BCS: from Atoms and Nuclei to the Cosmos

Gordon Baym

University of Illinois

BCS theory has had a profound impact on physics well beyond laboratory superconductors and superfluids. This talk will describe the influence of the theory -- spanning more than 20 decades of energy scales -- from nuclear physics, neutron stars, and quark matter, to ultracold trapped atoms.

Fifty Years of BCS Theory

APS March Meeting, Denver

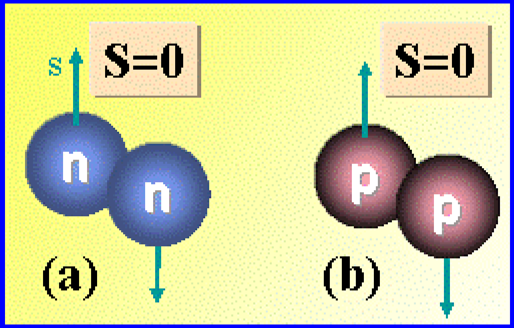
5 March 2007

Elementary-Particle Physics: Revealing the Secrets of Energy and Matter (NRC,1998)

energy, whose properties determine much of the behavior of the system? The so-called Landau-Ginzburg model is a loose description, or a "toy model," of superconductivity. This model was superseded by a full and complete dynamical theory by Bardeen, Cooper, and Schrieffer (BCS). The BCS theory is one of the most remarkable dynamical models in physics. With it, conventional superconductors are understood.

Analogously, EPP, by attempting to understand the origin of quark, lepton, and gauge boson masses, is asking a very similar question: What is the structure of the vacuum, also the state of lowest energy, again whose properties determine why particles have masses and why weak forces become weak? (The vacuum in quantum mechanics is not nothing!) The vacuum pervading the entire universe can be thought of as a kind of superconductor, involving mechanisms that we are just now on the threshold of understanding. The Standard Model assumes that something like the Landau-Ginzburg toy model (slightly modified and redubbed the Higgs mechanism) is applicable. This gives a description of the mass generation of all quarks, leptons, and gauge bosons, and the rest of the machinery of the Standard Model performs beautifully in all experimental tests to date. Yet, the Higgs mechanism is really just a "black box" concealing a deeper mechanism that we do not yet understand, just as the Landau-Ginzburg model was a black box containing the BCS theory. Thus, EPP—with the Standard Model—finds itself today in a kind of "pre-BCS" era. The exciting aspect of all this is that we are on the threshold of understanding what is really happening by deeper examination of the physics currently accessible to Fermilab's Tevatron, and the LEP at the European Laboratory for Particle Physics (CERN) and eventually the LHC in the next decade.

Neutron and proton pairing in nuclei



Pairing of even numbers of neutrons or protons outside closed shells

*David Pines to Niels Bohr's Institute in Copenhagen, Summer 1957, just as BCS was being finished in Urbana.

*Aage Bohr, Ben Mottelson and Pines (57) suggest BCS pairing in nuclei to explain energy gap in single particle spectrum

– odd-even mass differences

*Rehovat Conference, Sept. 1957

*Pairing gaps deduced from odd-even mass differences:

$\Delta \simeq 12\ A^{-1/2}$ MeV for both protons and neutrons

B. Mottelson, M. Goeppert-Mayer, H. Jensen, Aa. Bohr

Conference on Nuclear Structure, Weizmann Institute, Sept. 8-14, 1957

PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. Bohr, B. R. Mottelson, and D. Pines*
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

THE nuclear structure exhibits many similarities with the electron structure of metals. In both cases, we are dealing with systems of fermions which may be characterized in first approximation in terms of independent particle motion. For instance, the statistical level density, at not too low excitation energies, is expected to resemble that of a Fermi gas. Still, in both systems, important correlations in the particle motion arise from the action of the forces between the particles and, in the metallic case, from the interaction with the lattice vibrations. These correlations decisively influence various specific properties of the system. We here wish to suggest a possible analogy between the correlation effects responsible for the energy gaps found in the excitation spectra of certain types of nuclei and those responsible for the observed energy gaps in superconducting metals.

proximately[1]

$$\delta \approx 50 A^{-1}\ \text{Mev}, \tag{1}$$

where A is the number of particles in the nucleus.

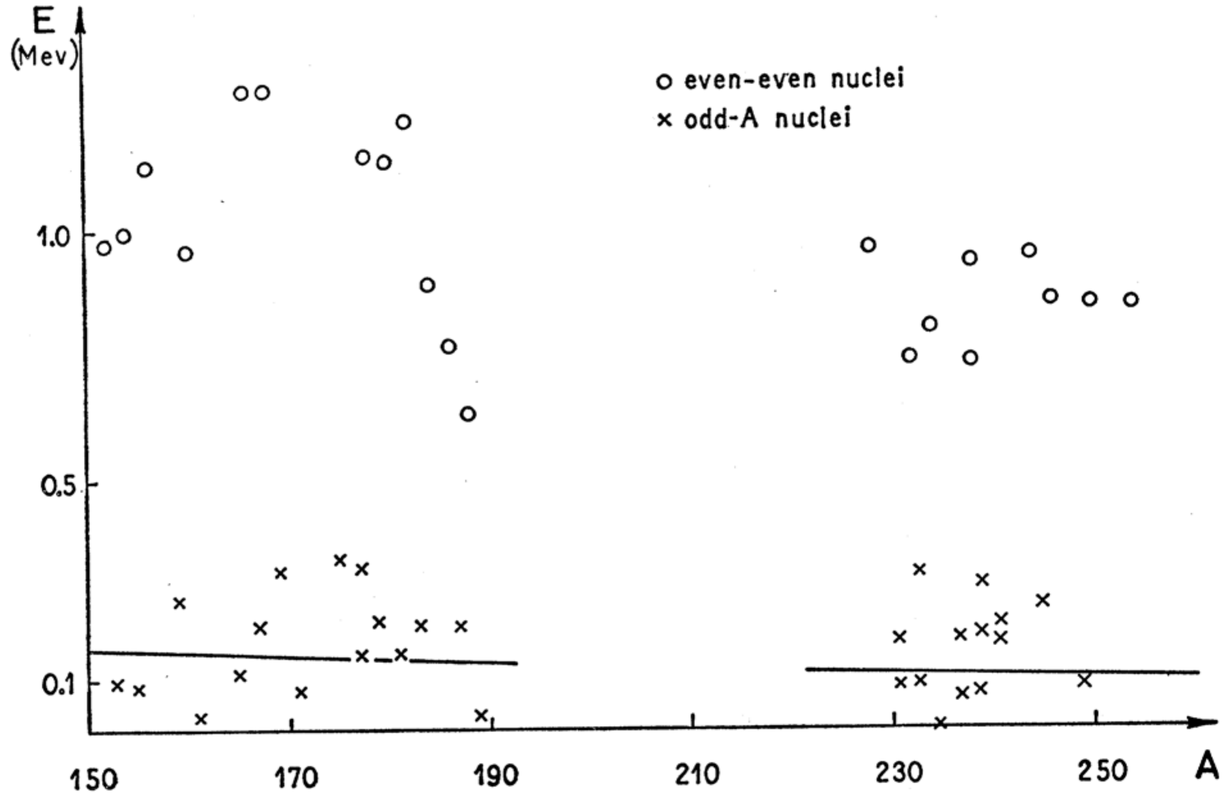
If the intrinsic structure could be adequately described in terms of independent particle motion, we would expect, for even-even nuclei, the first intrinsic excitation to have on the average an energy $\frac{1}{2}\delta$, when we take into account the possibility of exciting neutrons as well as protons. Empirically, however, the first intrinsic excitation in heavy nuclei of the even-even type is usually observed at an energy of about 1 Mev (see Fig. 1). The only known examples of intrinsic excitations with appreciably smaller energy are the $K=0-$ bands which occur in special regions of nuclei, and which may possibly represent collective octupole vibrations.[2]

