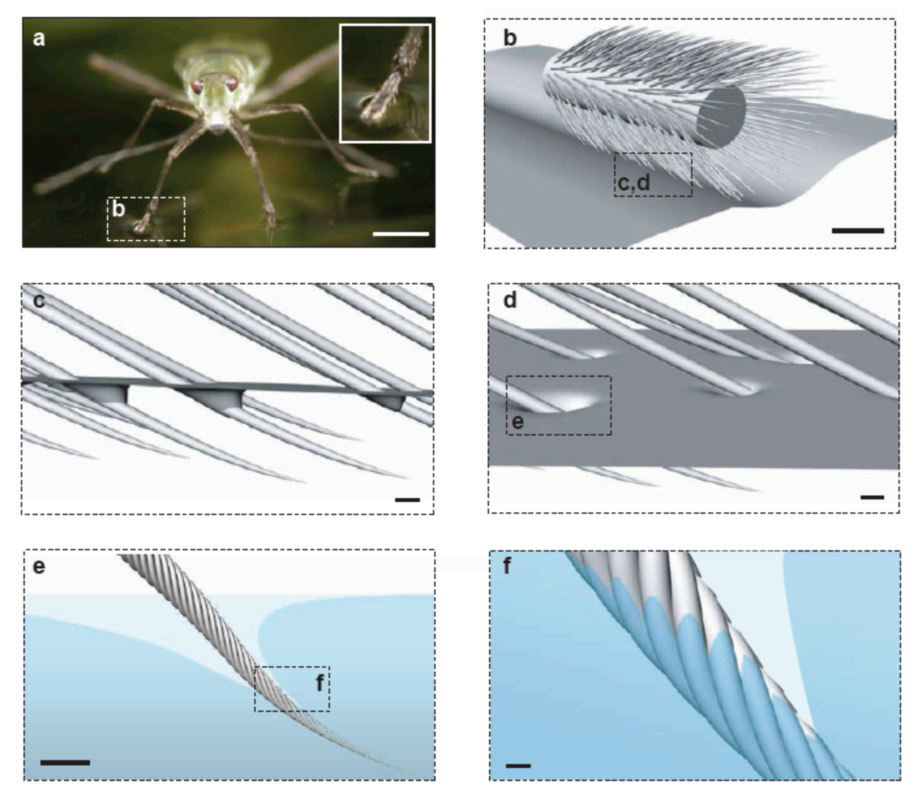
- superhydrophobic material with $\beta \sim 1 \mu m$ would require a diving bell with characteristic area $100\, m^2$

Conclusions

- brief overview of some problems arising in Entomological Fluids
- require consideration of broad range of scales (Nm - cm)

An irony

- early work on wetting inspired by pesticide design
- surviving insects now inform and inspire biomimetic design



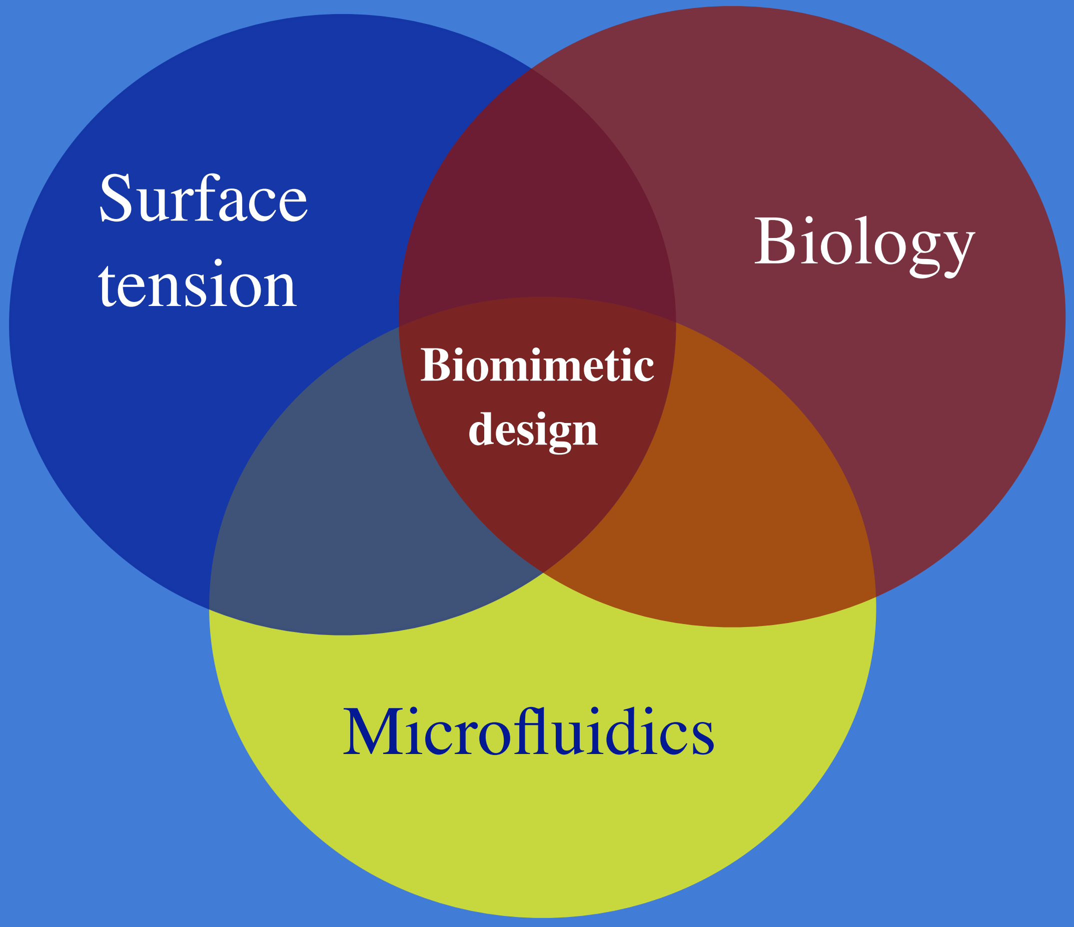
Fundamental issues raised

- impact at low Bond number (with J. Aristoff, M. Hancock)
- dynamic water-repellency (with M. Prakash, D. Quere)
- adhesion/detachment of a soft solid at an interface (with P. Reis)

BIGGER PICTURE

- a philosophical point: on mechanism in biology

"If you can imagine it, it exists."



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In all things of Nature, there is something of the marvelous.
- Aristotle