Pattern Formation and Turbulence in Convection: The Legacy of Henry Benard

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Santa Barbara CA USA

Henri Benard 1874-1939

Otto Laporte Lecture, Nov. 18 2007

My many co-workers over four decades:

Students (29)
Avinoam Kornblit
Wing-Yim Tam
Marco Dominguez-Lerma
Alan Singsaas
Anneli Aitta
Robert Duncan
Christopher Meyer
Li Ning
Michael Dennin
Lori Goldner
Melora Larson
Yu-Chou Hu
Kristina Lerman
Steve Trainoff
Kerry Kuehn
Brian Naberhuis
Marcus Linek
Leif Thomas
Nathan Currier
Xiaochao Xu
Kim Thompson
Woravat Meevasana
John Royer
Patrick O'Neill
Nathan Becker
Alexei Nikolaenko
Eric Brown
James Hogg
Matthew Schreiner
Francois Heber

Postdocs (37)
Alan Evenson
Dennis Greywall
Robert Behringer
Robert Walden
Victor Steinberg
Ravi Mehrotra
Ingo Rehberg
Richard Heinrichs
Tim Sullivan
Joseph Niemela
Norbert Mulders
Ken Babcock
John de Bruyn
Eberhard Bodenschatz
Stephen Morris
Lars Inge Berge
Feng-Chuan Liu
Andy Kahn
Mingming Wu
Hanan Baddar
Haiying Fu
Jun Liu
Kapil Bajaj
A. Schegolev
Urs Bisang
Nathalie Mukolobwiez
Edgar Genio
Sarabjit Mehta
Dan Murphy
Michael Scherer
Jaechul Oh
Denis Funfschilling
Xin-Liang Qiu
Sheng-Qi Zhou
Tahar Aouaroun
Jin-Qiang Zhong

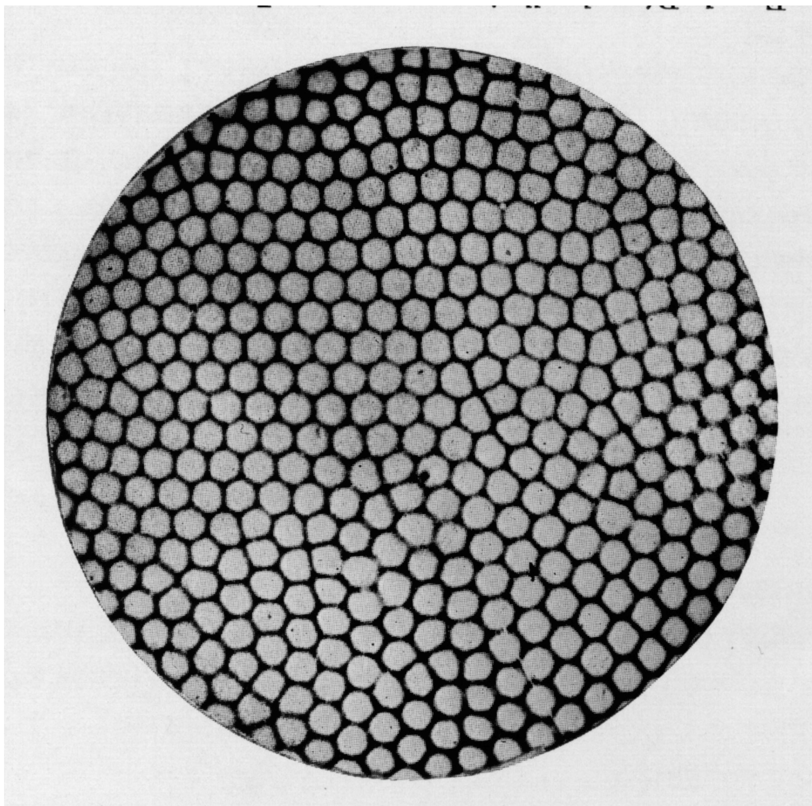
Others (10)
Michael Jefferson
Martin Treiber
A. Tschammer
Frank Horner
Janet Scheel
Francisco Fontenele Araujo
Karl Mueller
S.M. Zoldi
Paul Finley
Sergei Jerebets
Enrico Calzavarini
Kazuyazu Sugiyama

Italic: 8 undergraduate students
Bold: 16 w. academic careers

Colleagues (25)
Frank Pobell
Amnon Aharony
Mike Cross
Pierre Hohenberg
Sam Safran
Henry Greenside
David Cannell
Manfred Lucke
Jack Swift
Helmut Brand
Robert Deissler
Ingo Rehberg
Morten Tveitereid
Robert Ecke
Shinichi Sakurai
Ronnie Mainieri
Lorenz Kramer
Werner Pesch
Siegfried Grossmann
Detlef Lohse
Jose Ortiz de Zarate
Jan Sengers
Nandor Eber
Agnes Buka
Yuanming Liu

102 creative individuals from 21 countries !!

Henri Benard (Ph.D. thesis, 1900)



Carried out the first systematic and quantitative study of convection in a shallow layer heated from below, and studied the associated formation of convection **PATTERNS** systematically and quantitatively

E. Bouty : "*Bénard did not make any effort to provide general theoretical explanations* … ".

The report of the thesis committee stated "*…. though Bénard's main thesis was very peculiar, it did not bring significant elements to our knowledge. … the thesis should not to be considered as the best of what Bénard could produce.*"

PHILOSOPHICAL MAGAZINE

AND

JOURNAL OF SCIENCE.

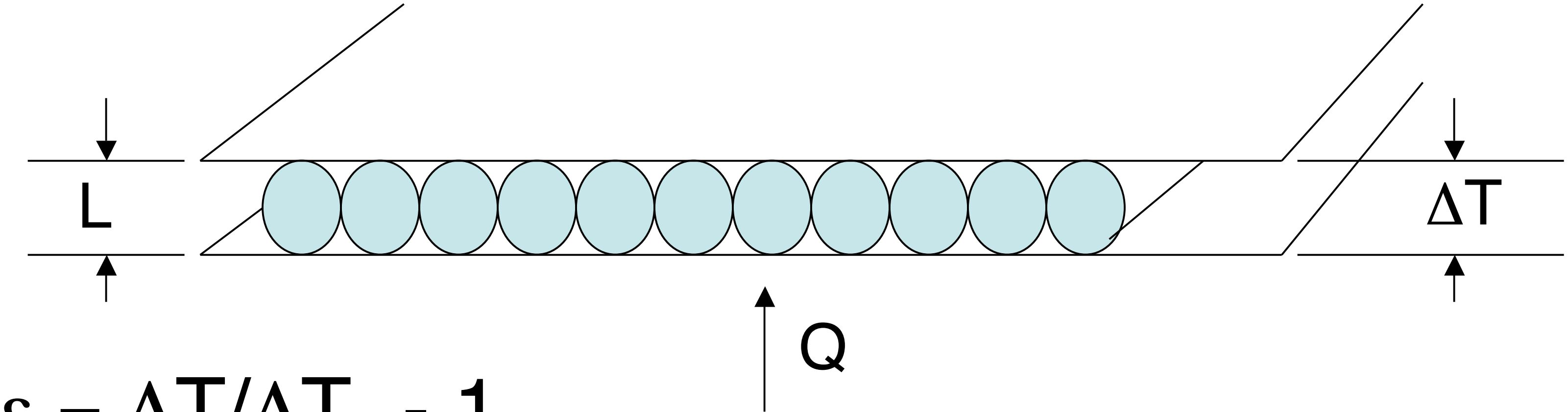
[SIXTH SERIES]

DECEMBER 1916.

LIX. *On Convection Currents in a Horizontal Layer of Fluid, when the Higher Temperature is on the Under Side.* *By* Lord RAYLEIGH, *O.M., F.R.S.**

THE present is an attempt to examine how far the interesting results obtained by Bénard † in his careful and skilful experiments can be explained theoretically. Bénard worked with very thin layers, only about 1 mm. deep, standing on a levelled metallic plate which was maintained at a uniform temperature. The upper surface was usually f

Lord Rayleigh



$\varepsilon = \Delta T/\Delta T_c - 1$

Stability analysis of the quiescent fluid layer.
Slip boundary conditions at top and bottom.

Benard's convection actually was driven primarily by surface-tension gradients.
[J.R.A. Pearson, JFM **4** , 489 - 500 (1958)].

Rayleigh's results:

$$R = (\alpha / \kappa \nu)\, g\, L^3\, \Delta T$$

α = isobaric thermal expansion coefficient

g = acceleration of gravity

ΔT = applied temperature difference

L = layer thickness

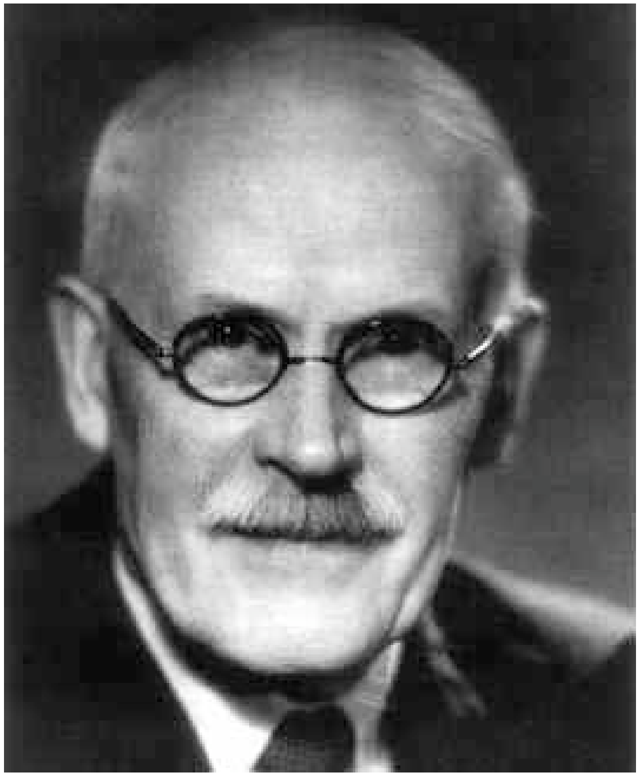
ν = kinematic viscosity

κ = thermal diffusivity

Instability at finite $\Delta T = \Delta T_c$ and finite $k = k_c$

Stationary instability (real eigenvalues)

$$R_c = \frac{27}{4}\pi^4 \qquad k_c = \frac{\pi}{\sqrt{2}}$$



Sir Harold
Jeffreys, 1891 - 1989

No-slip (rigid)
Boundary conditions

$$R_c = 1708$$

$$k_c = 3.117$$

Next milestones:

nonlinear effects:

- nature of the bifurcation
- pattern just above onset
  - Malkus and Veronis (1958): slip (free) BCs
    - super-critical
    - rolls (or stripes)

Next milestones:

nonlinear effects:

nature of the bifurcation

pattern just above onset

Malkus and Veronis (1958): slip (free) BCs

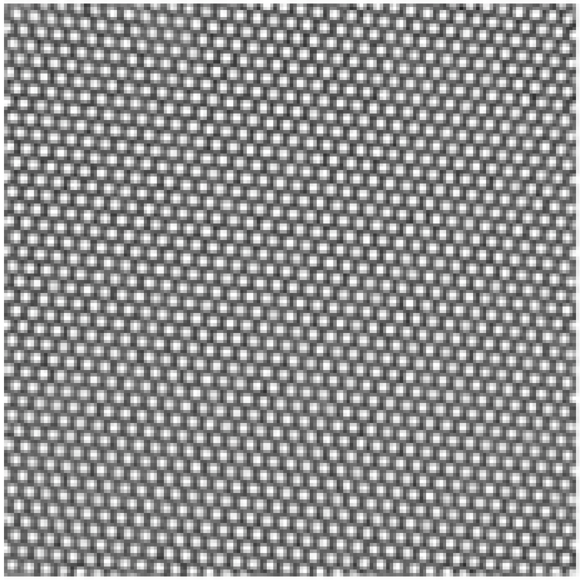
super-critical

rolls (or stripes)

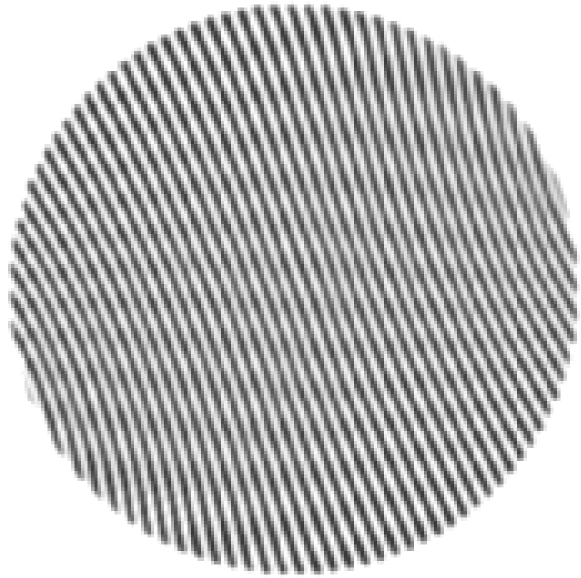
Schluter, Lortz, and Busse (1965): ridgid BCs

super-critical

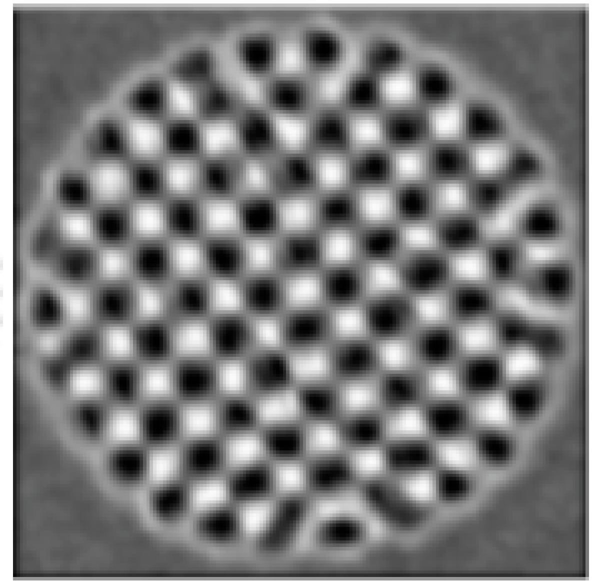
rolls (or stripes)



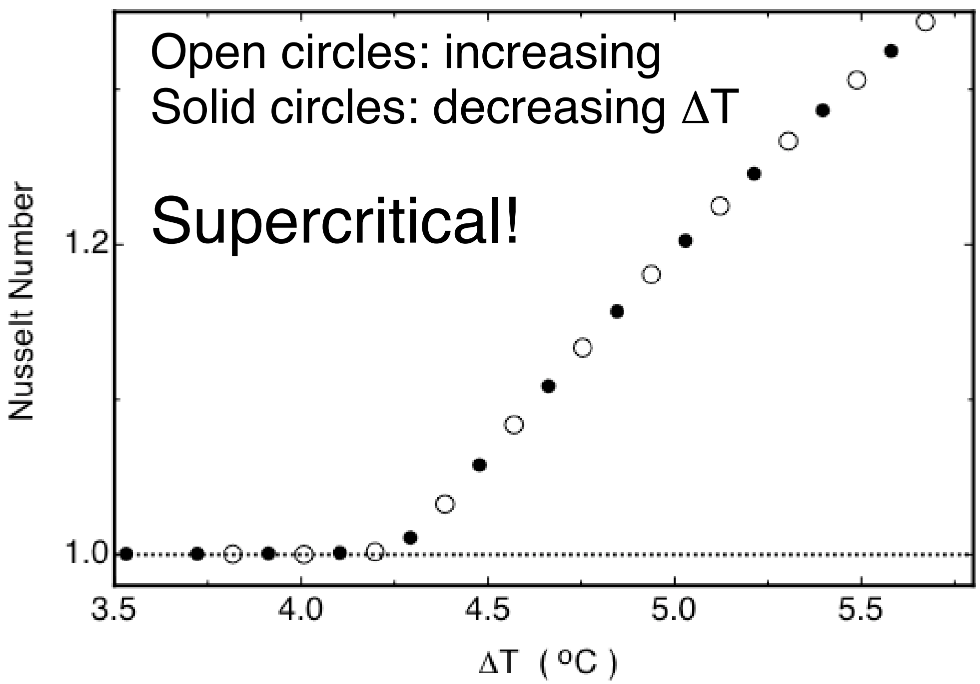
Non-Boussinesq



Boussinesq



Binary mixtures
(diffusion driven)