Energies of first excited states: even-even vs. odd A nuclei

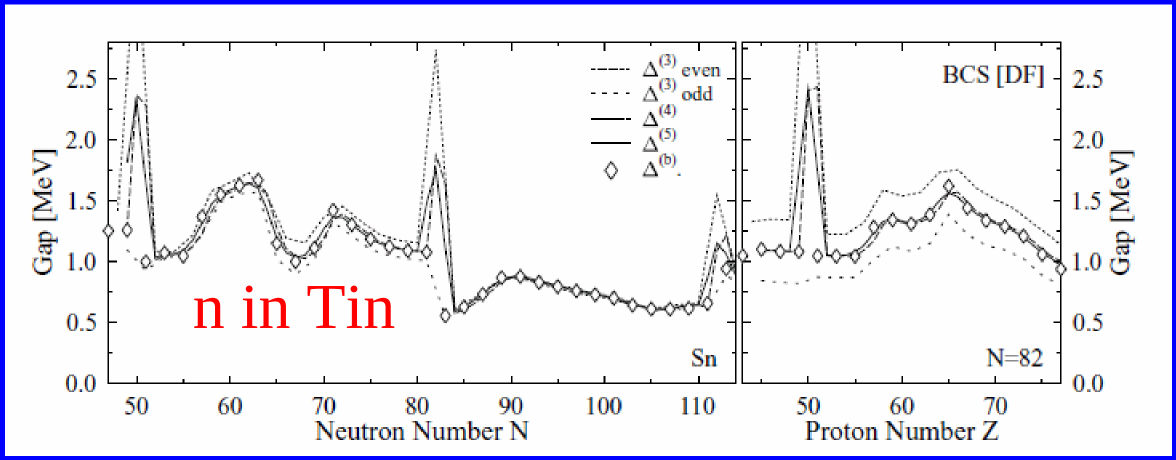
FIG. 1. Energies of first excited intrinsic states in deformed nuclei, as a function of the mass number. The experimental data may be found in *Nuclear Data Cards* [National Research Council, Washington, D. C.] and detailed references will be contained in reference 1 above. The solid line gives the energy $\delta/2$ given by Eq. (1), and represents the average distance between intrinsic levels in the odd-A nuclei (see reference 1).

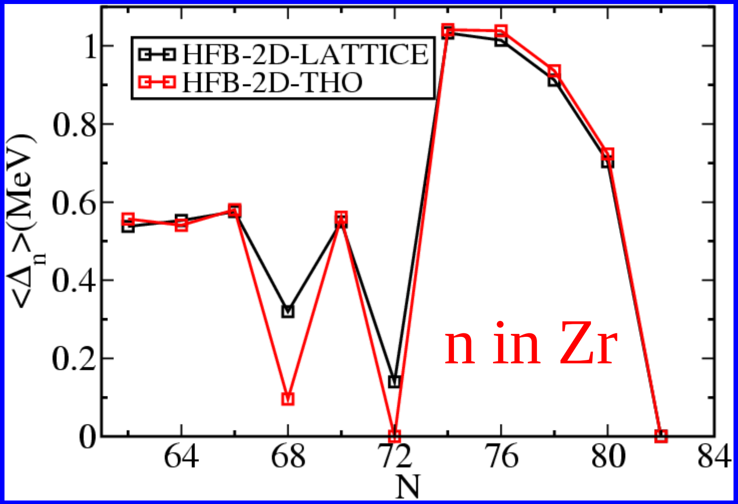
The figure contains all the available data for nuclei with $150<A<190$ and $228<A$. In these regions the nuclei are known to possess nonspherical equilibrium shapes, as evidenced especially by the occurrence of rotational spectra (see, e.g., reference 2). One other such region has also been identified around $A=25$; in this latter region the available data on odd-A nuclei is still represented by Eq. (1), while the intrinsic excitations in the even-even nuclei in this region do not occur below 4 Mev.

We have not included in the figure the low lying $K=0$ states found in even-even nuclei around Ra and Th. These states appear to represent a collective odd-parity oscillation.



Typical calculated nuclear pairing gaps





Rotational spectra of nuclei: $E = J^2/ 2I$, indicate moment of inertia, I, reduced from rigid body value, I_{cl} .

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Reduction of moment of inertia due to BCS pairing = analog of Meissner effect. Detailed calculations by Migdal (59).

Element	β [7]	x_p	x_n	$\left(\frac{\mathscr{J}}{\mathscr{J}_0}\right)_{\text{rect.}}$	$\left(\frac{\mathscr{J}}{\mathscr{J}_0}\right)_{\text{osc.}}$	$\left(\frac{\mathscr{J}}{\mathscr{J}_0}\right)_{\text{exper.}}$ [7]
Nd^{150}	0.26	0.54	0.94	0.15	0.38	0.35
Sm^{152}	0.24	0.65	1.02	0.17	0.43	0.38
Gd^{154}	0.26	0.52	0.88	0.13	0.35	0.36
Gd^{156}	0.33	0.87	1.37	0.22	0.57	0.48
Gd^{157}	0.29	0.93	1.60	0.22	0.64	0.60
Dy^{162}	0.30	0.84	1.43	0.23	0.57	0.50
Hf^{179}	0.20	0.99	1.75	0.27	0.66	0.52
Os^{186}	0.18	0.44	0.69	0.09	0.26	0.28
Th^{230}	0.22	0.63	0.95	0.15	0.40	0.43
Th^{232}	0.22	0.84	1.42	0.24	0.60	0.44
U^{238}	0.24	0.83	1.29	0.22	0.54	0.43

1.C:
1.E.6

Nuclear Physics **13** (1959) 655—674; © *North-Holland Publishing (*

SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL
Atomic Energy Institute of USSR, Academy of Sciences, Moscow

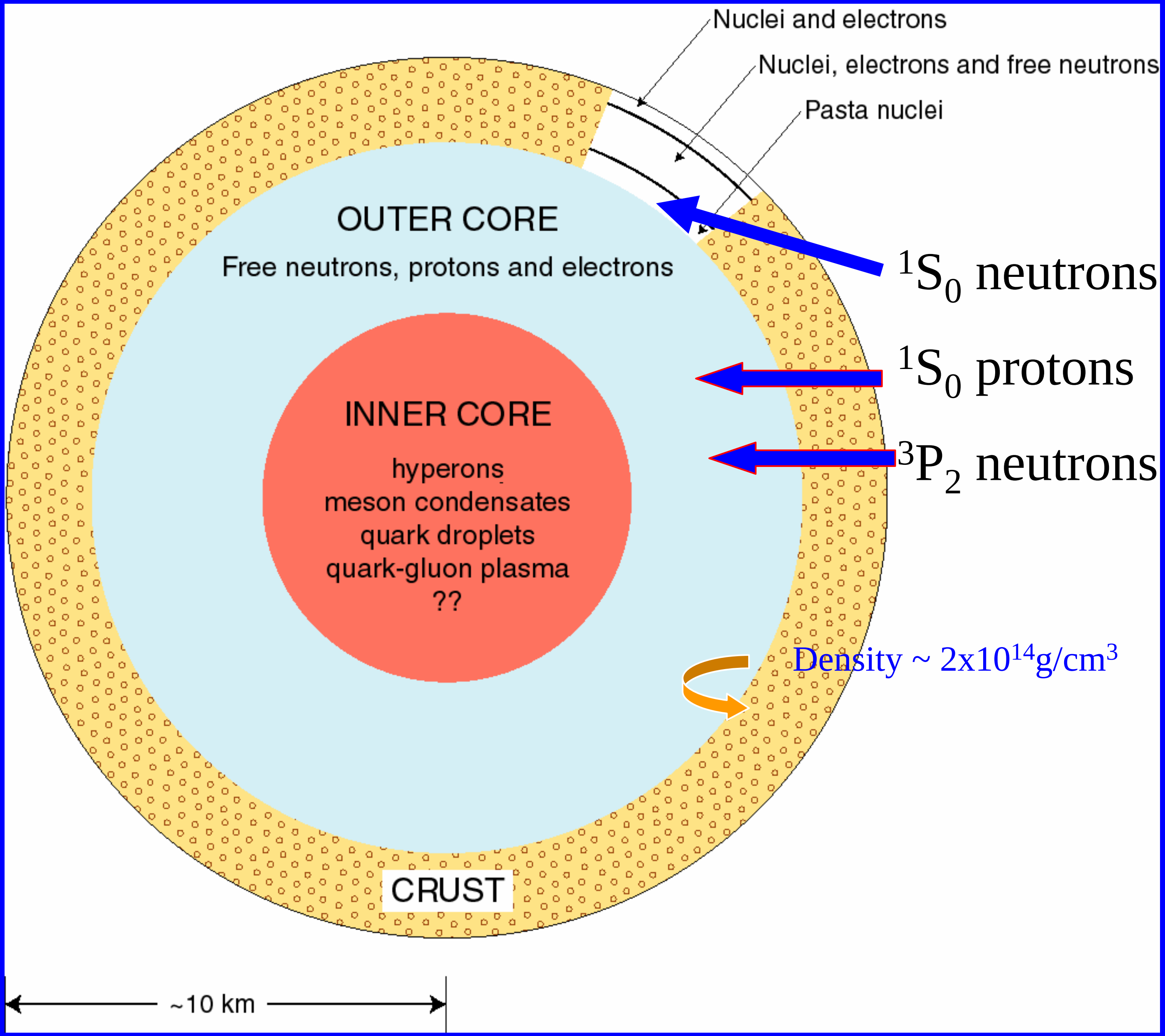
Received 11 April 1959

Abstract: A method is presented which permits one to study superfluidity in finite size systems. Moments of inertia are computed by this method in the quasi-classical approximation and satisfactory agreement with the observed values is obtained. The calculated increase of the moment of inertia upon transition from even to odd-mass nuclei and also the gyromagnetic ratio for rotating nuclei are in agreement with the experiments. These results thus confirm the assumption of superfluidity of nuclear matter.