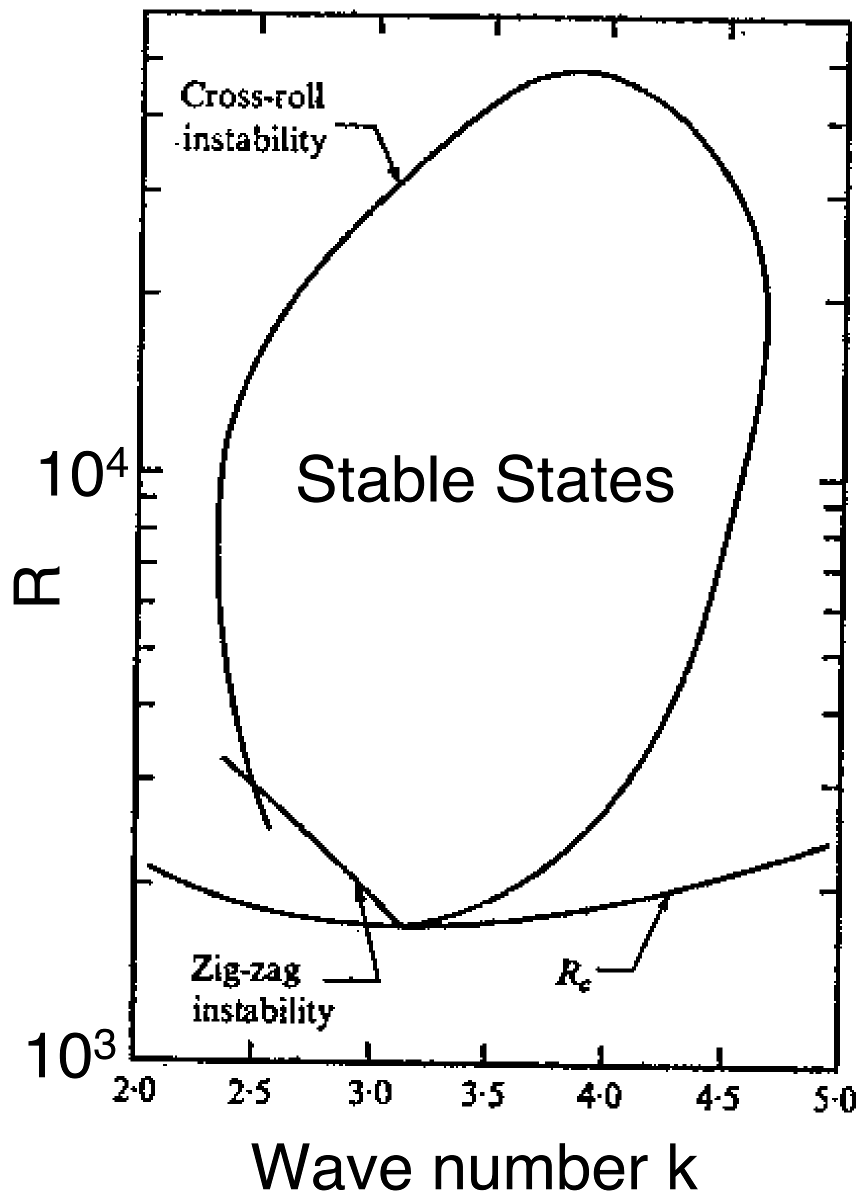
G.A., unpublished

G. A., in "Dynamics of spatio-temporal cellular structures - Henri Benard centenary review", edited by I. Mutabazi, Jose E. Wesfreid, and E. Guyon (Springer Tracts in Modern Physics, V. 207), 2006.

Swift and Hohenberg showed that the bifurcation becomes **subcritical** in the presence of additive (thermal) noise !!!

J. Swift and P.C. Hohenberg, "Hydrodynamic fluctuations at the convective instability", Phys. Rev. A **15**, 319 (1977).

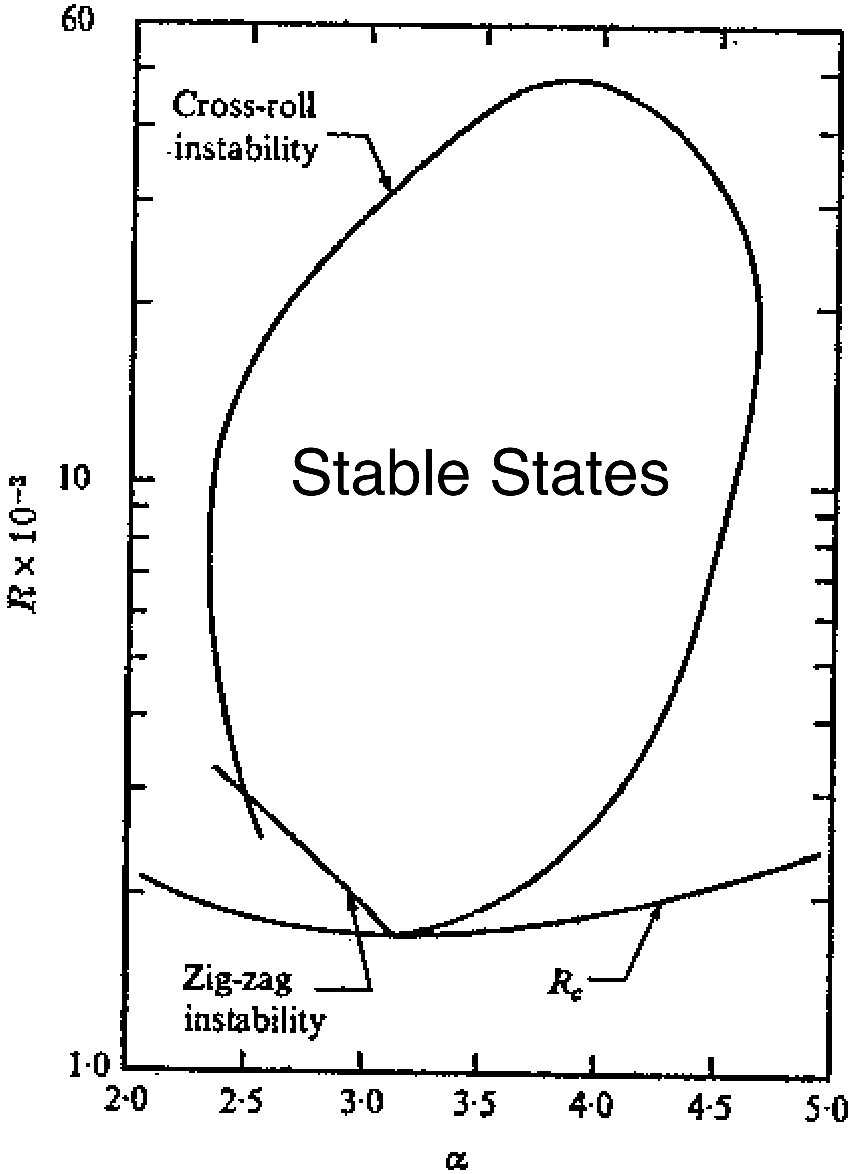
For most systems, this effect was expected to be observable only for $\varepsilon < 10^{-6}$ and out of reach of the experimentalists.



In the 1970's and thereafter Busse and Clever calculated the stability boundaries of rolls above onset. The "Busse Balloon" became the "playground" of experimentalists and theorists with an interest in nonlinear physics and pattern formation

J. Fluid Mech. **65**, 625 (1974); **91**, 319 (1979); and several other papers.

Fritz Busse, FD Prize recipient 2000, and his Balloon!



No extremum principle !

Any state inside the Busse Balloon is attainable if the phase of the pattern is pinned, e.g. by sidewalls, in an experiment.

Wave-number selection processes can occur when the phase can slip at some point in the pattern.

Four uniquely nonlinear issues:

1.) Fluctuations near the onset of RBC

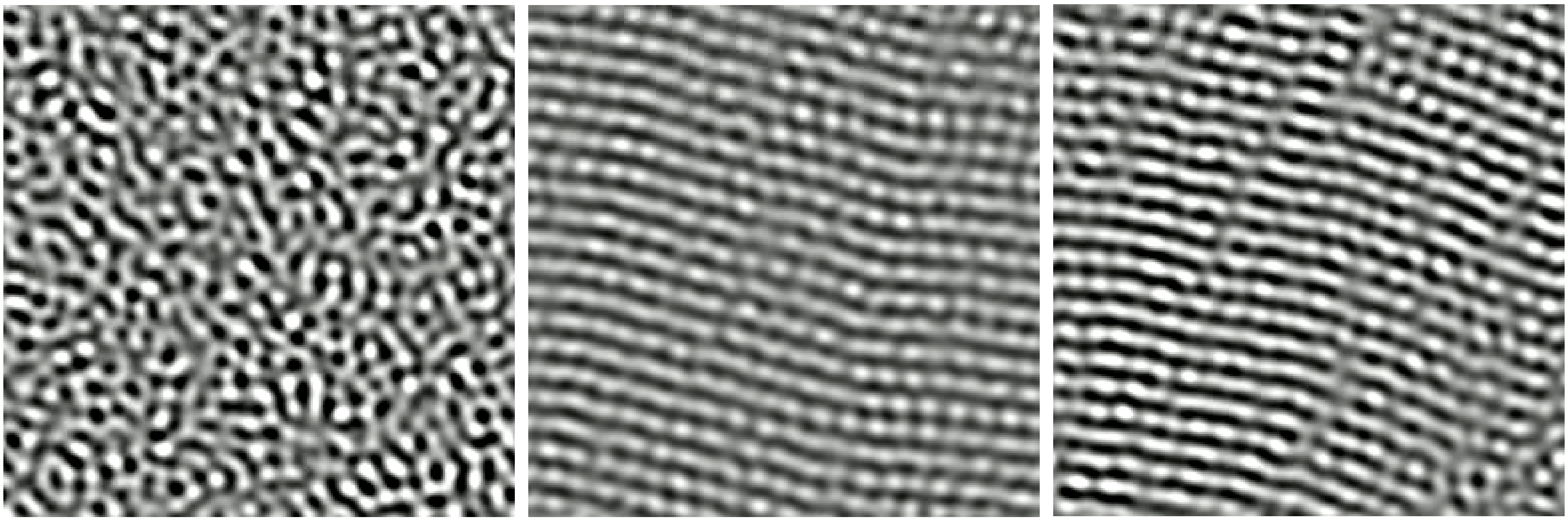
2.) Wavenumber selection

3.) Spatio-temporal chaos (STC)

4.) Localized structures (Pulses)

Fluctuations near Onset

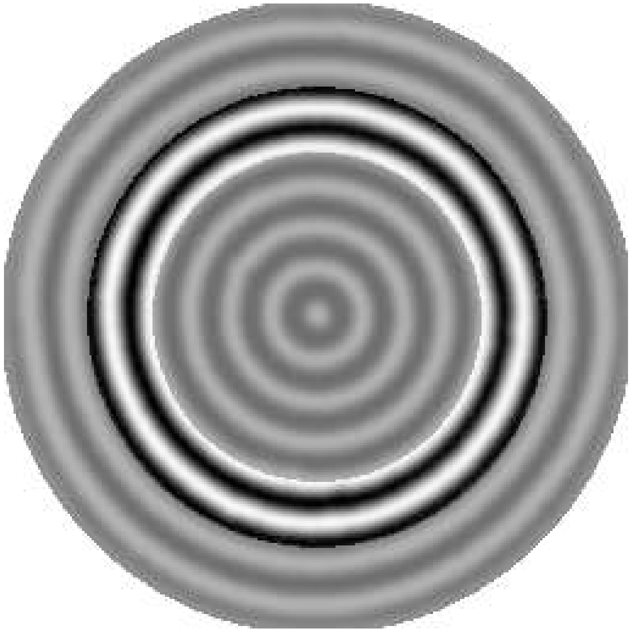
$\varepsilon = -0.066$ $\varepsilon = 0.000$ $\varepsilon = 0.003$



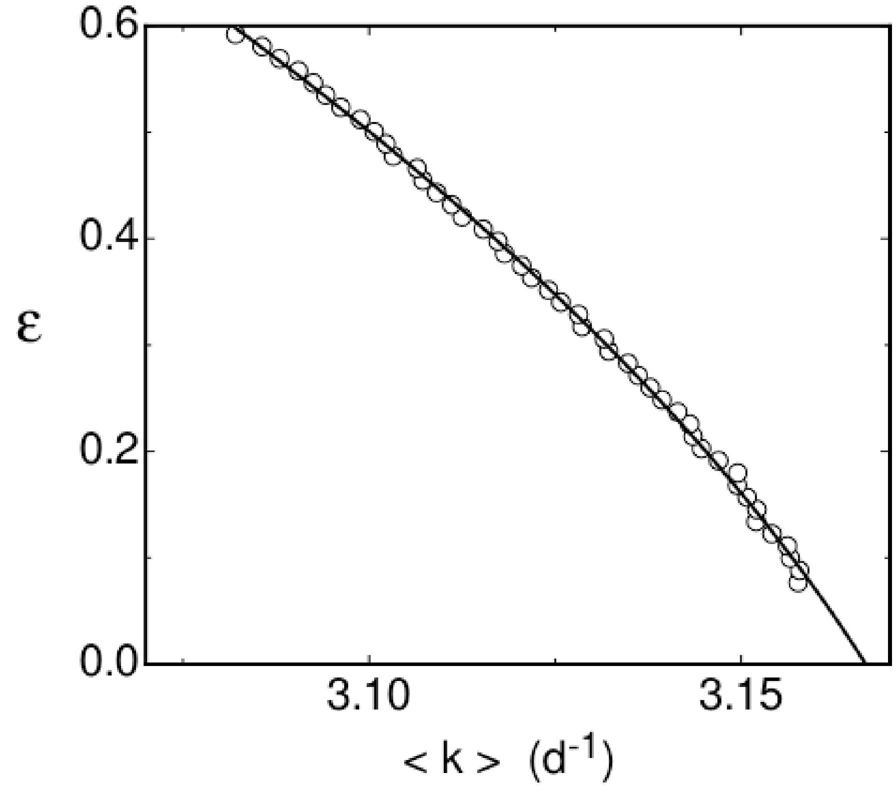
J. Oh and G.A., Phys. Rev. Lett. **91**, 094501 (2003).

$\varepsilon = R / R_c - 1$

Convection in SF_6



J.R. Royer, P. O'Neill, N. Becker, and G. A., Phys. Rev. E **70** , 036313 (2004).

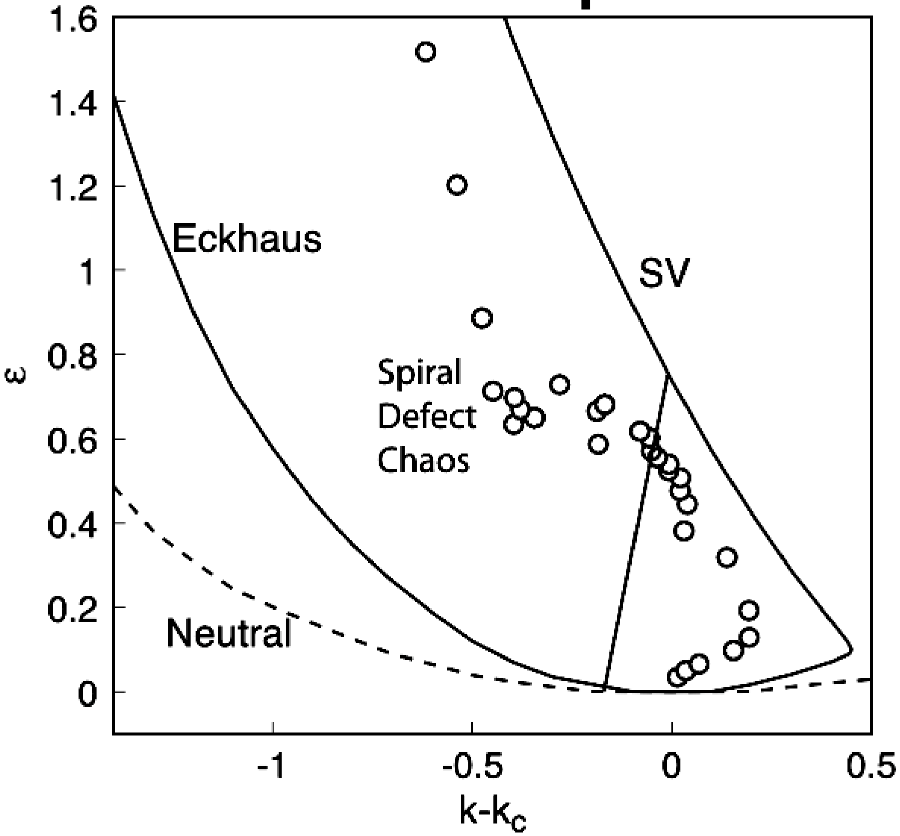


Wave-number selection by curved rolls:

Curved rolls normally induce mean flow. For target patterns mean flow can not occur. Thus the mean flow must be balanced by a pressure gradient. This condition leads to a unique wave number unrelated to any extremum principle.

L. Koschmieder and S. Pallas, Int. J. Heat Mass Trans. **17**, 991 (1974); M. Cross, Phys. Rev. A **27**, 490 (1983); P. Manneville and J.M. Piquemal, Phys. Rev. A **28**, 1774 (1983); M.C. Cross and A.C. Newell, Physica D **10**, 299 (1984); J. Buell and I. Catton, Phys. Fluids **29**, 1 (1986); A.C. Newell, T. Passot, and M. Souli, Phys. Rev. Lett. **64**, 2378 (1990); J. Fluid Mech. **220**, 187 (1990).

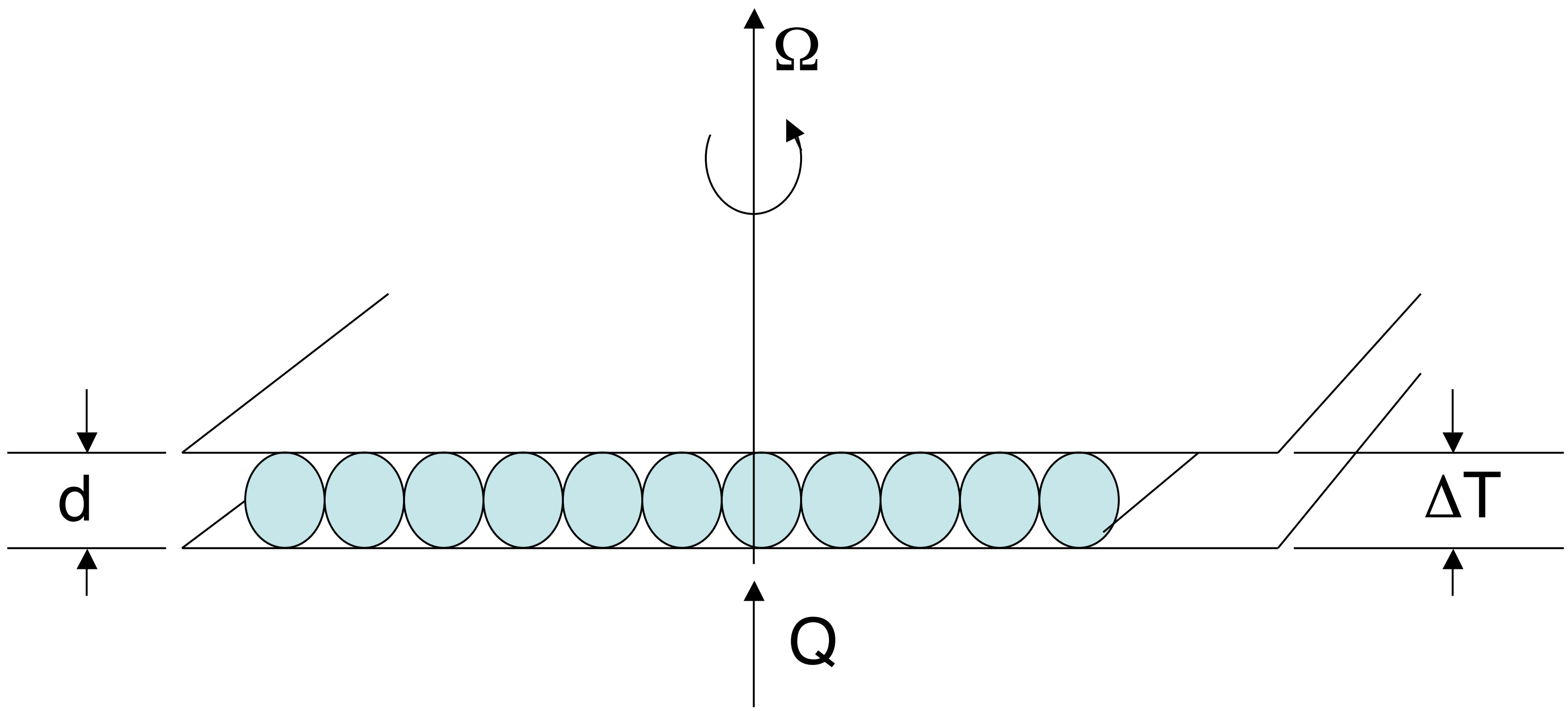
Spatio-temporal chaos
Spiral-Defect-Chaos



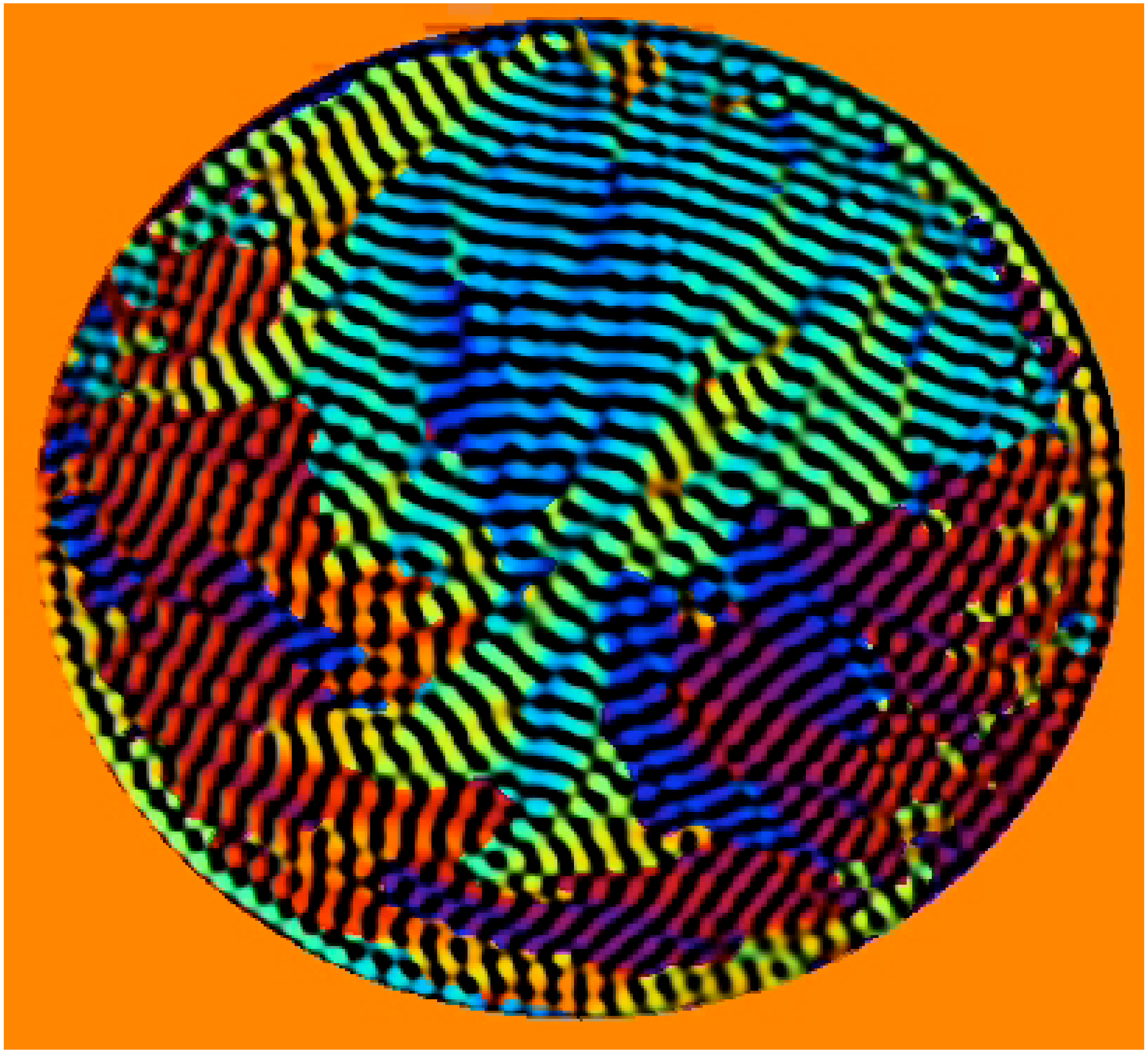
Bi-stability (straight rolls and Spiral-Defect-Chaos)
Dynamics driven by competition between different wavenumber-selection mechanisms and mean flow

[Chiam, M. R. Paul, M. C. Cross, and H. S. Greenside, Phys. Rev. **E67**, 056206 (2003)].
Experiment: S. Morris, E. Bodenschatz, D.S. Cannell, and G.A., Phys. Rev. Lett. 71, 2026 (1993).

Rotation about a vertical axis
Domain Chaos
Kuppers-Lortz instability



G. Kuppers and D. Lortz, J. Fluid Mech. **35**, 609 (1969);
Clever and Busse, J. Fluid Mech. **94**, 609 (1979).



Movies by N. Becker and G.A.

Prandtl = 0.9 CO_2 Omega = 17

Spatio-temporal chaos at onset above a supercritical bifurcation

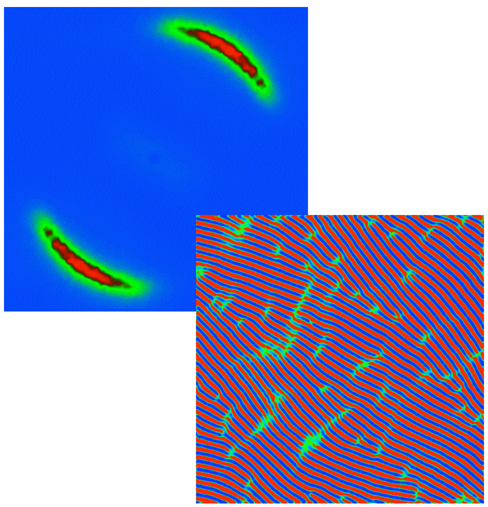
In critical phenomena, the universality class of e.g. a Curie point
is determined in part by the symmetry of the underlying crystal lattice:

Ising, XY, or Heissenberg

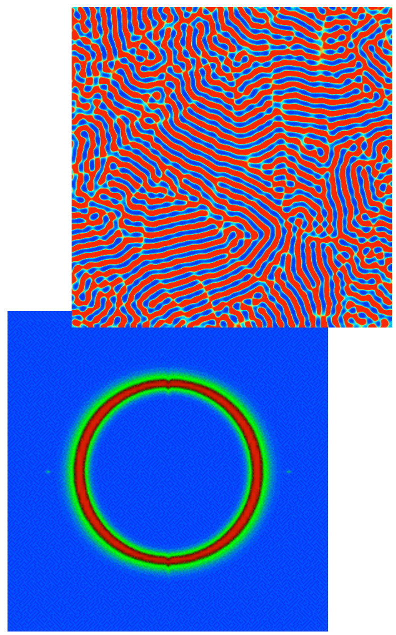
This symmetry is reflected in the structure factor.

Suggestion:

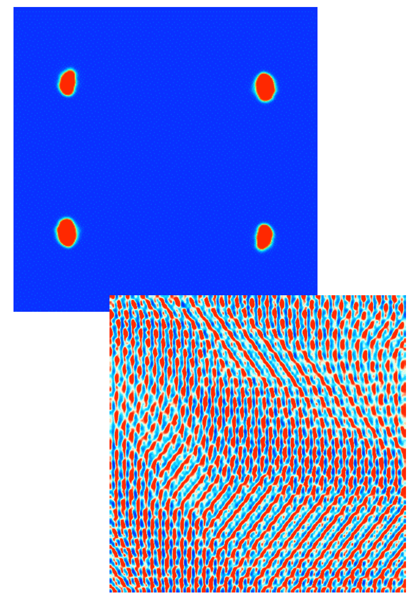
The universality class of a given example of spatio-temporal chaos is reflected in the symmetry properties of the structure factor.



Homeotropic
Electro-conv.

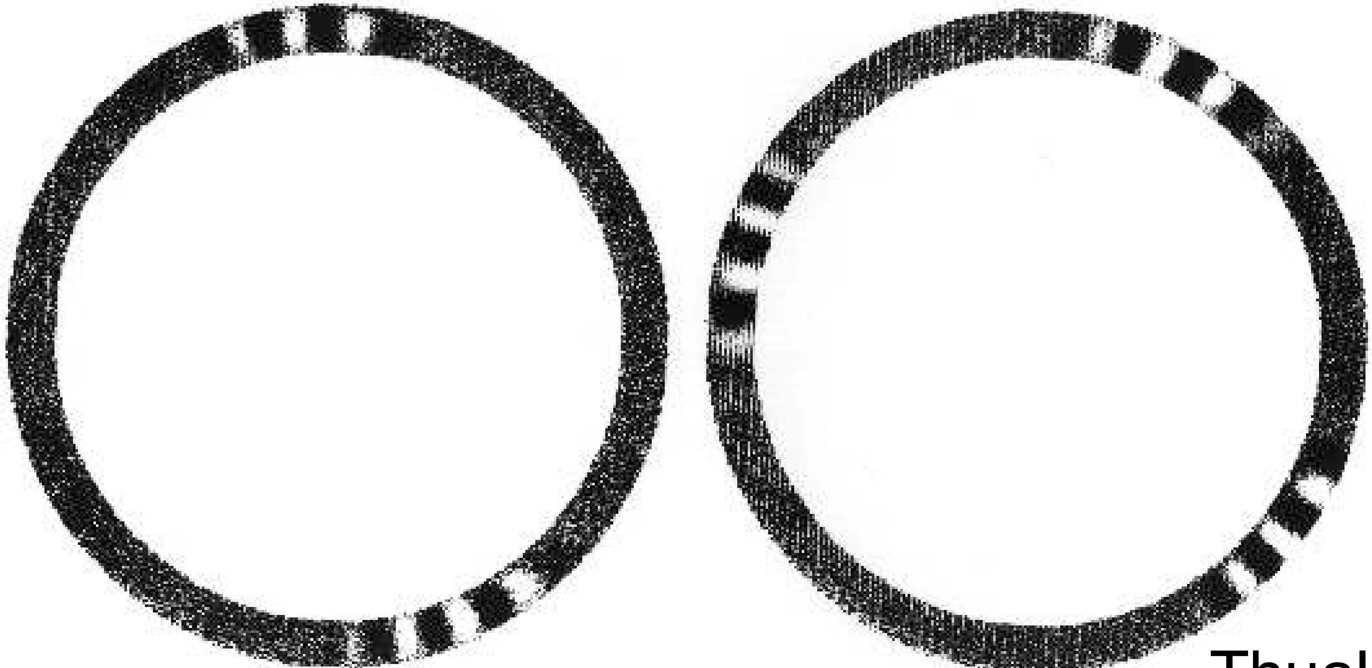


Rotating Rayleigh-
Benard (Domain Chaos)



Planar
Electro-conv.

Localized structures (Pulses, dissipative solitons)

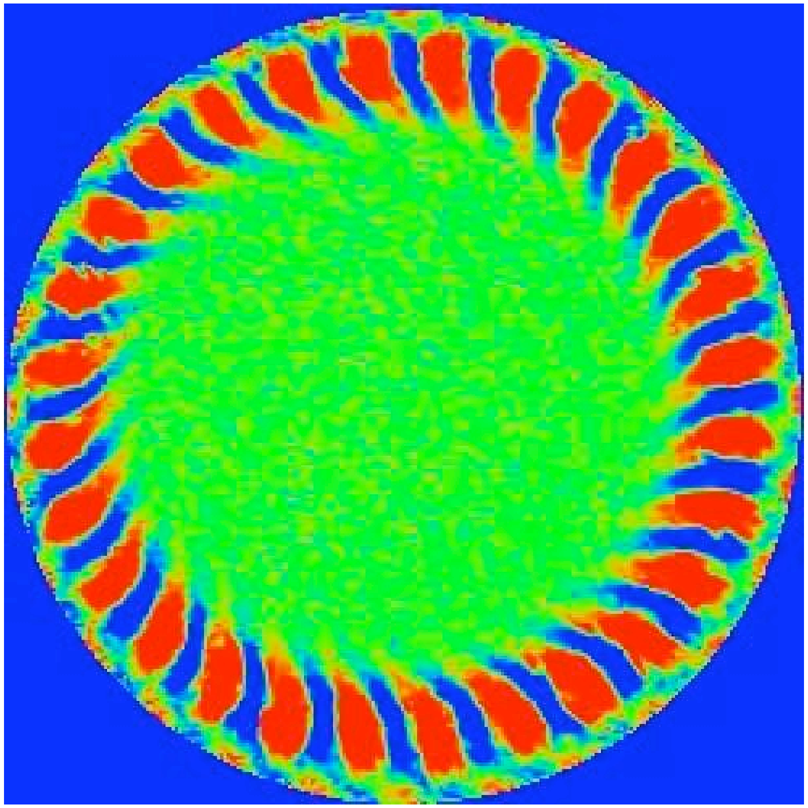


Binary-mixture convection in an annulus

Experiments:
E. Moses, J. Fineberg, and V. Steinberg, Phys. Rev. A **35**, 2757 (1987);
R. Heinrichs, G.A., and D.S. Cannell, Phys. Rev. A **35**, 2761 (1987);
J.J. Niemela, G.A., and D.S. Cannell, Phys. Rev, Lett. **64**, 1365 (1990);
Numerous subsequent papers by Kolodner and others.
Theory:
O. Thual and S. Fauve, Europhys. Lett. **49**, 749 (1988);
and numerous papers thereafter.