BCS pairing of nucleons in neutron stars

Mass ~ 1.4 M_{sun}
Radius ~ 10-12 km
Temperature
~ 10^6-10^9 K

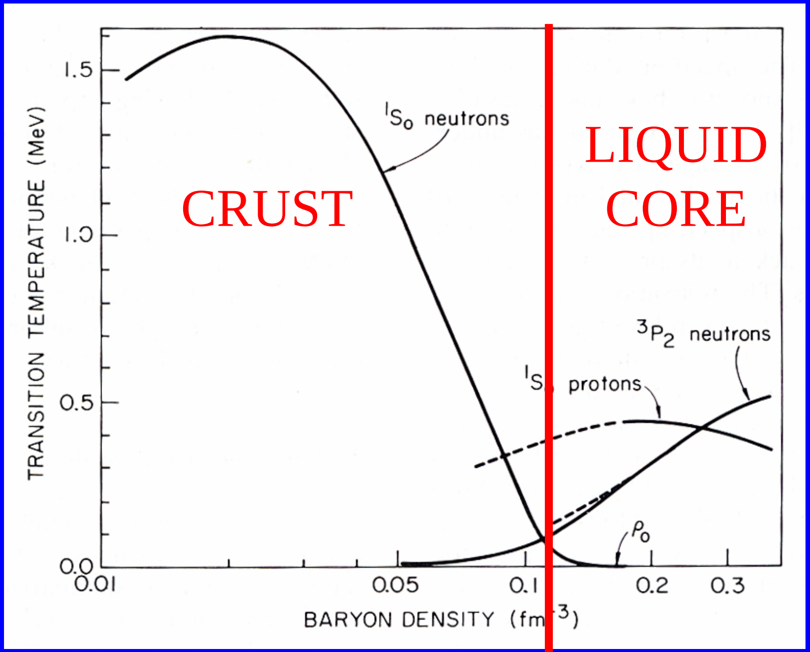
Surface gravity
~10^{14} that of Earth
Surface binding
~ 1/10 mc^2
Mountains < 1 mm



Superfluidity of nuclear matter in neutrons stars

Migdal 1959, Ginzburg & Kirshnits 1964; Ruderman 1967;GB, Pines & Pethick, 1969

First estimates of pairing gaps based on scattering phase shifts



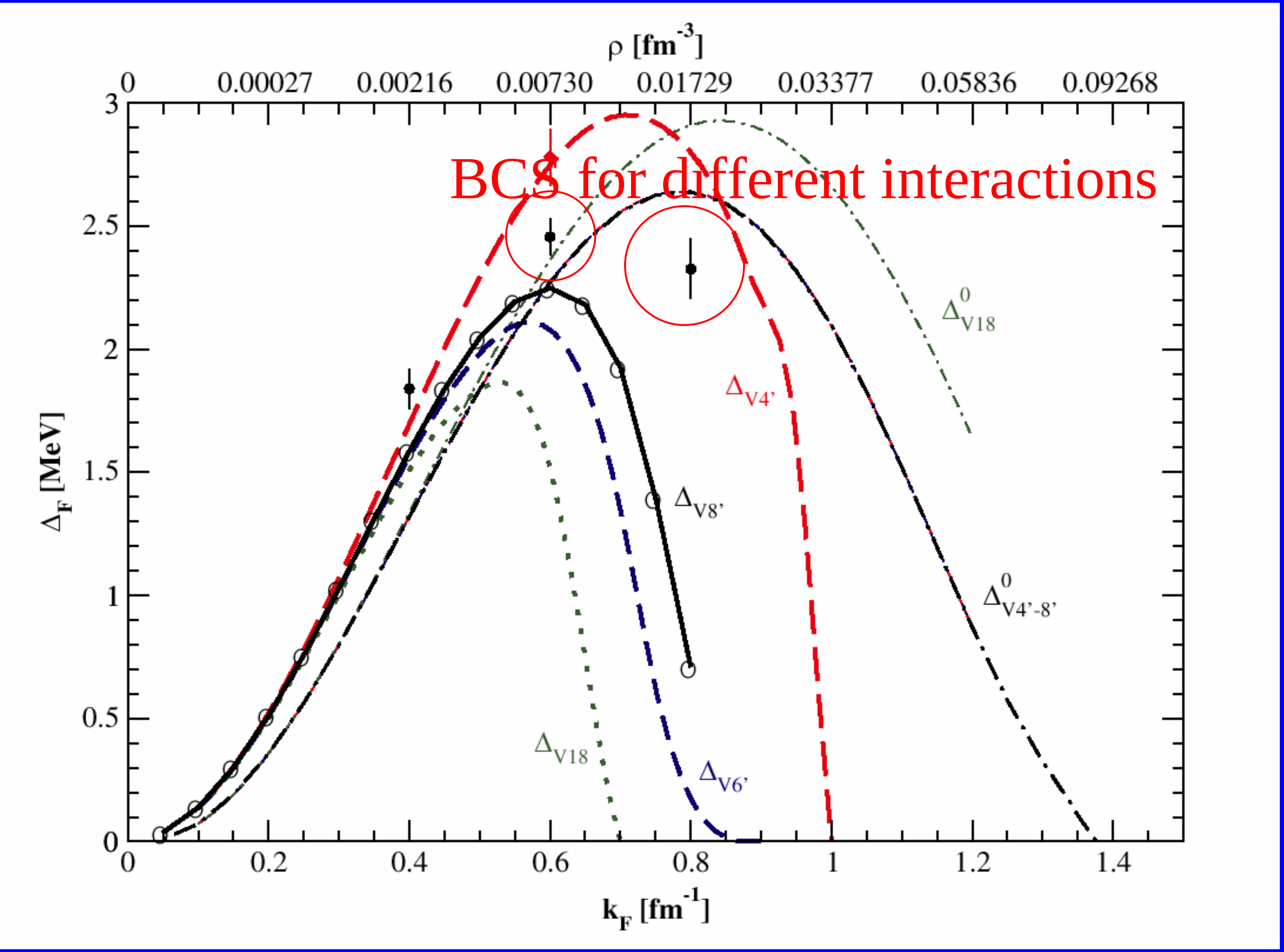
Neutron fluid in crust BCS-paired in relative $^1S_0$ states

Neutron fluid in core $^3P_2$ paired
Proton fluid $^1S_0$ paired

n=Hoffberg et al. 1970, p=Chao et al. 1972

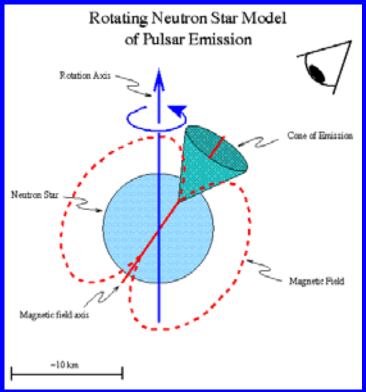
Quantum Monte Carlo calculations of $^1S_0$ nn gap:

Fabrocini et al, PRL 95, 192501 (2005)

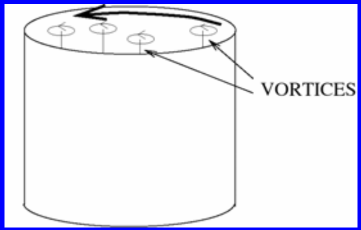


QMC (black points) close to standard BCS (upper curves)

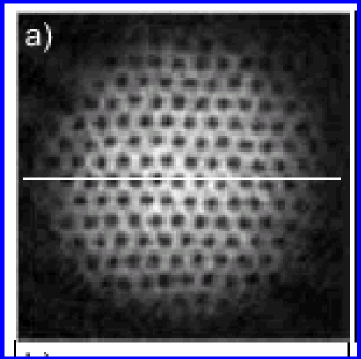
Rotating superfluid neutrons



Rotating superfluid threaded by triangular lattice of vortices parallel to stellar rotation axis



Bose-condensed ^{87}Rb atoms
Schweikhard et al., PRL92 040404 (2004)



Quantized circulation of superfluid velocity about vortex:

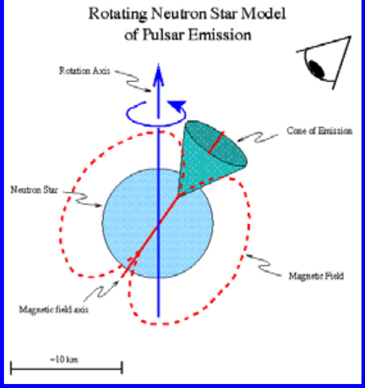
$$\oint_C \mathbf{v_s} \cdot d\ell = \frac{2\pi\hbar}{2m_n}$$

Vortex core $\sim$ 10 fm
Vortex separation $\sim 0.01P(s)^{1/2}$cm; Vela contains $\sim 10^{17}$ vortices

Angular momentum of vortex $=N\hbar(1-r^2/R^2)$ decreases as vortex moves outwards => to spin down must move vortices outwards

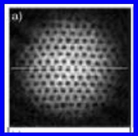
Superfluid spindown controlled by rate at which vortices can move against barriers, under dissipation

Superconducting protons in magnetic field



Even though superconductors expel magnetic flux, for magnetic field below critical value, flux diffusion times in neutron stars are >> age of universe. Proton superconductivity forms with field present.