Thual + Fauve: Pulses are related to the solitons of the nonlinear Schroedinger equation

Localized structures (Pulses, dissipative solitons)



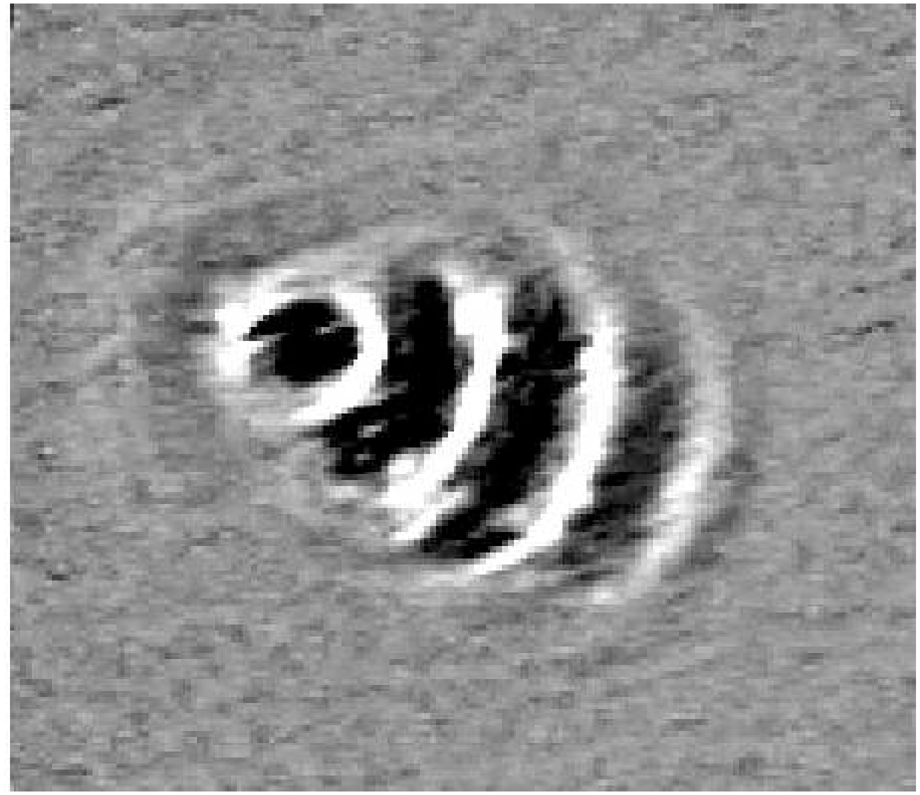
Binary mixture convection

Experiment:

K. Lerman, D.S. Cannell, and G. A.,
Phys. Rev. E **53**, R2041 (1996).

Theory:

I. Mercader, M. Net, and E. Knobloch,
Phys. Rev. E **51**, 339 (1995).



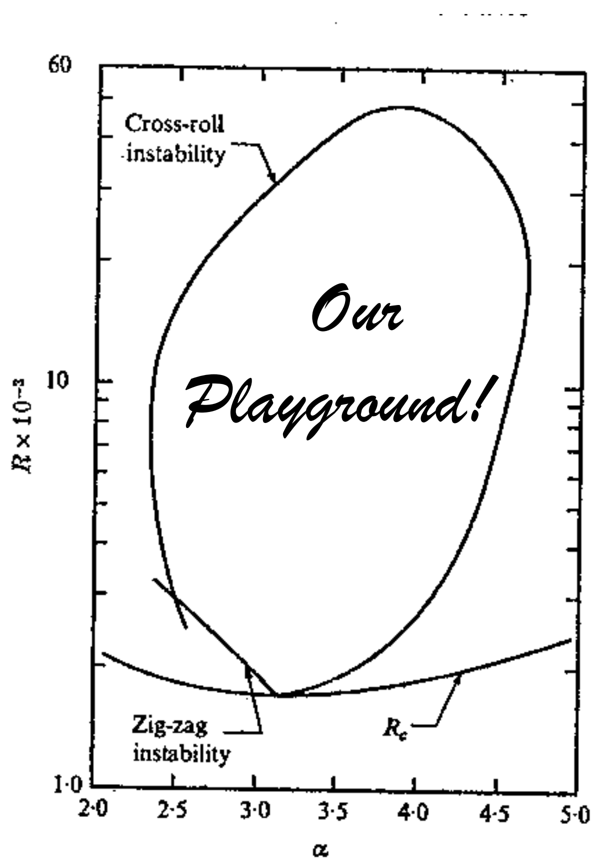
Binary mixture convection

Experiment:

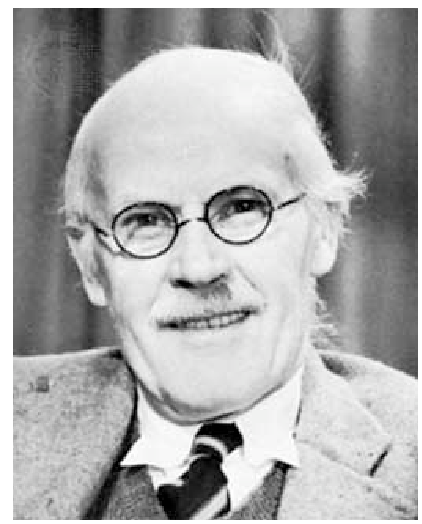
K. Lerman, E. Bodenschatz, D.S. Cannell, and G. A,
Phys. Rev. Lett. 70, 3572 (1993).

Our Heroes:

Henri Benard



Lord Rayleigh

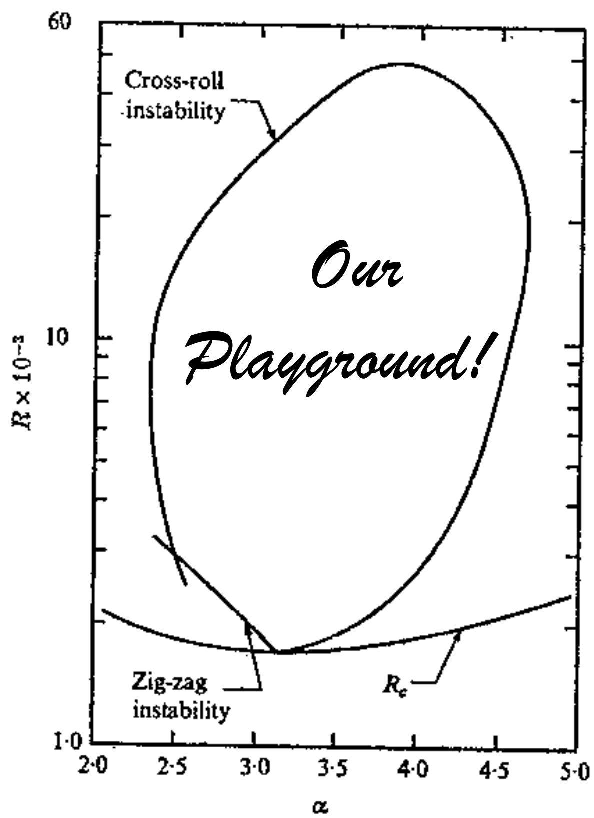


Sir Harold

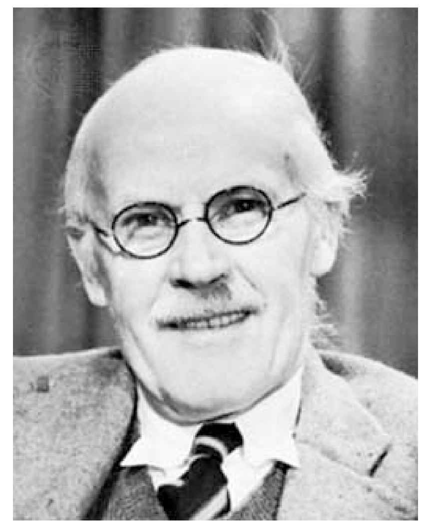
Where do we go next?

Our Heroes:

Henri Benard



Lord Rayleigh

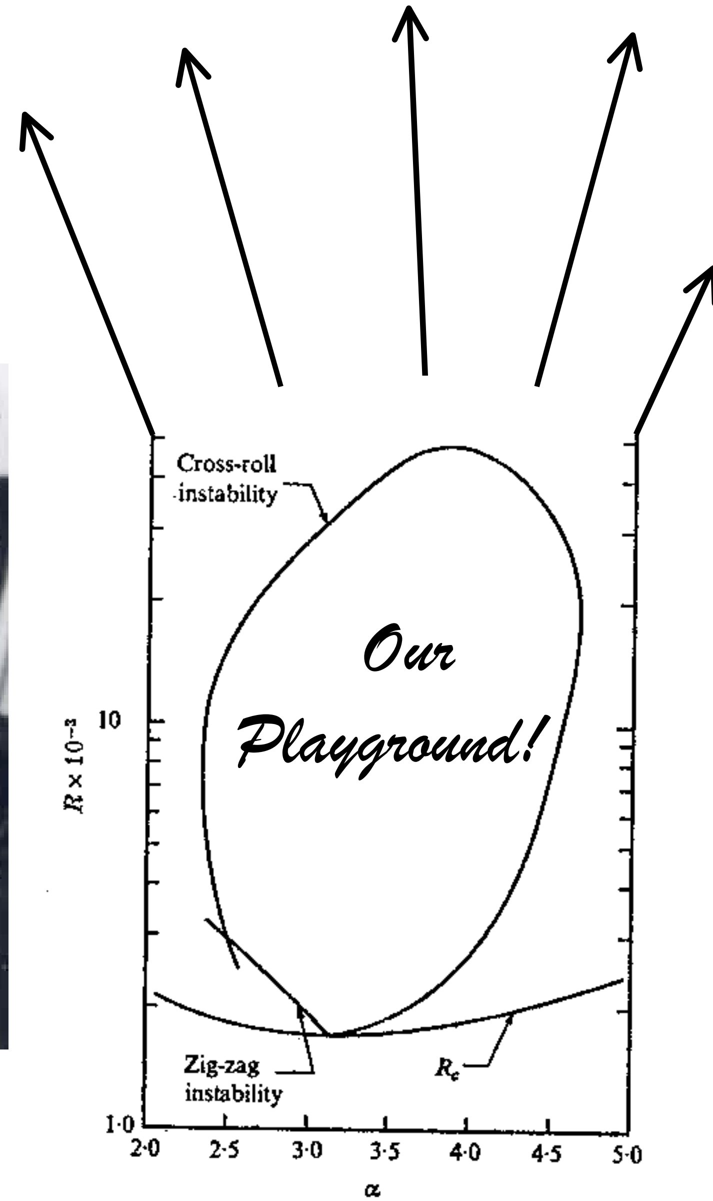


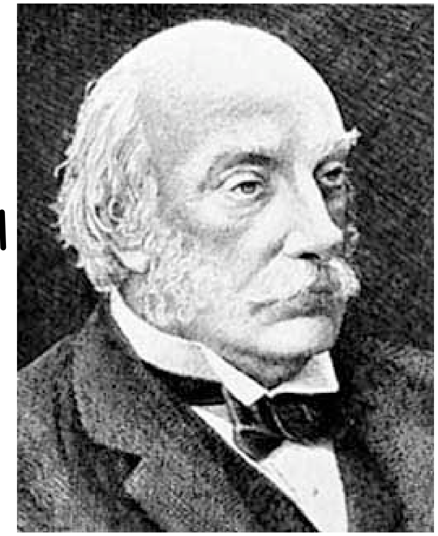
Sir Harold

$R = 10^{12}$! 10^{15}!! 10^{20}!!!

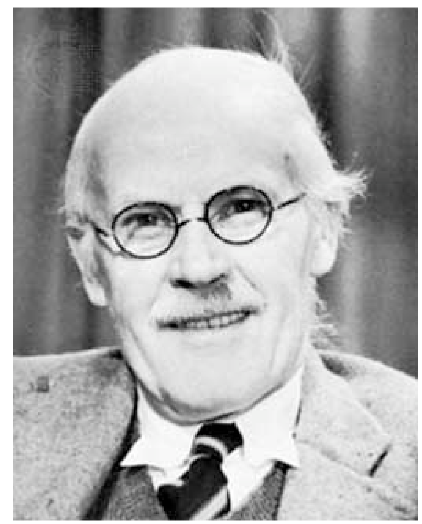
Our Heroes:

Henri Benard



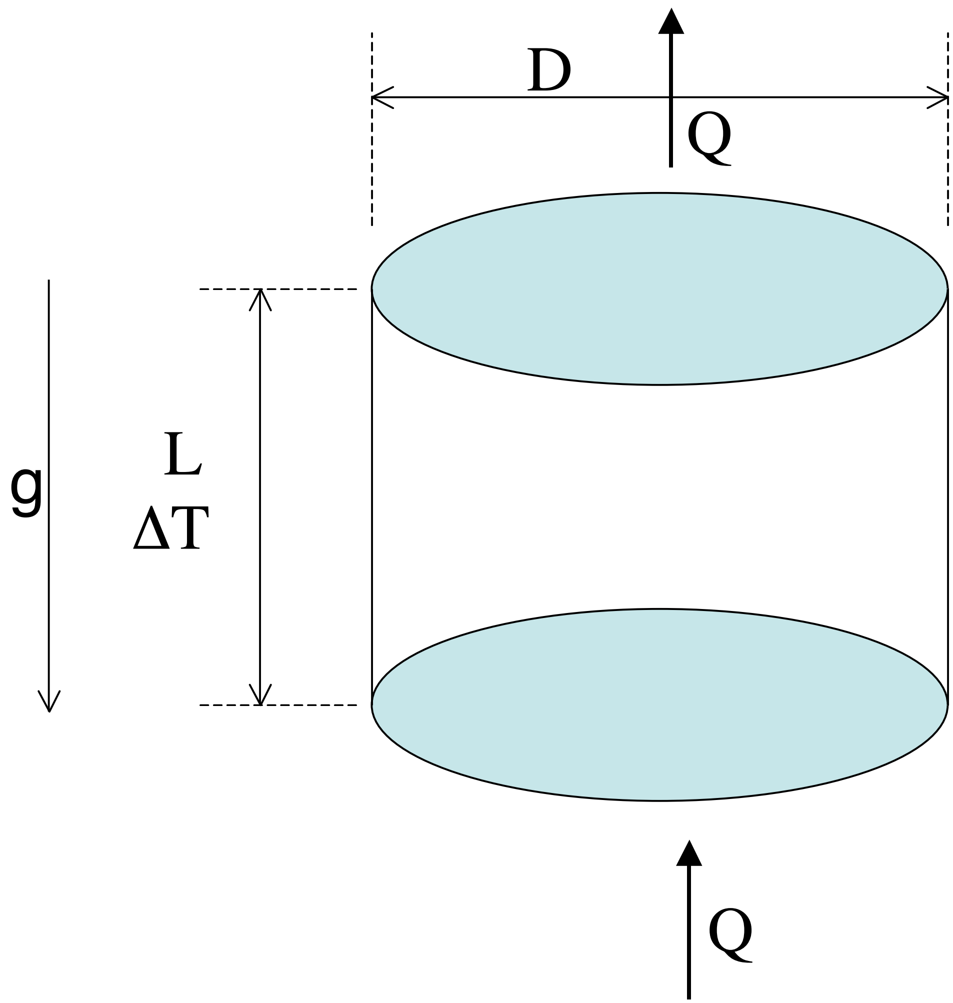


Lord Rayleigh



Sir Harold

Turbulent Rayleigh-Benard Convection



$\Gamma = D/L = 1.0$

$$R = \alpha\, g\, L^3\, \Delta T \,/\, \kappa\, \nu$$

$10^8 < R < 10^{12}$

Fluid: H_2O

E. Brown, A, Nikolaenko, D. Funfschilling, + G.A., Phys. Fluids **17**, 075108 (2005).

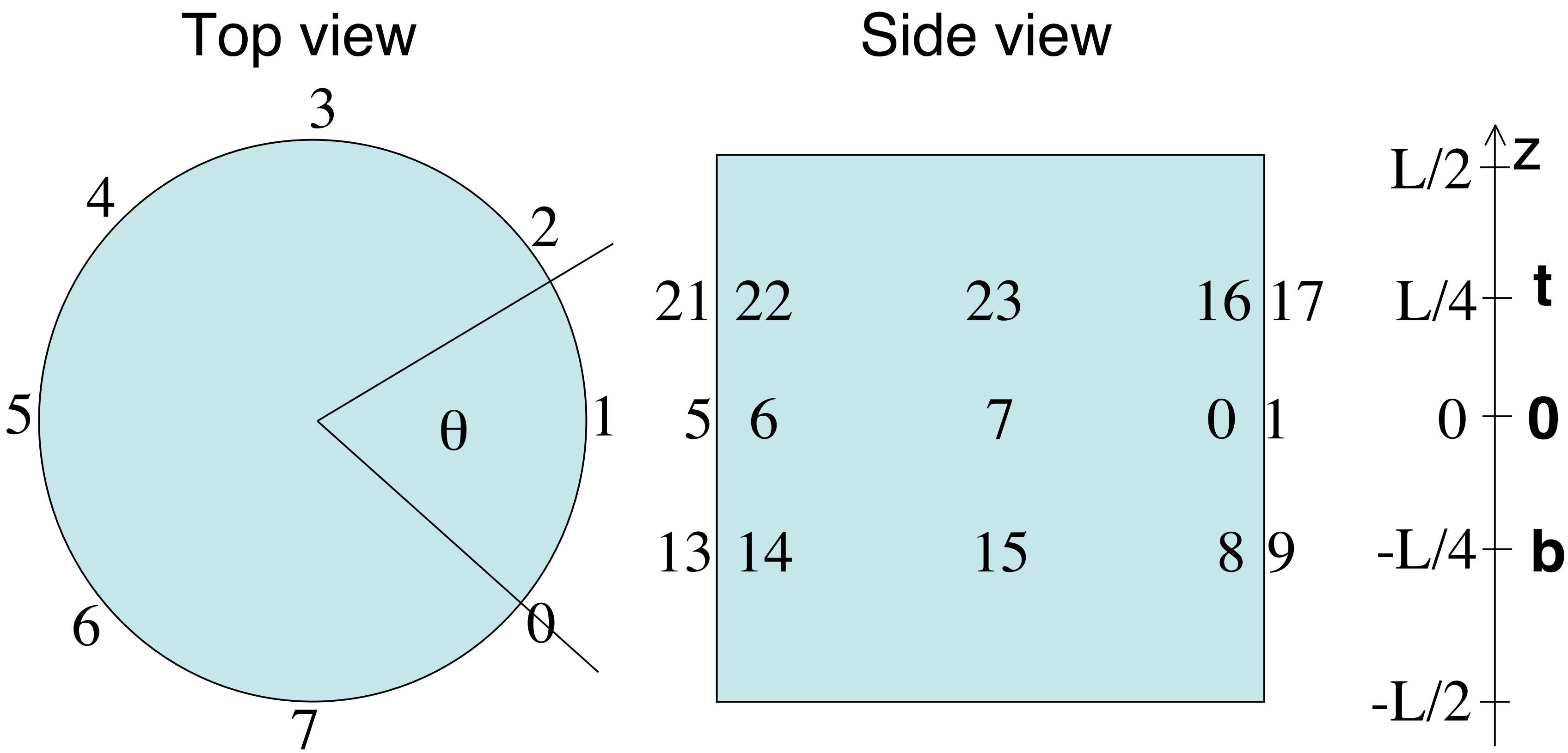
There exists a Large-Scale Circulation (LSC)

Movie from the group of K.-Q. Xia, Chinese Univ., Hong Kong

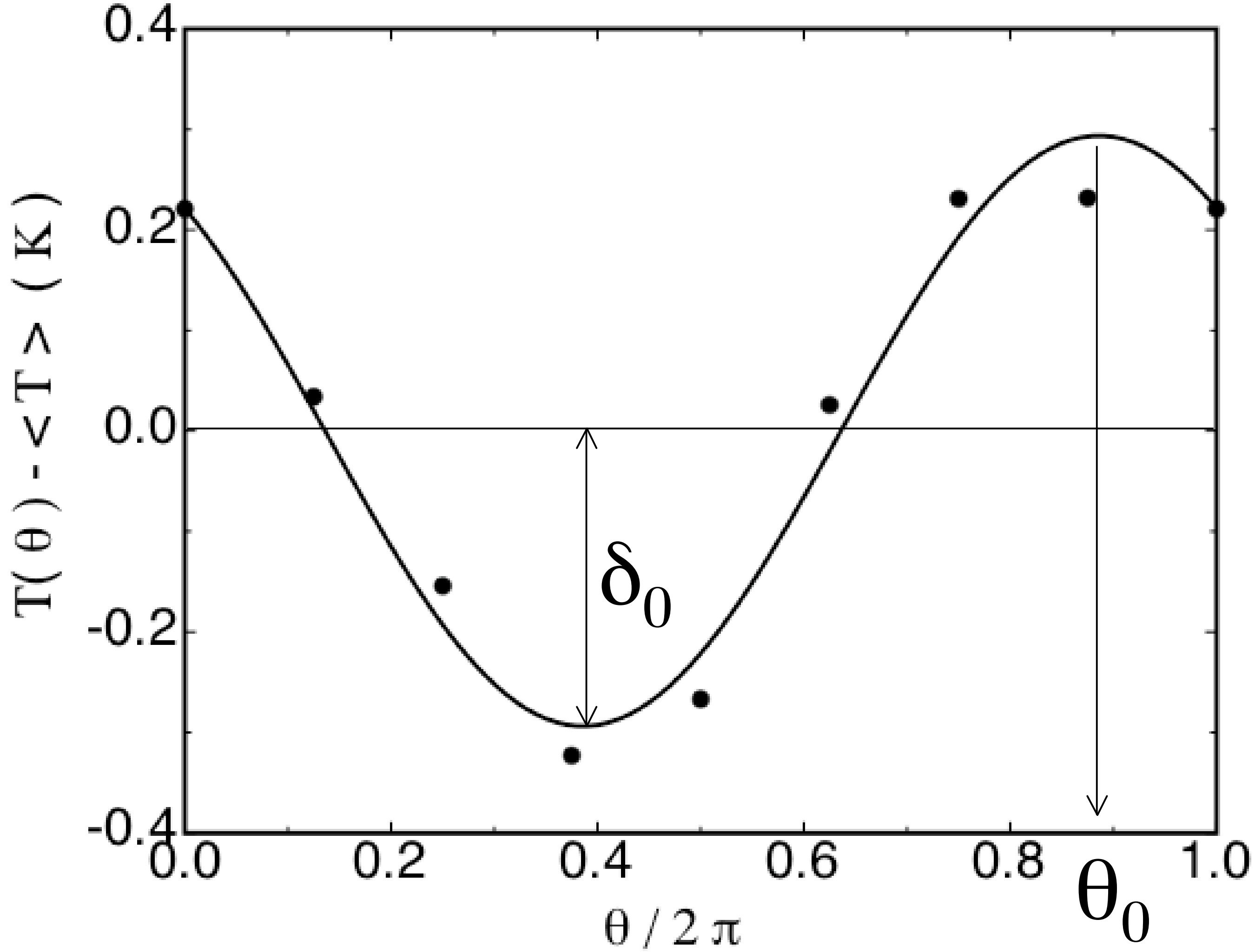
There is a near-vertical circulation plane with azimuthal orientation $\theta_0(t)$. Its dynamics includes

1.) Oscillations
2.) Cessations
3.) Azimuthal diffusion
4.) Interaction with Earth's Coriolis force

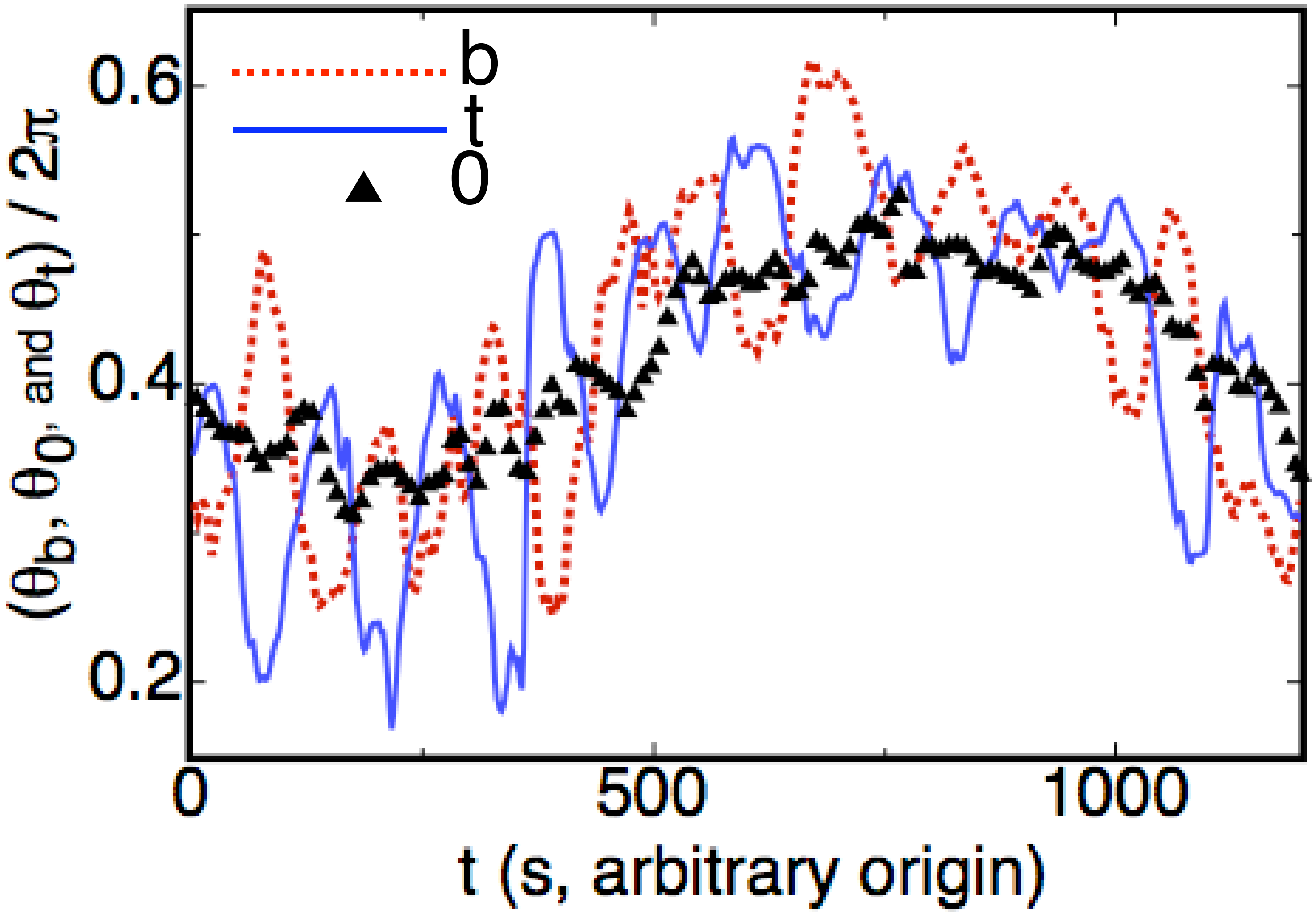
Infer the LSC orientation from the sidewall-temperature profile



$$T_i = \langle T\rangle + \delta_0 \cos(i\pi/4 + \theta_0),\ \ i = \ \ 0, \ldots, 7$$

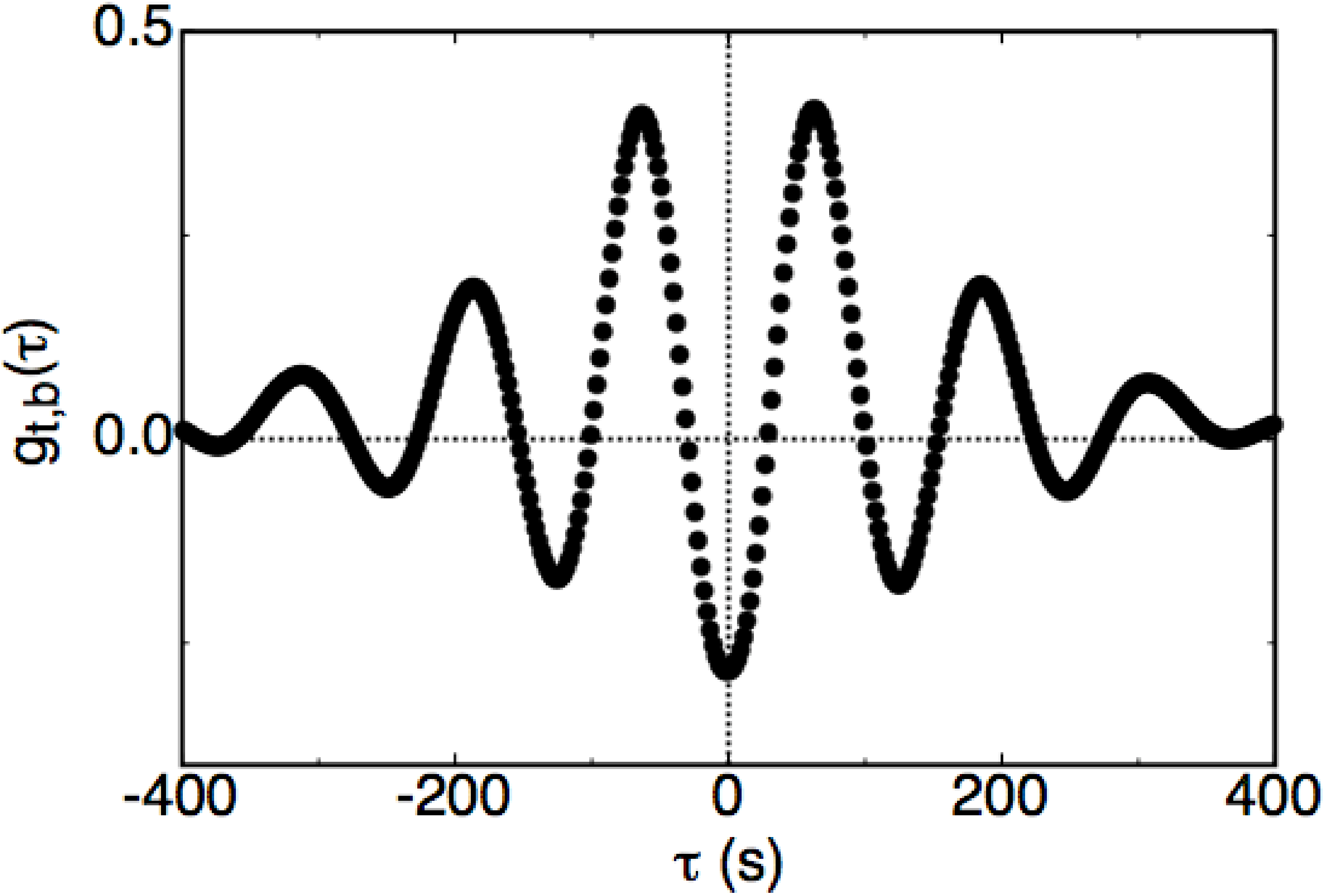


R = 3x10^{10}



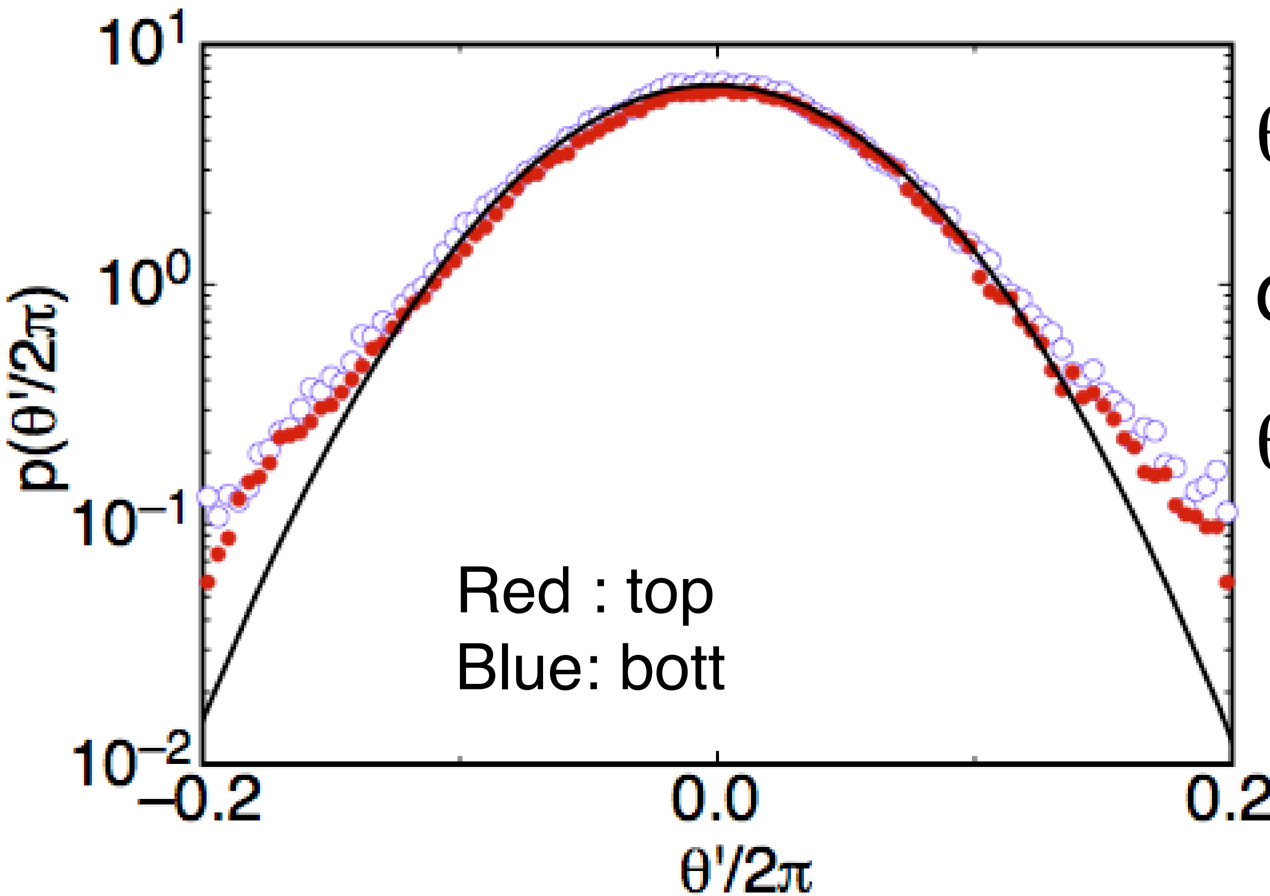
D. Funfschilling, E. Brown, and G.A., to be pub.
D. Funfschilling and G.A., Phys. Rev. Lett. **92**, 194502 (2004).

Cross-correlation between top and bottom



$$g_{t,b}(\tau) = \langle [\theta_t(t) - \theta_0(t)] \times [\theta_b(t+\tau) - \theta_0(t+\tau)] \rangle$$

Anti-correlation implies **torsional oscillation**.