Proton fluid threaded by triangular (Abrikosov) lattice of vortices parallel to magnetic field (for Type II superconductor)



Quantized magnetic flux per vortex:

$$\oint_C \mathbf{B} \cdot d\ell = \frac{2\pi\hbar c}{2e} = \phi_0 = 2\times 10^{-7}\text{G.}$$

Vortex core $\sim$ 10 fm,

$n_{vort} = B/\phi_0$ => spacing ~ 5×10^{-10} cm $(B/10^{12}\text{G})^{-1/2}$

Pulsar glitches

Sudden speedups in rotation period, relaxing back in days to years, with no significant change in pulsed electromagnetic emission

$\sim$ 90 glitches detected in $\sim$ 30 pulsars

Vela (PSR0833-45) Period=1/Ω=0.089sec

15 glitches since discovery in 1969

$\Delta\Omega/\Omega \sim 10^{-6}$ Largest = 3.14×10^{-6} on Jan. 16, 2000

Moment of inertia $\sim 10^{45}$ gcm^2 => $\Delta E_{rot} \sim 10^{43}$ erg

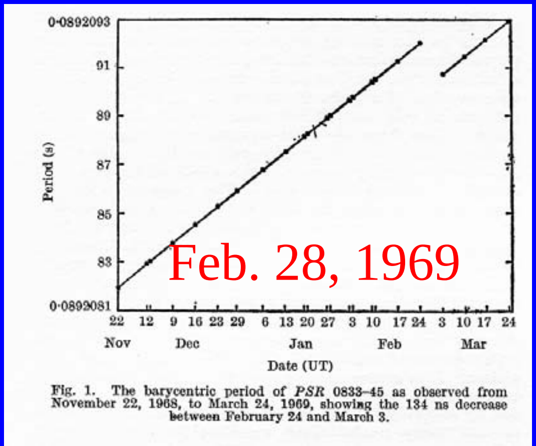
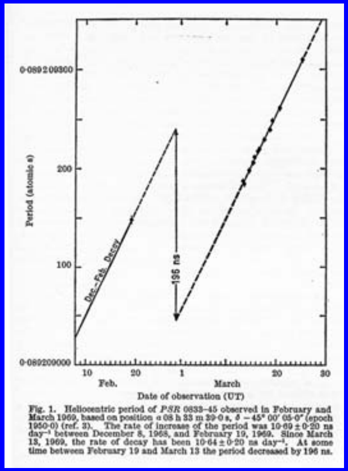
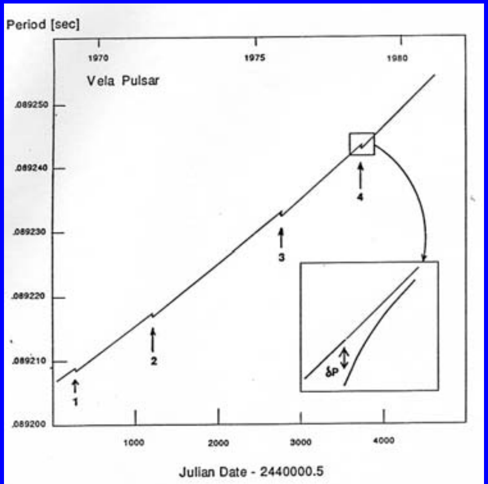


Fig. 1. The barycentric period of PSR 0833-45 as observed from November 22, 1968, to March 24, 1969, showing the 134 ns decrease between February 24 and March 3.

Reichley and Downs, Nature 1969





Radhakrishnan and Manchester, Nature 1969

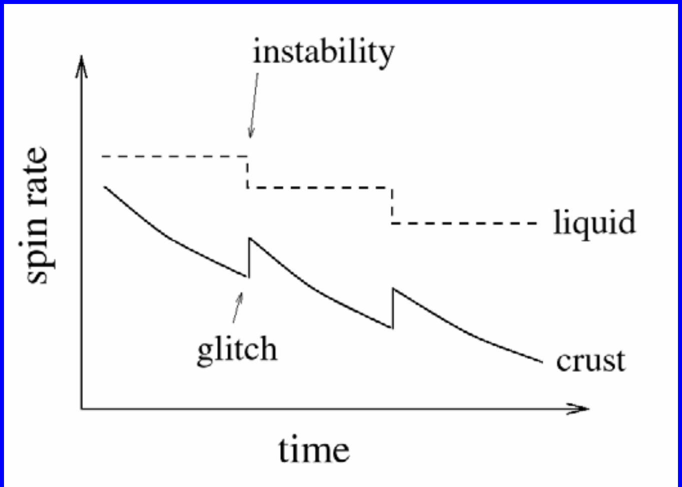
Crab (PSR0531+21) P = 0.033sec 14 glitches since 1969 $\Delta\Omega/\Omega \sim 10^{-9}$

Mechanism of glitches

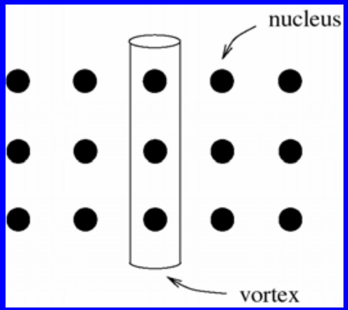
Pulse structure not notably affected by glitch => phenomenon internal in the neutron star. Long time scales for response (relaxation ~ months) => well-oiled machinery – superfluidity! [Metastable superfluid flow (Packard 1972).]

Pulses connected via magnetic field - to the crust.

Neutron liquids in star act as a reservoir of angular momentum L.
Crust neutron superfluid carries ~ 3% of total L.
Sudden transfer of L_{sf} to crustal solid speeds it up => glitch



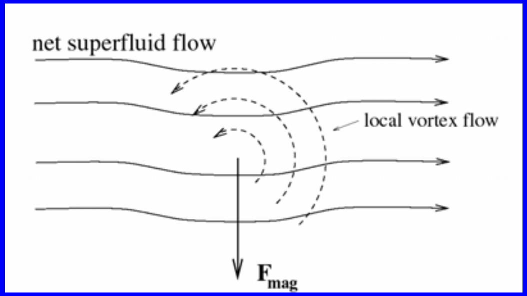
Vortex model of glitches



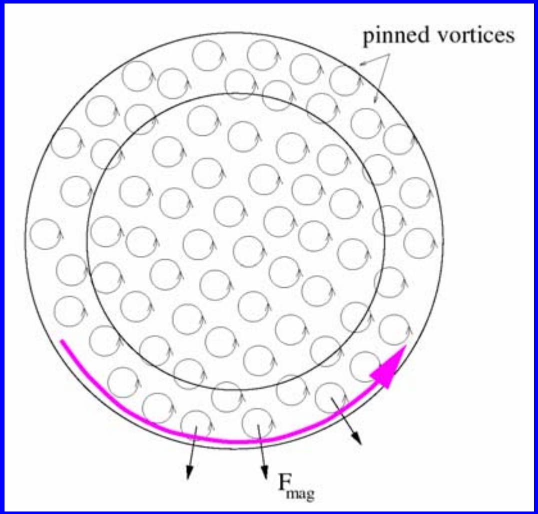
Pin vortices to (or between) nuclei in inner crust (*Anderson & Itoh 1975*). E$\sim$ 3Mev/nucleus.

$n_{vortices}$ fixed => $\Omega_{superfluid}$ fixed; Ω_{normal} decreases as star radiates.

As Ω_{sf} - Ω_{n} grows, Magnus force =$\rho_s \times (\mathbf{v}_{vortex}-\mathbf{v}_{superfl})$ drives unpinning (glitch) and outward relaxation.