$\theta' = \theta_t - \theta_0$

or

$\theta' = \theta_b - \theta_0$

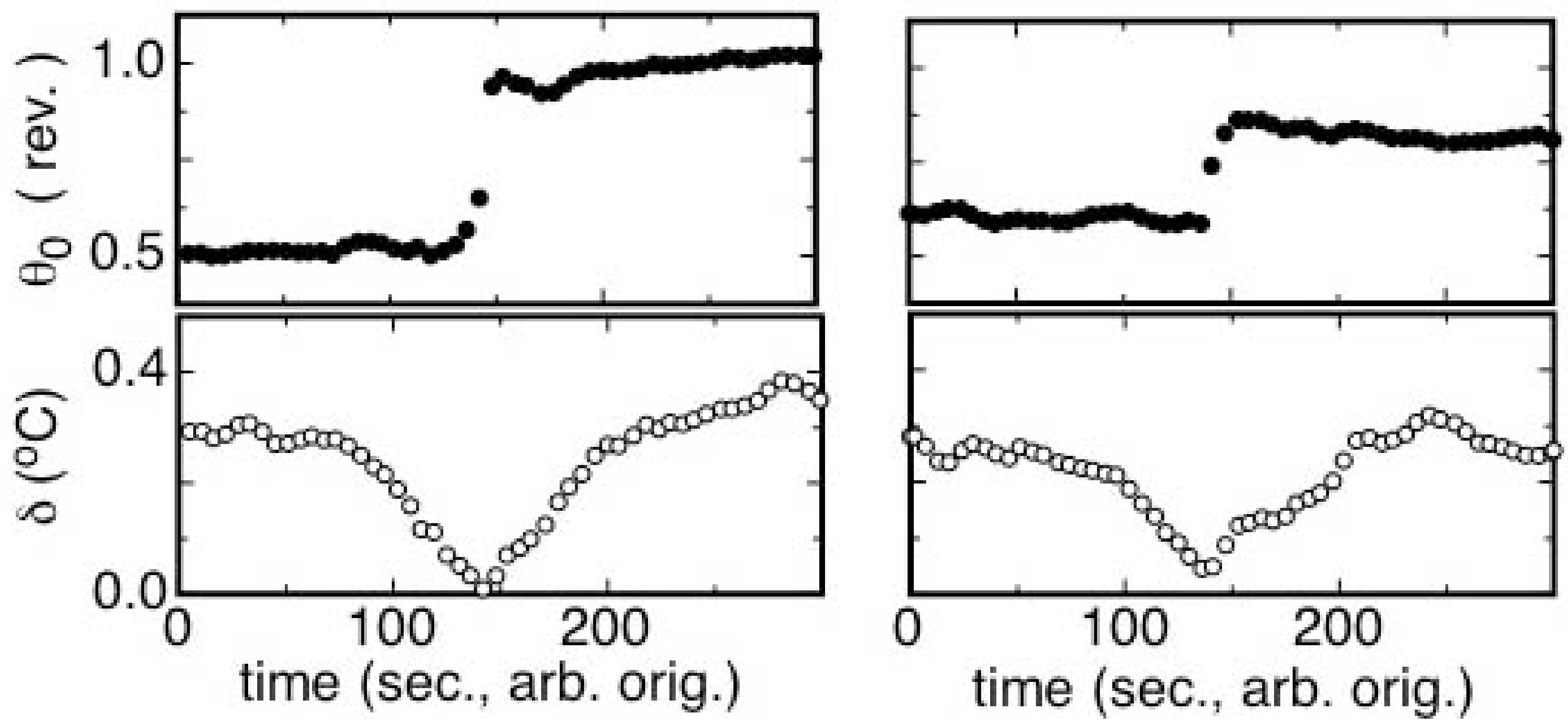
$$\ddot{\theta}' = -b\dot{\theta}' - \omega_0^2\theta' + g(t),$$

$g(t) =$ Gaussian white noise

Stochastically driven damped harmonic oscillator
NOT the result of a Hopf bifurcation.

Occasional reorientations of the LSC

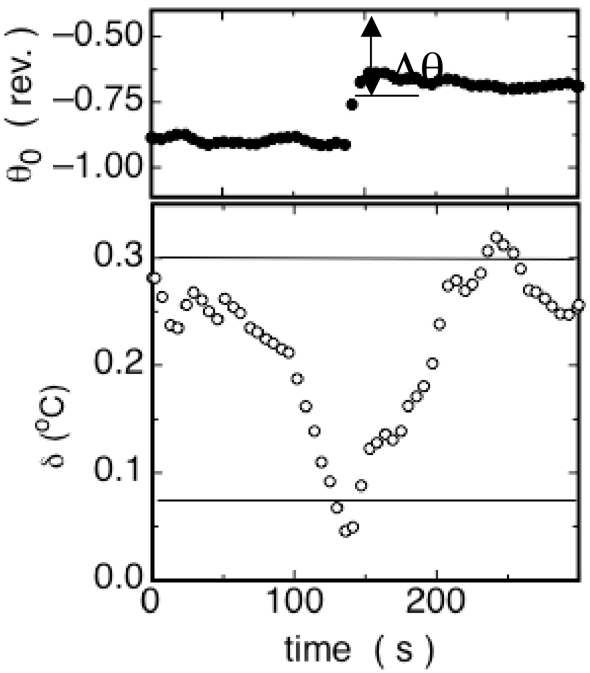
Cessation

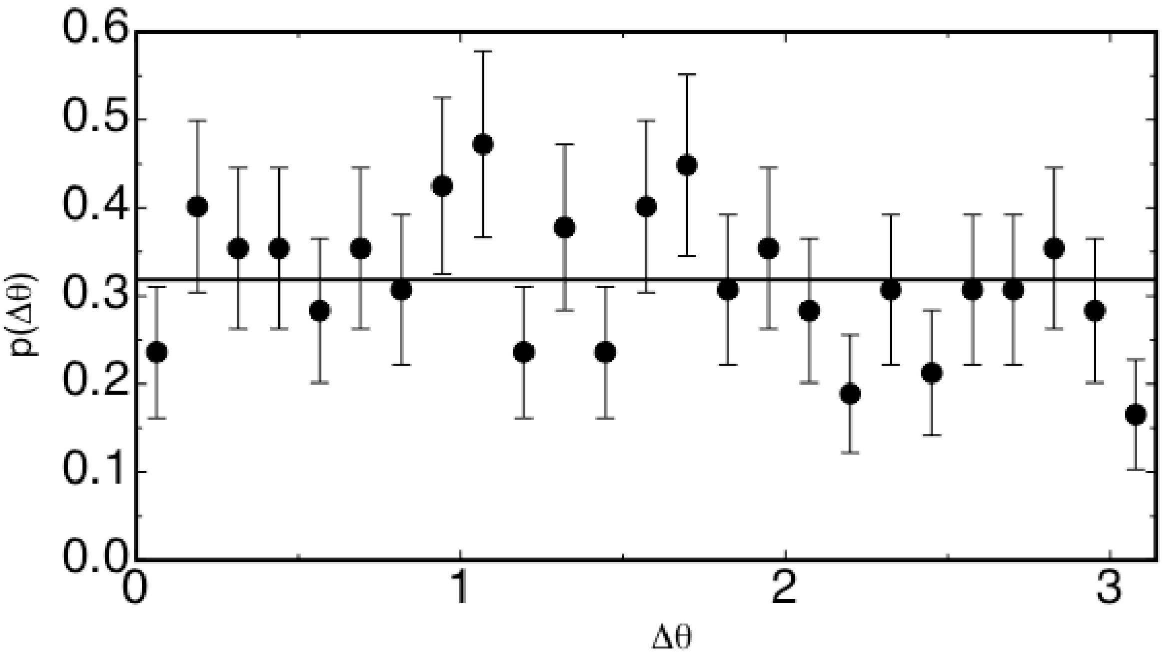


E. Brown and G. A., J. Fluid Mech. **568**, 351-386 (2006).

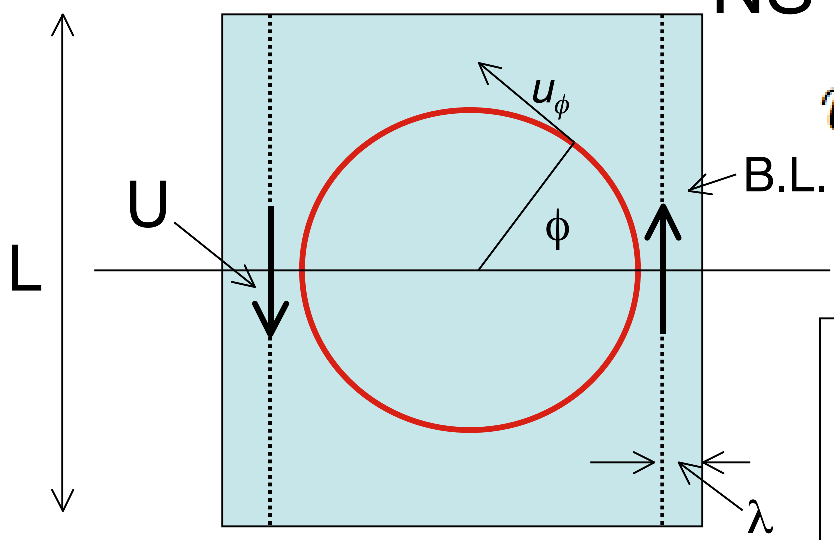
Cessations

probability distribution of $|\Delta\theta|$





NS inspired model:



$$\dot{u}_\phi = g\alpha(T - T_0) + \nu\nabla^2 u_\phi$$

Volume average:

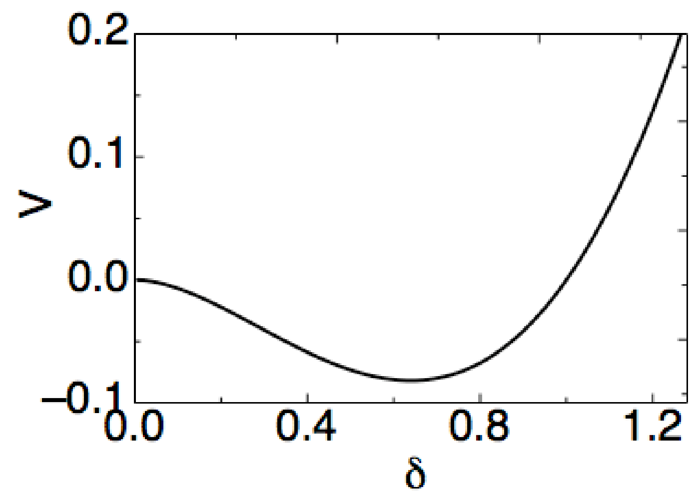
$$\dot{\delta} = \frac{\delta}{\tau_\delta} - \frac{\delta^{3/2}}{\tau_\delta\sqrt{\delta_0}}$$

$\delta_0 = 18\pi\Delta T\sigma R_e^{3/2}/R$; $\tau_\delta = L^2/(18\nu R_e^{1/2})$

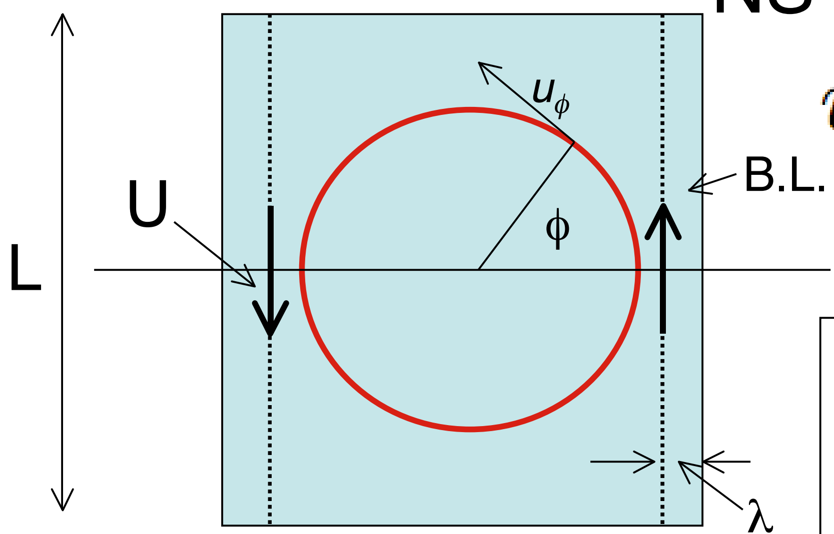
Two fixed points:

$$\delta = 0 \quad unstable$$

$$\delta = \delta_0 \quad stable$$



E. Brown and G.A., Phys. Rev. Lett. **98**, 134501 (2007).



NS inspired model:

$$\dot{u}_\phi = g\alpha(T - T_0) + \nu\nabla^2 u_\phi$$

Volume average:

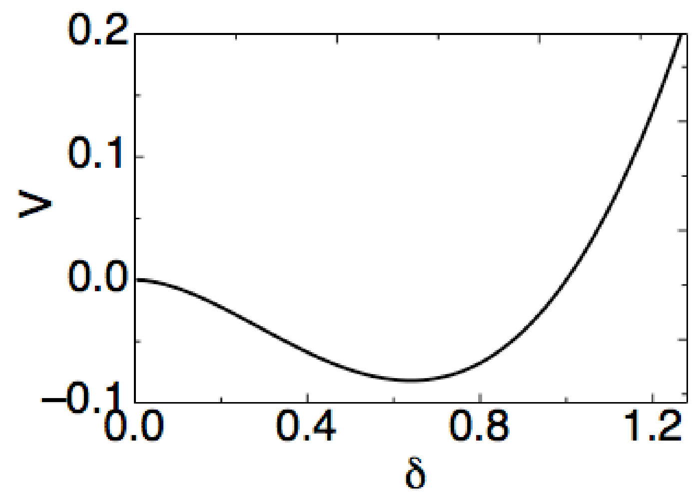
$$\dot{\delta} = \frac{\delta}{\tau_\delta} - \frac{\delta^{3/2}}{\tau_\delta\sqrt{\delta_0}} + f_\delta(t)$$

$$\delta_0 = 18\pi\Delta T\sigma R_e^{3/2}/R\ ;\ \tau_\delta = L^2/(18\nu R_e^{1/2})$$

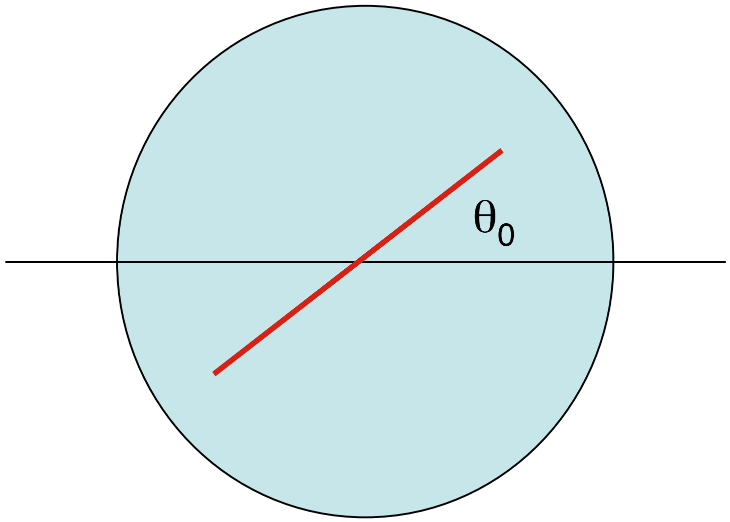
Two fixed points:

$$\delta = 0 \quad unstable$$

$$\delta = \delta_0 \quad stable$$



E. Brown and G.A., Phys. Rev. Lett. **98**, 134501 (2007).



$$\dot{u}_\theta + (\vec{u} \cdot \vec{\nabla}) u_\theta = 0$$

Nonlinear term describes the angular momentum of the LSC and provides coupling to the δ equation. Volume average:

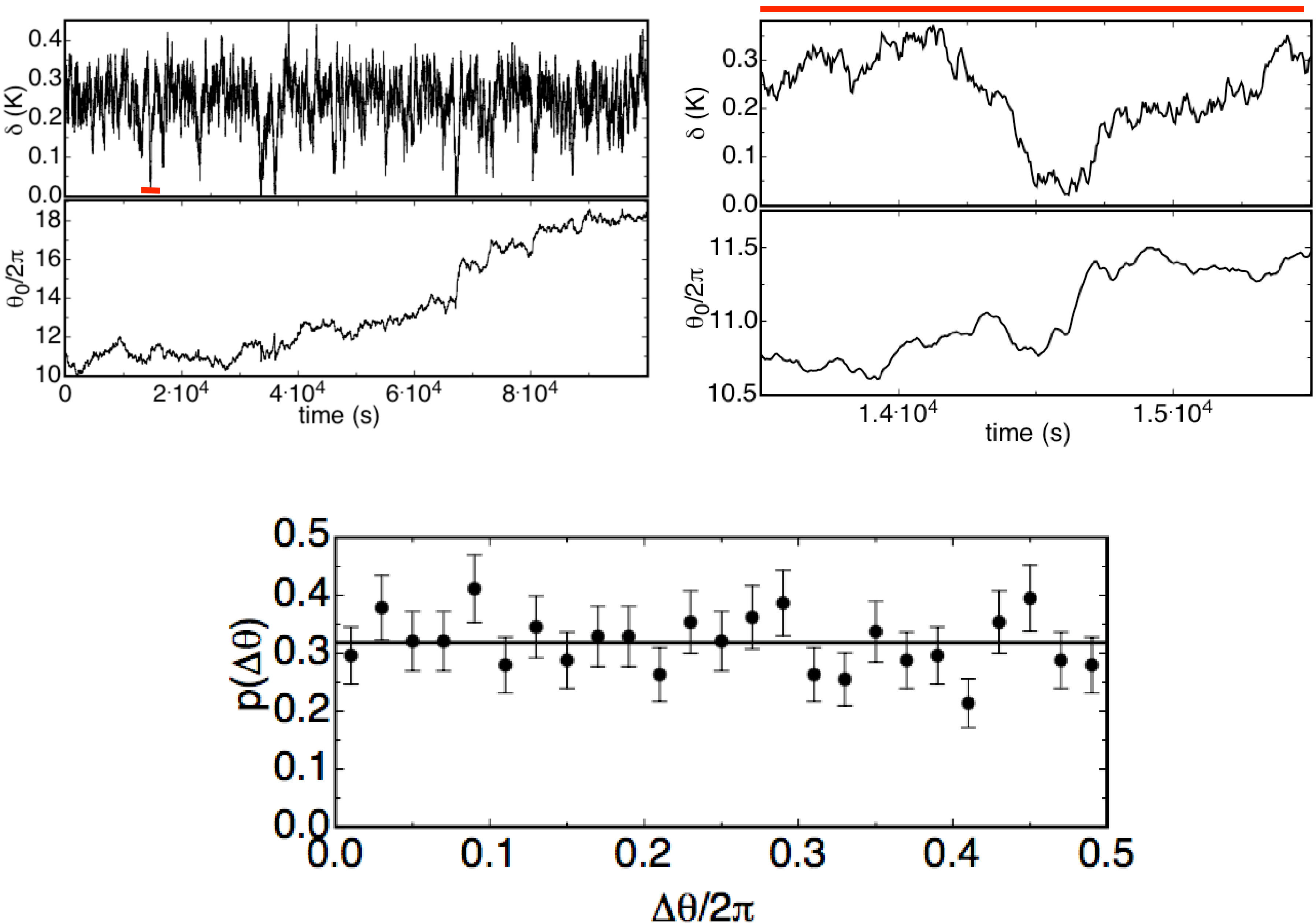
$$\ddot{\theta}_0 = -\frac{\dot{\theta}_0 \delta}{\tau_{\dot{\theta}} \delta_0} + f_{\dot{\theta}}(t)$$

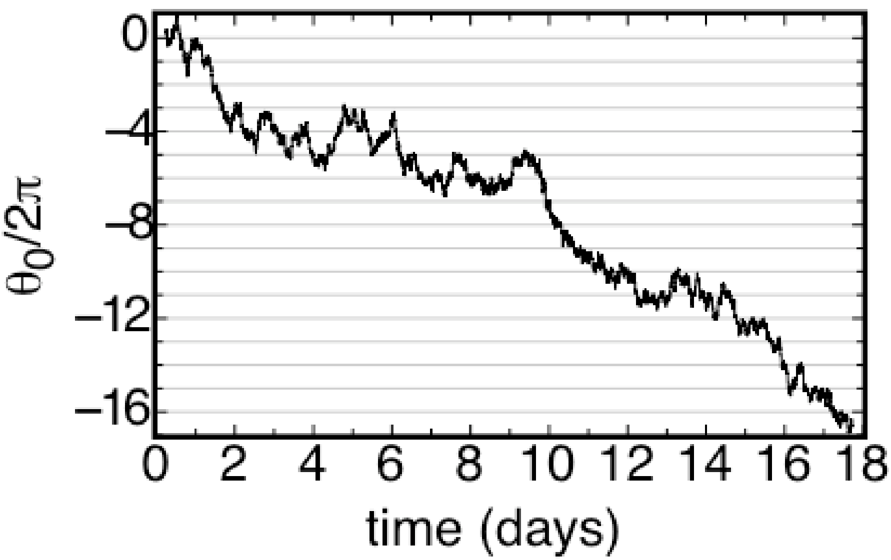
$$\dot{\delta} = \frac{\delta}{\tau_\delta} - \frac{\delta^{3/2}}{\tau_\delta \sqrt{\delta_0}} + f_\delta(t)$$

$$\tau_\delta = \frac{L^2}{18 \nu R_e^{1/2}} \qquad \tau_{\dot{\theta}} = \frac{L^2}{2 \nu R_e}$$

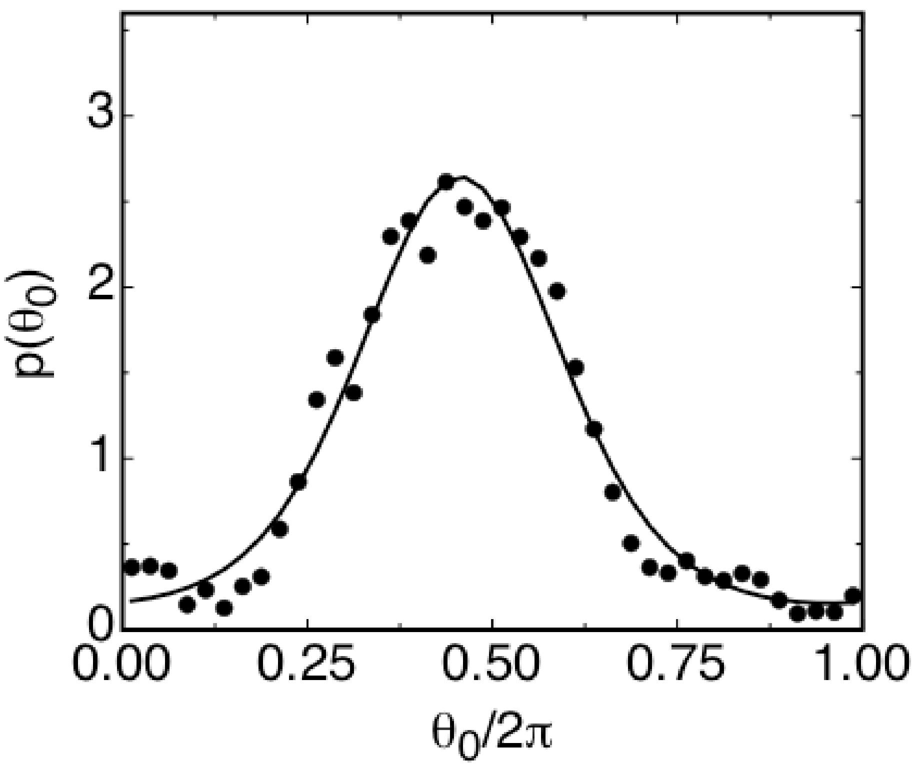
$\delta_0 = \langle \delta \rangle_t$ from experiment

$f_{\dot{\theta}}$ and f_δ from the measured $D_{\dot{\theta}}$ and D_δ


δ (K)
θ0/2π
time (s)
δ (K)
θ0/2π
time (s)
p(Δθ)
Δθ/2π



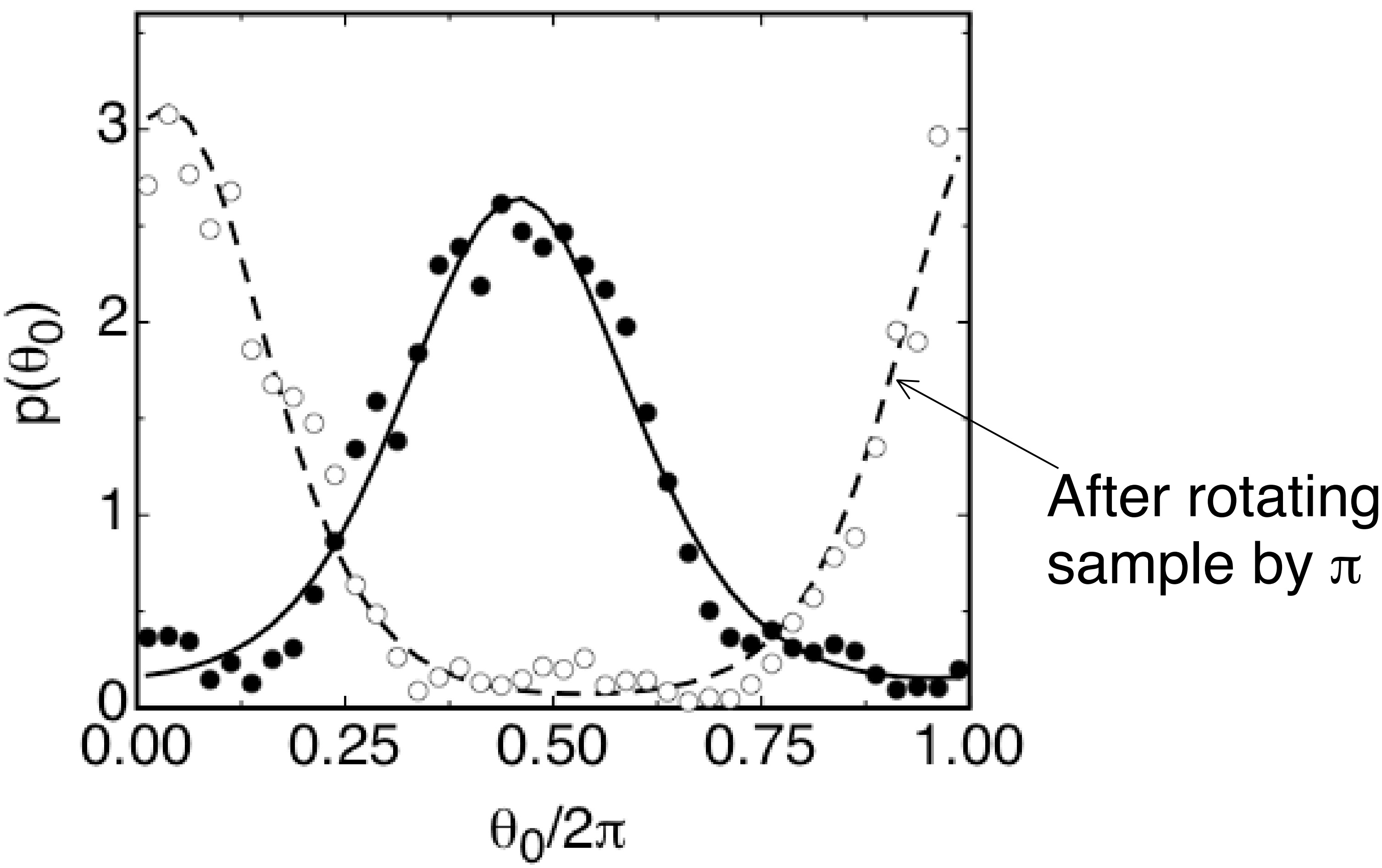
Should be UNIFORM !!!



Effect of Earth's Coriolis force On the LSC

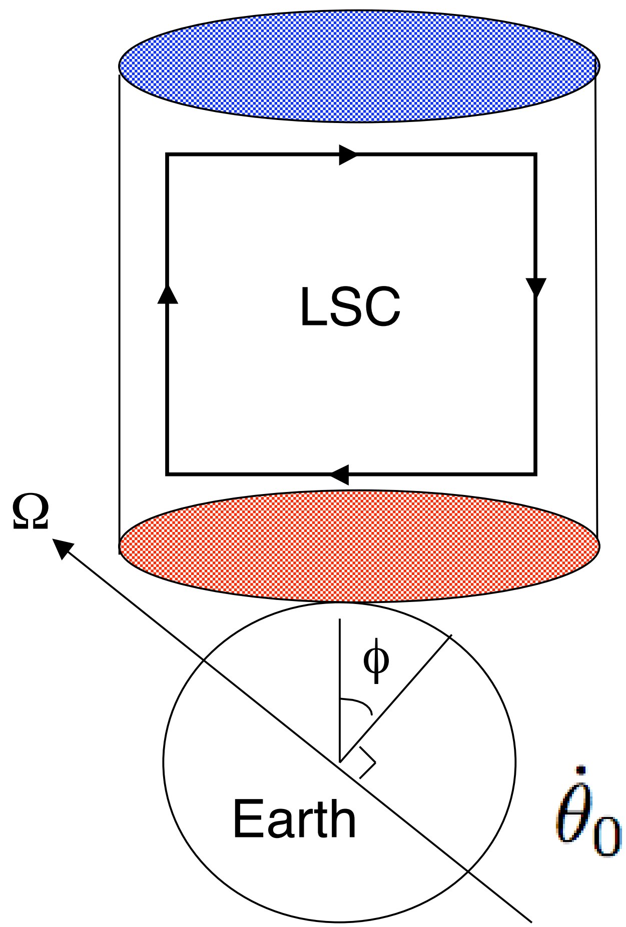
E. Brown and G. A., Phys. Fluids **18**, 125108 (2006).

Preferred Orientation



R = $9x10^9$

Coriolis force model: Langevin equation



ϕ= 34° in Santa Barbara

Coriolis force:

$$\frac{d\vec{u}}{dt} = -2\left(\vec{\Omega} + \frac{d\vec{\theta}_0}{dt}\right) \times \vec{u} - \frac{d^2\vec{\theta}_0}{dt^2} \times \vec{r}\ .$$

Coriolis force on vertical legs causes θ_0 to align towards west

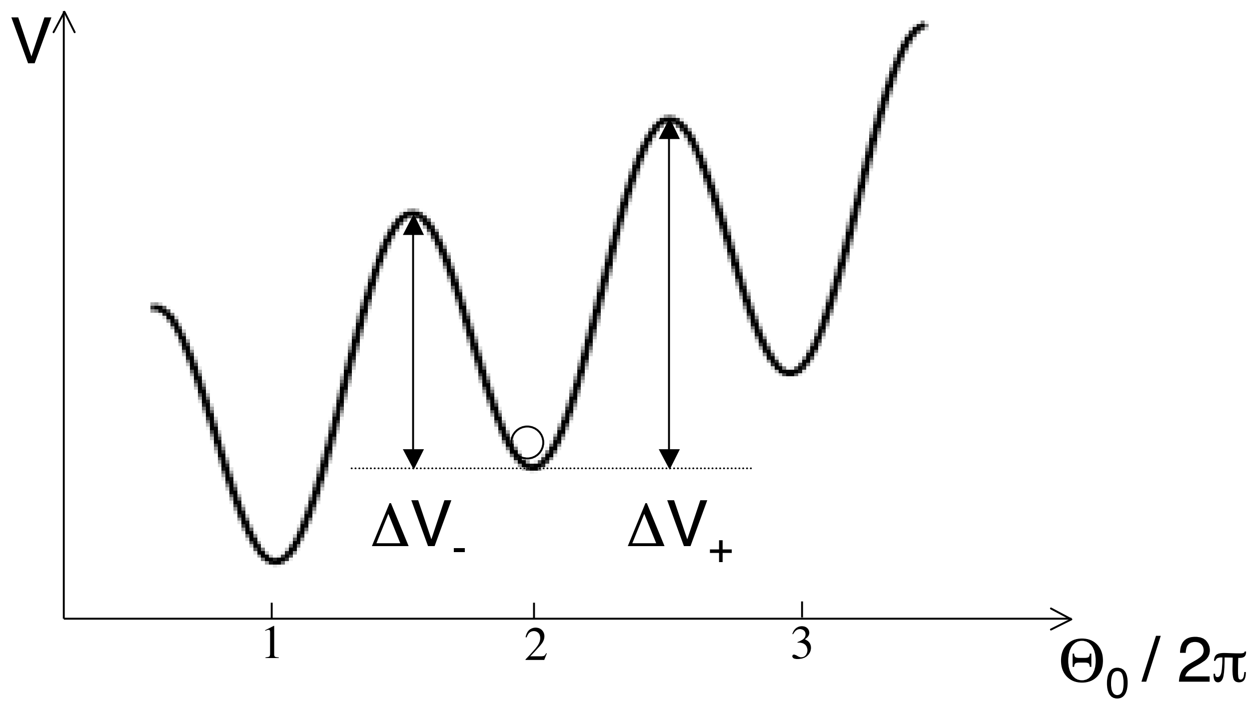
Coriolis force on horizontal legs causes net rotation

$$\dot{\theta}_0 \simeq 2\Omega\cos(\phi)\sin\theta_0 - \Omega\sin\phi + f_\theta(t)$$

diffusive noise term

Diffusion in a ("washboard") potential

$$V = -\int \dot{\theta}_0 d\theta_0 = 2\Omega \cos\phi \cos\theta_0 + \Omega\theta_0 \sin\phi$$



Arrhenius-Kramers problem

See also charge-density waves, Josephson junction, etc.