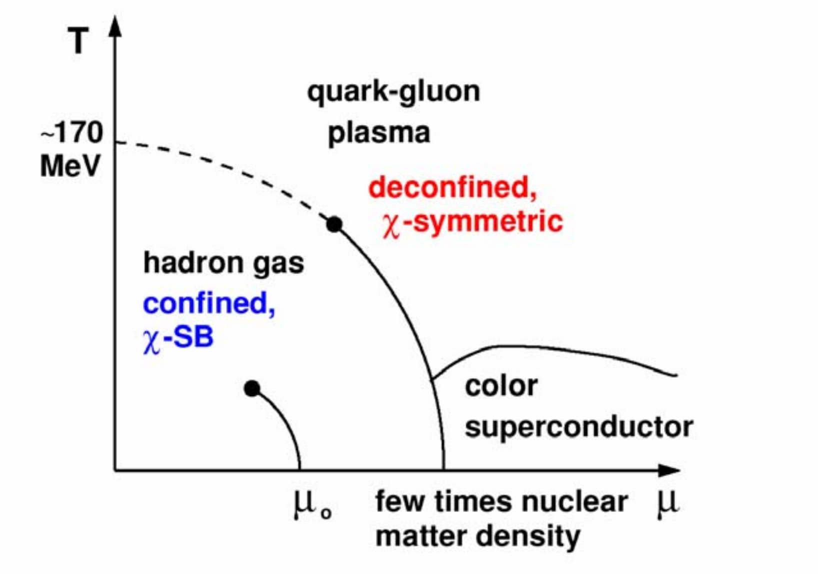
Collective outward motion of many ($\sim 10^{14}$) vortices produces large glitch



Pairing in high energy nuclear/particle physics

Vacuum condensates: quark-antiquark pairing underlies chiral SU(3)×SU(3) breaking of vacuum=>

$$\langle \bar{q} q \rangle_{\text{vacuum}} \neq 0$$



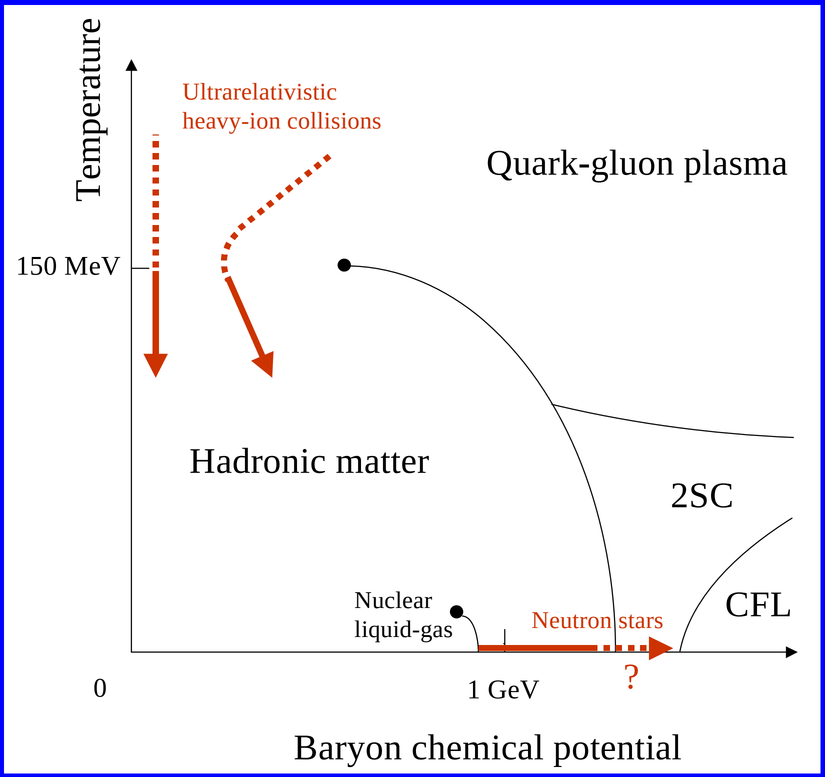
Karsch & Laermann, hep-lat/0305025

Experimental Bose-Einstein decondensation

* **BCS pairing of degenerate quark matter** – color superconductivity

Color pairing in quark matter

Review: Rajagopal & Wilczek, hep-ph/0011333

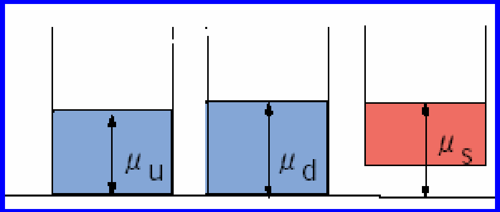


Superfluidity

condensate of paired quarks => superfluid baryon density (n_s)

Color Meissner effects

transverse color fields screened on spatial scale ~ London penetration depth ~ $(\mu/g^2 n_s)^{1/2}$



Two interesting phases:

2SC (u,d)

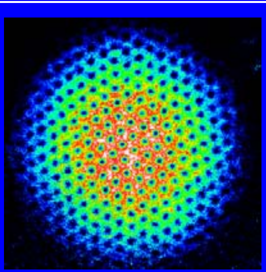
$\langle u_R d_B \rangle$

Color-flavor locked (CFL) ($m_u = m_d = m_s$)

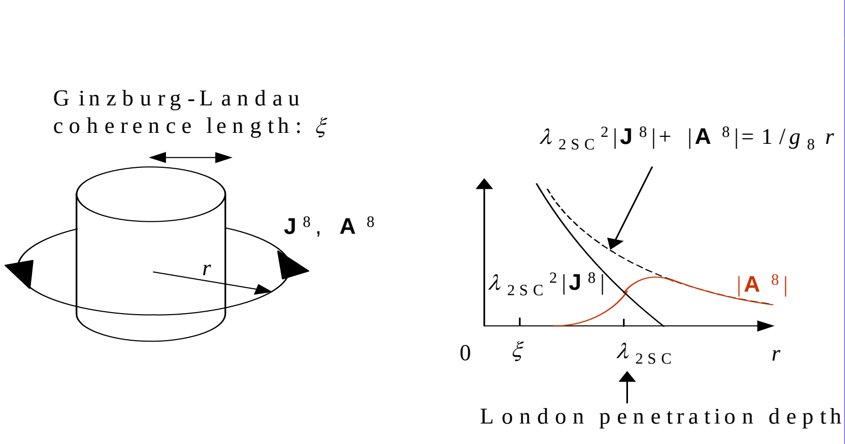
$\langle u d \rangle = \langle d s \rangle = \langle s u \rangle$

Responses of superfluid quark matter to magnetic fields and rotation

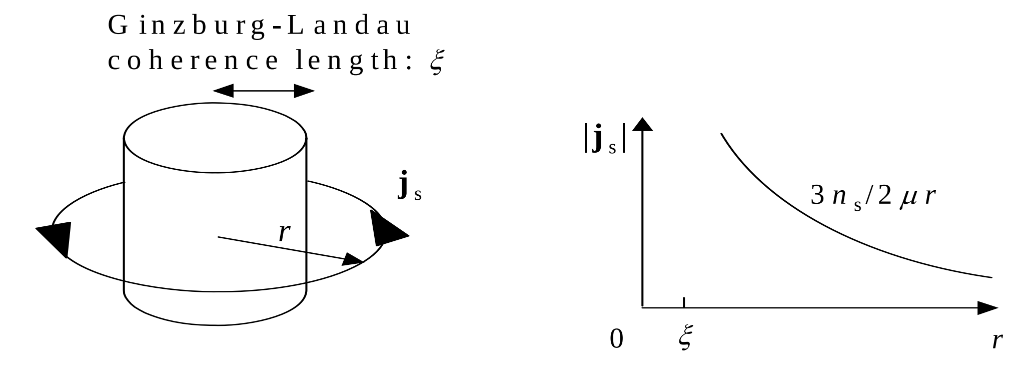
Phase	Magnetic fields	Rotation
CFL	Partial screening $SU(3)_{c+f}$ vortices (Type II)	$U(1)_B$ vortices
2SC	Partial screening $U(1)_{em}$ vortices (Type II)	London B field

Lattice of quantized vortices: 