Using the Coriolis-force potential and the measured diffusivities, we can write the

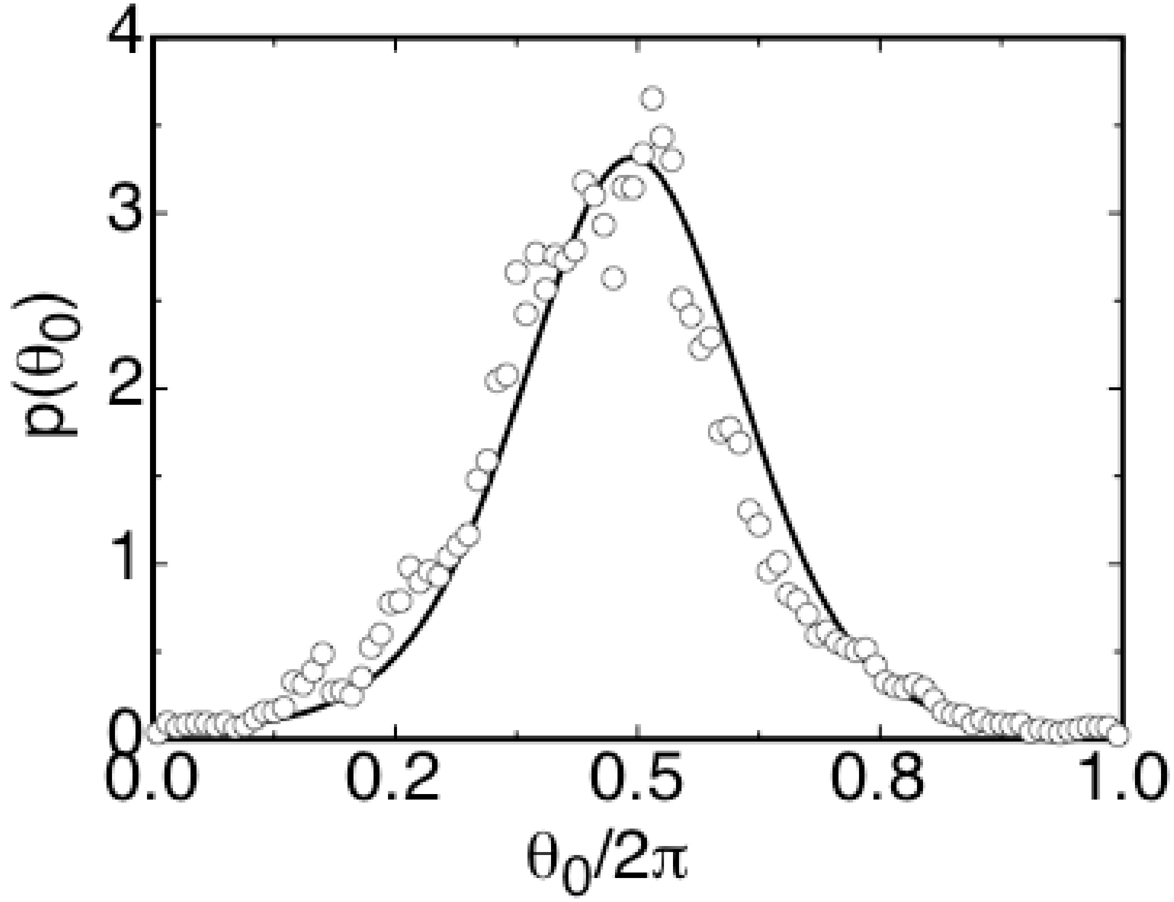
Fokker-Planck equation

$$\frac{dp(\theta_0)}{dt} = \nabla[-p(\theta_0)\dot{\theta}_0(\theta_0) + D_\theta \nabla p(\theta_0)]$$

Assume a steady state and integrate:

Probability distribution of θ_0

$R = 5x10^9$



No adjustable parameters !

In an inertial frame this distribution should be UNIFORM !

A.) There are azimuthal oscillations, out-of-phase at top and bottom, that are due to the stochastic driving by the small-scale turbulent fluctuations.

B.) There are re-orientations by cessation. The probability distribution of the angular change for cessations it is a constant. Cessations are Poisson distributed in time.

C.) The azimuthal meandering of the LSC is diffusive; the diffusion is interpreted to be a consequence of the action of the turbulent fluctuations on the LSC.

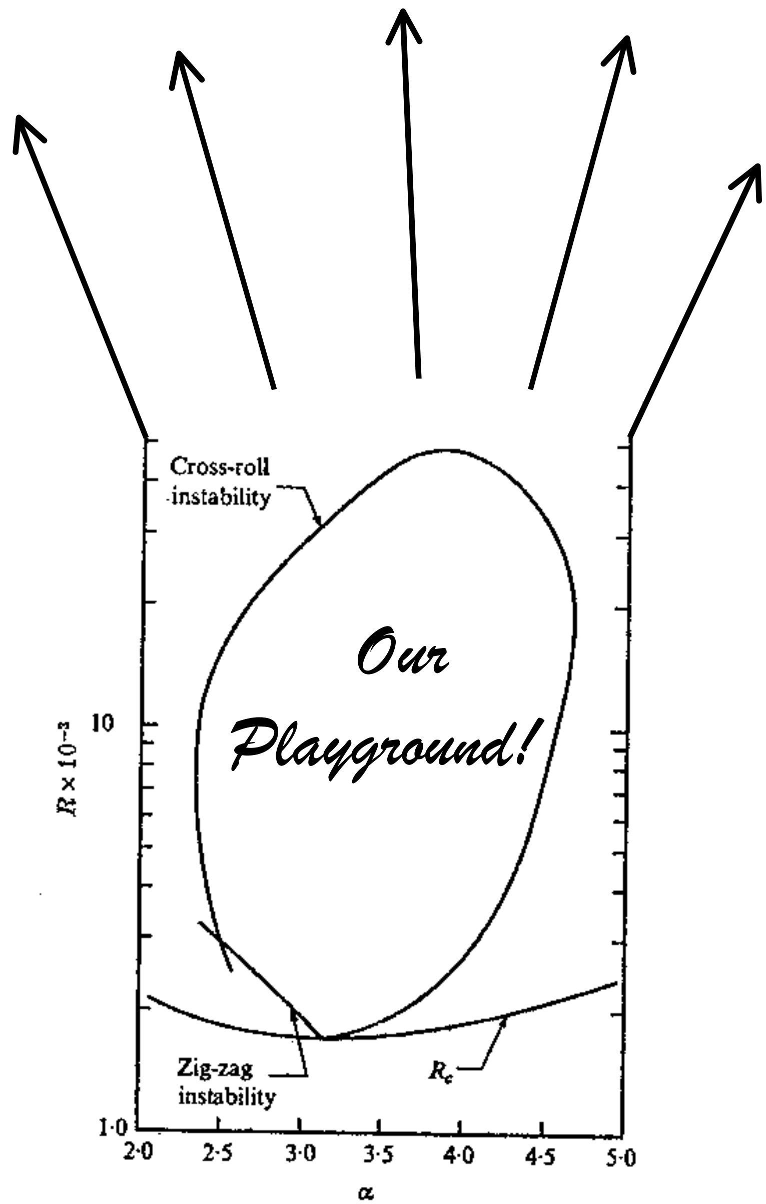
D.) B, and C could be reproduced by a simple NS based model of the LSC.

E.) There is a preferred orientation [a maximum of $p(\theta_0)$] of the LSC, fixed in the laboratory frame in the West, that could be explained quantitatively from a model (“washboard”) potential due to Earth’s Coriolis force, using the measured diffusivity and the Fokker-Planck equation.

And we move on to Higher Rayleigh Numbers !!!

Remember

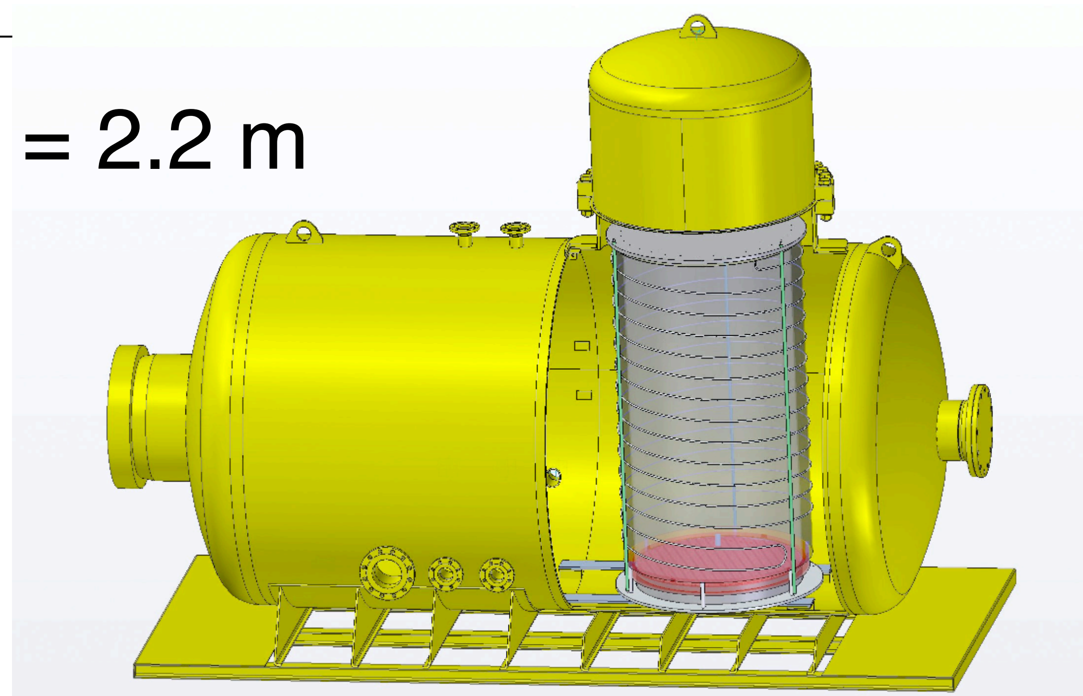
$$R = \alpha\, g\, L^3\, \Delta T / \kappa\, \nu$$



In collaboration with
E. Bodenschatz,
D. Funfschilling, et al.

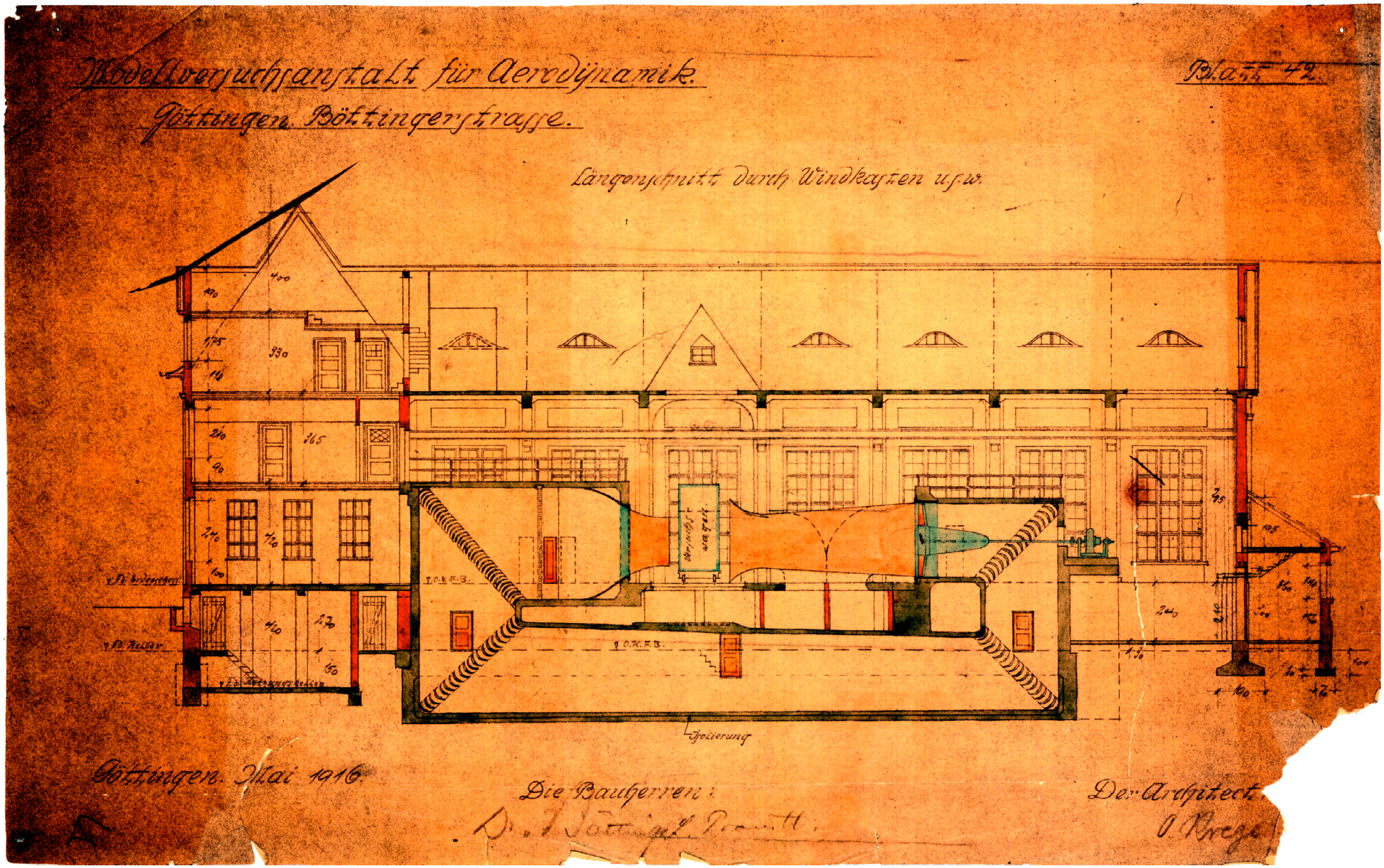
The “U-boat of Goettingen”

L = 2.2 m



SF_6 15 bar $R \sim 10^{15}$

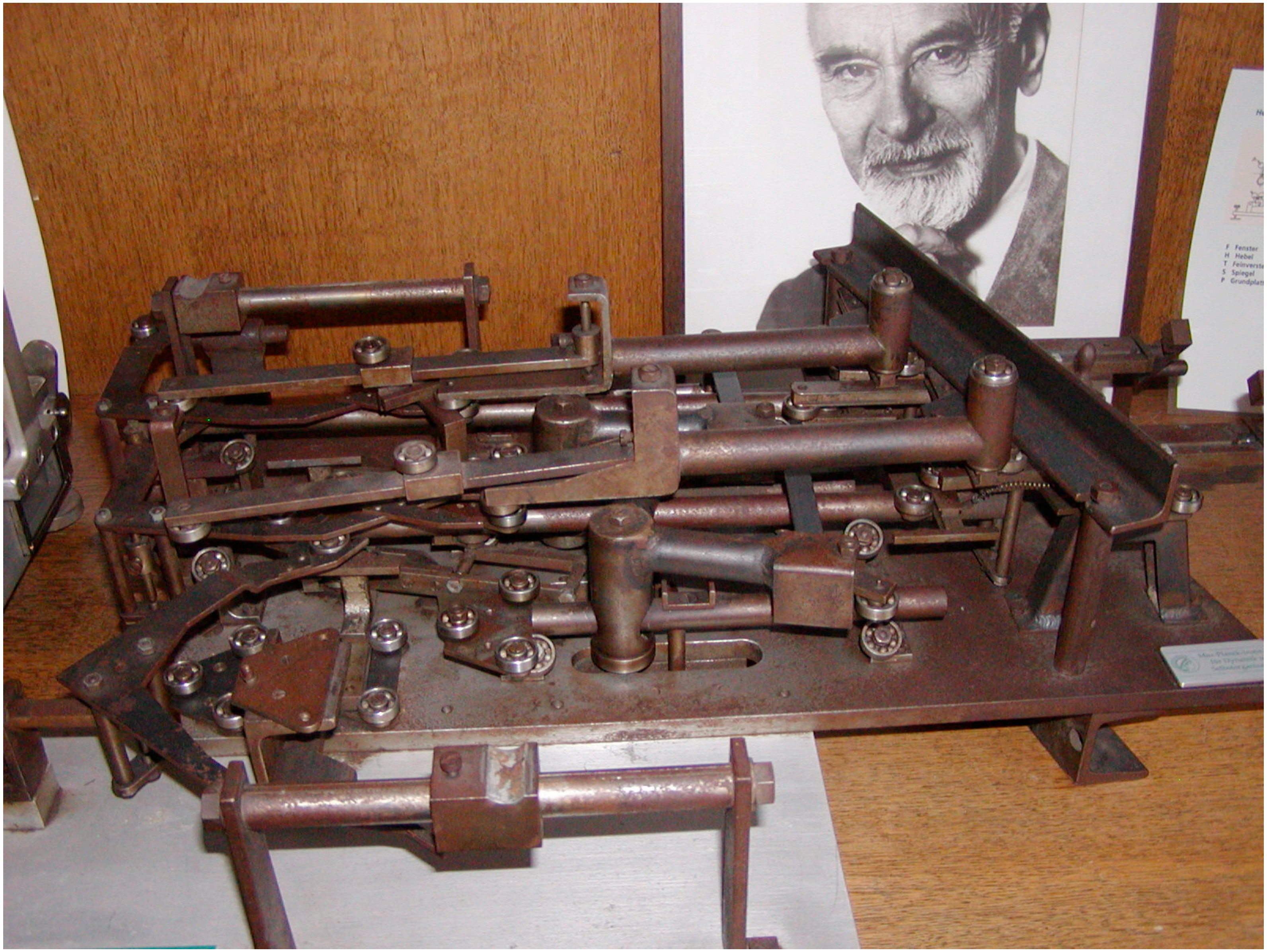
Max-Planck-Institut für Dynamik und Selbstorganisation, Goettingen
Eberhard Bodenschatz, Director



The wonderful fluid mechanics history in Goettingen !

A drawing of Prandtl’s building, including his large wind tunnel.

Prandtl's office and old furniture



An analog computer for solving
a boundary-layer problem

Prandtl’s small wind tunnel

The End