K.Iida



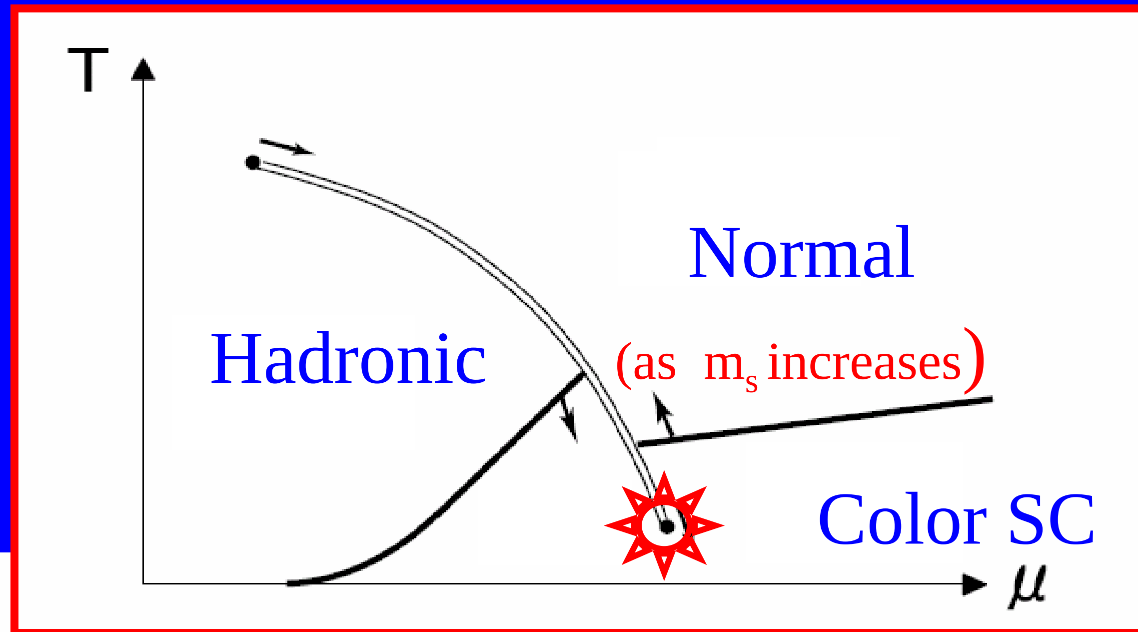
Magnetic vortex in the 2SC phase

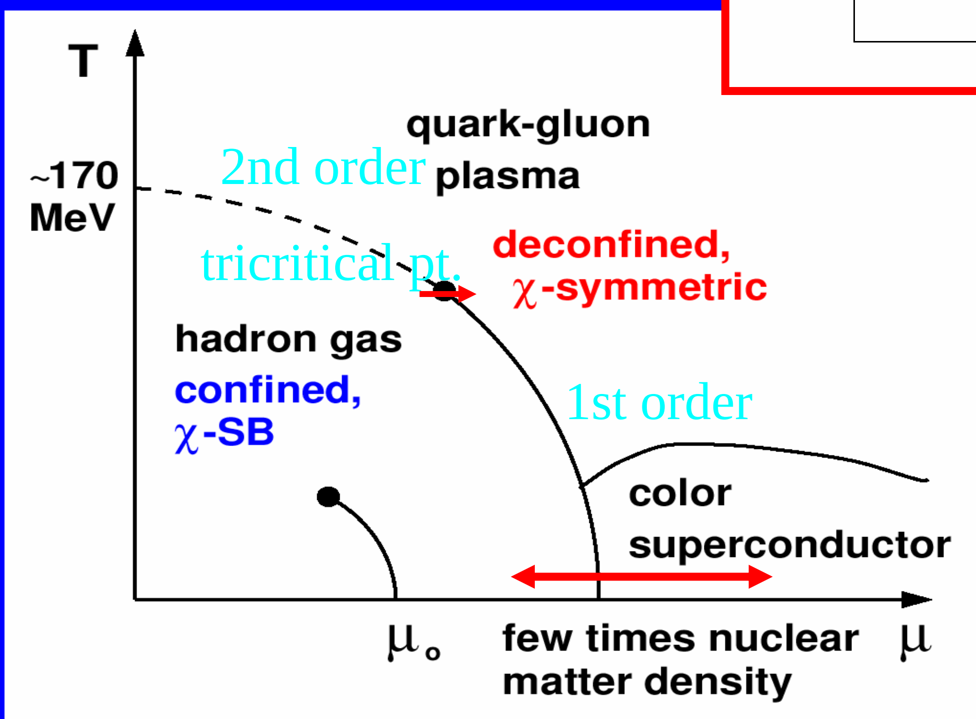


Rotational vortex in the CFL phase

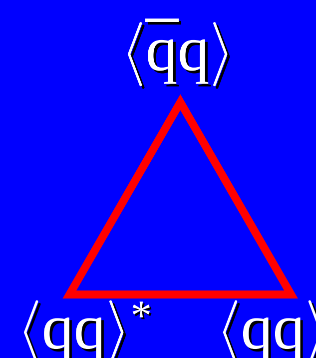
New critical point in phase diagram:

Induced by coupling of chiral condensate and diquark pairing via axial anomaly

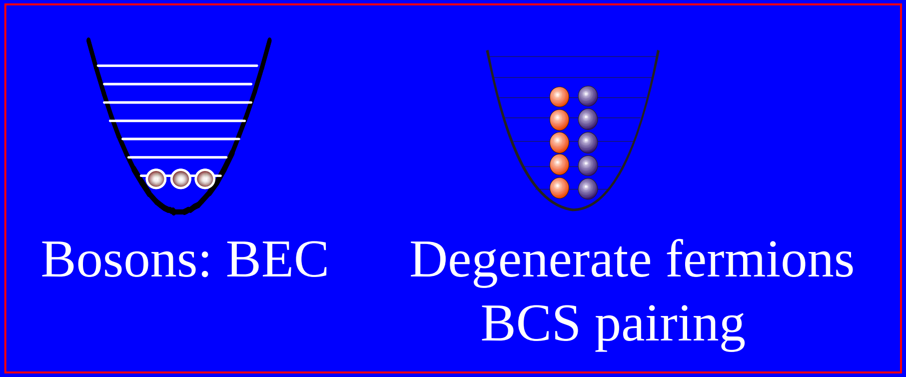


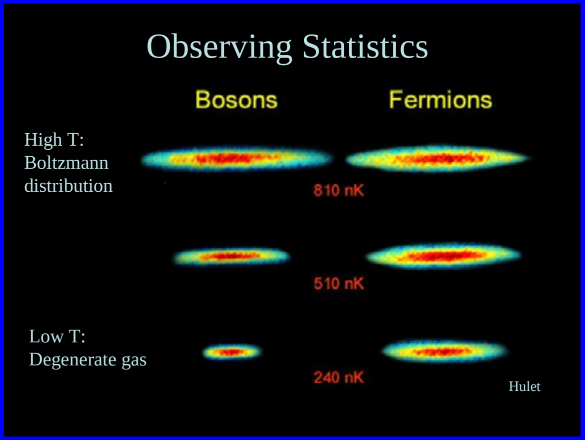


Hatsuda, Tachibana, Yamamoto & GB, PRL 97, 122001 (2006)



BCS paired fermions: a new superfluid





^{7}Li vs. ^{6}Li

Produce trapped degenerate Fermi gases: ^{6}Li, ^{40}Increase attractive interaction with Feshbach resonance

At resonance have “unitary regime”: no length scale – “resonance superfluidity”

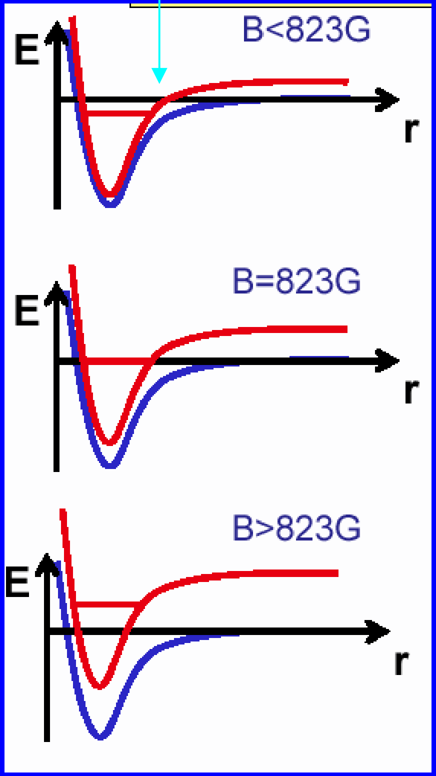
Experiments: JILA, MIT, Duke, Innsbruck, ...

Controlling the interparticle interaction

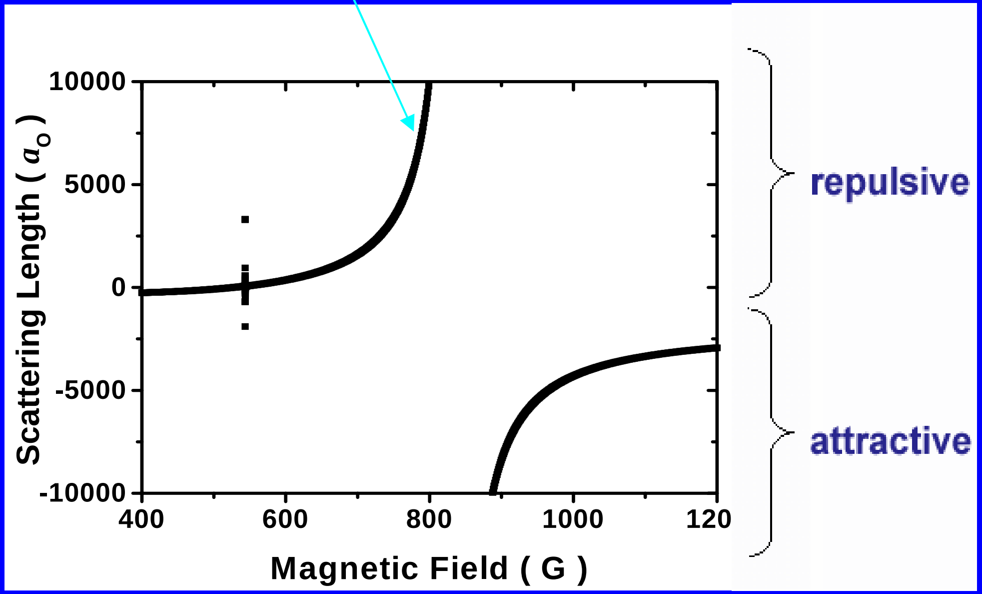
Effective interparticle interaction short range s-wave:

$V(r_1-r_2) = (4\pi\hbar^2 a/m)\,\delta\,(r_1-r_2)$; a= *s-wave atom-atom scattering length*

weakly bound molecule in closed channel



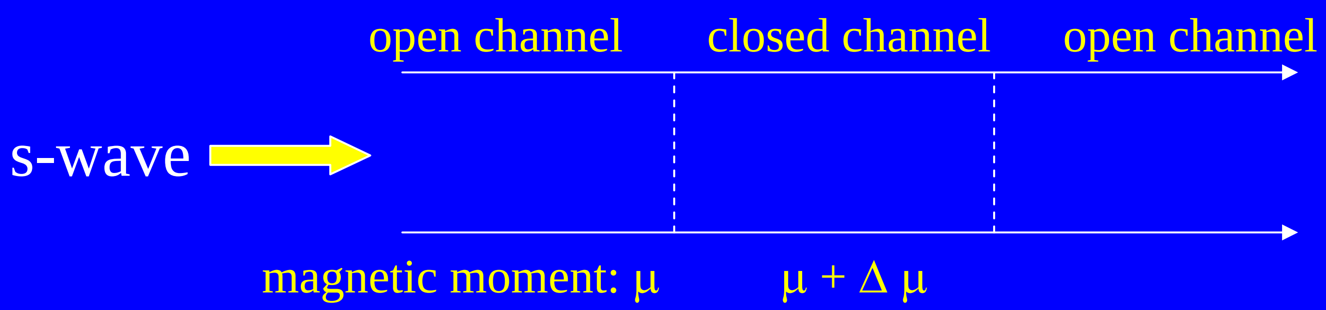
^{6}Li



Broad resonance around 820-830 Gauss

Increasing magnetic field through resonance changes interactions from repulsive to attractive; very strong in neighborhood of resonance

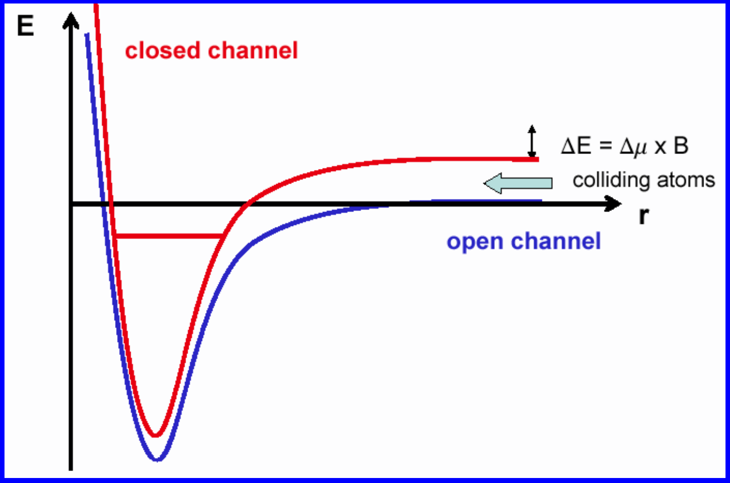
Feshbach resonance in atom-atom scattering



Scattering amplitude $\propto \dfrac{|M|^2}{E_c - E_o}$

$E_c\text{-}E_0 \sim \Delta\mu\, B + \ldots$

Low energy scattering dominated by bound state closest to threshold



Adjusting magnetic field, B, causes level crossing and resonance, seen as divergence of s-wave scattering length, a:

$$a(B) = a_{bg}\left(1 - \frac{\Delta}{B - B_{Feshbach}}\right)$$

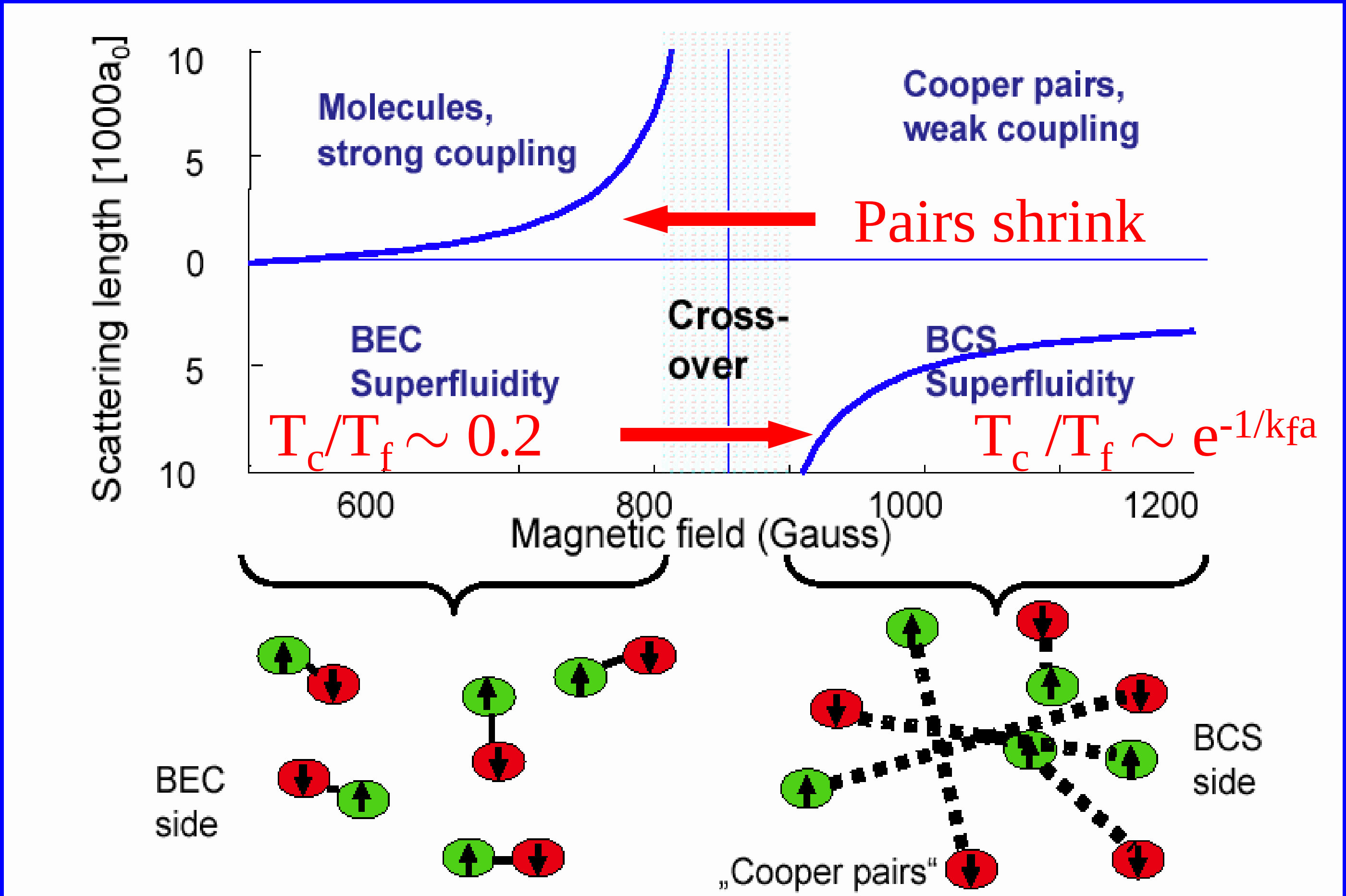
BEC-BCS crossover in Fermi systems

Continuously transform from molecules to Cooper pairs:

D.M. Eagles (1969)

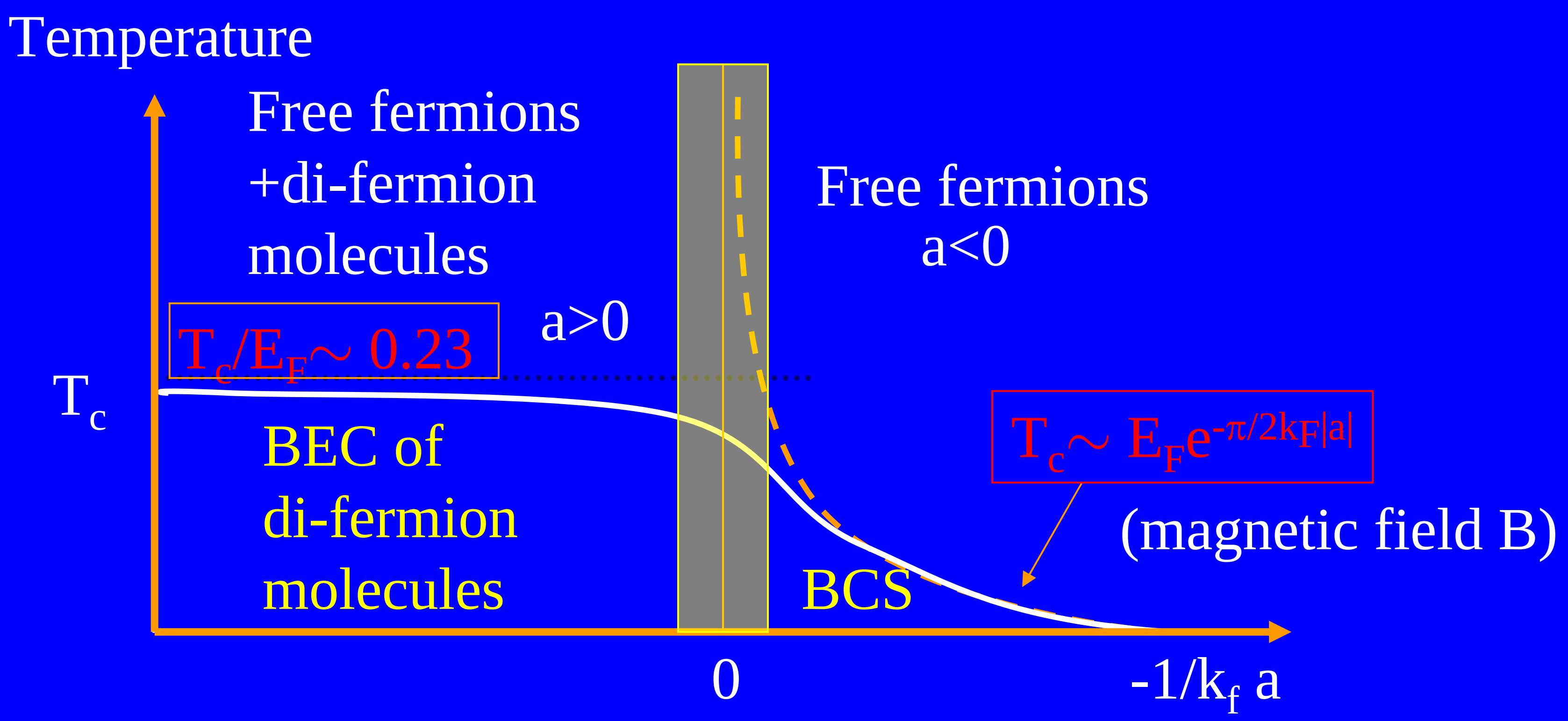
A.J. Leggett, J. Phys. (Paris) C7, 19 (1980)

P. Nozières and S. Schmitt-Rink, J. Low Temp Phys. 59, 195 (1985)



^{6}Li

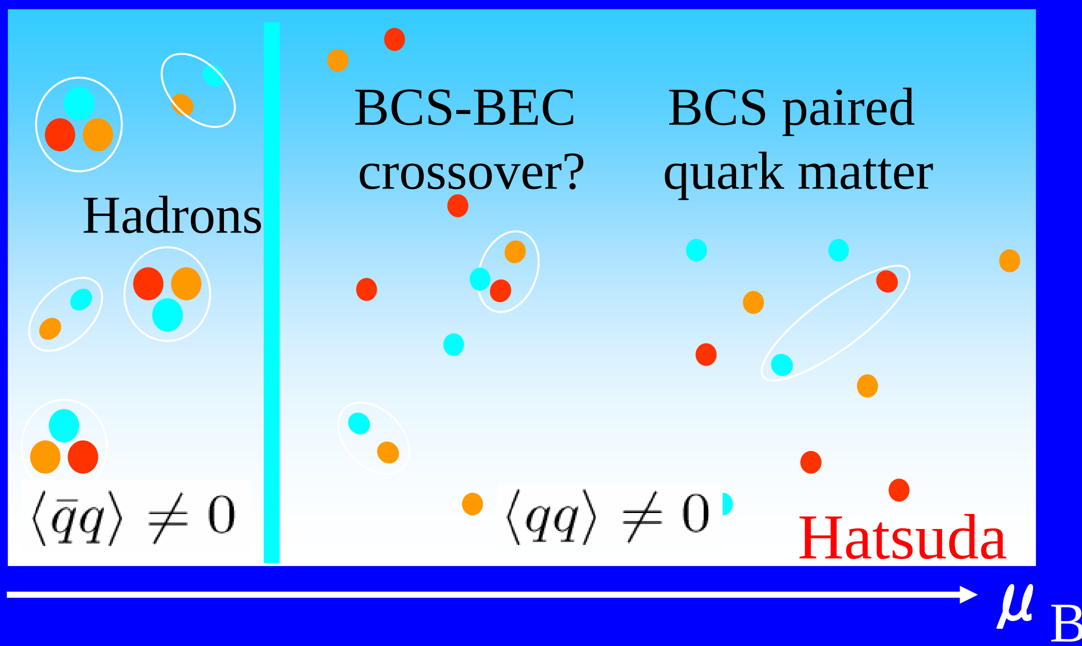
Phase diagram of cold fermions vs. interaction strength

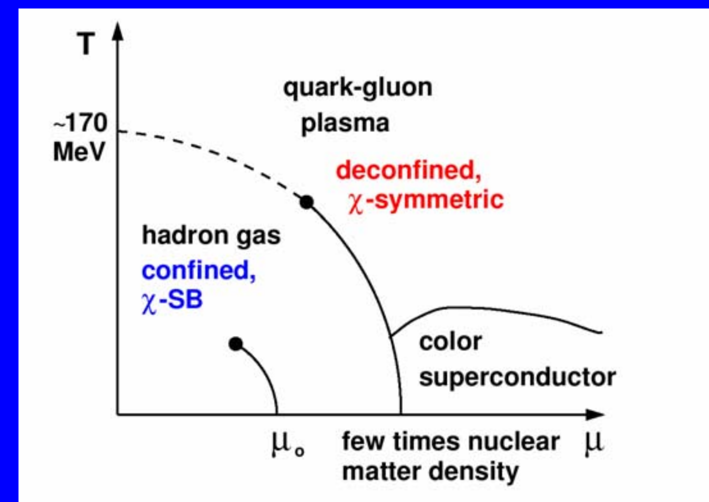


Unitary regime -- crossover

No phase transition through crossover

Hadron-quark matter deconfinement transition vs. BEC-BCS crossover





Abuki, Itakura & Hatsuda,
PRD65, 2002

In $SU(2)_C$:

hadrons <=> 2 fermion molecules,

paired deconfined phase <=> BCS paired fermions

Strongly coupled regime is scale free

Only length-scale for cold atoms near resonance is density. No microscopic parameters enter equation of state

$$\frac{E}{N} = \frac{3}{5} E_F (1+\beta) \sim n^{2/3}/m$$

(free Fermi energy)

β is universal parameter. No systematic expansion

Fixed Node Green's Function Monte Carlo, Carlson et al. (2003,5): β = -0.56 -> – 0.58

Diagrammatic. Perali, Pieri & Strinati (2004) β = -0.545

Experiment:

Rice: -0.54(5), Duke: -0.26(7), ENS: -0.3, JILA: -0.4, Innsbruck: 0.68(1)

BCS transition temperature $\simeq$ 0.24-0.26 T_f $\sim$ submicrokelvin

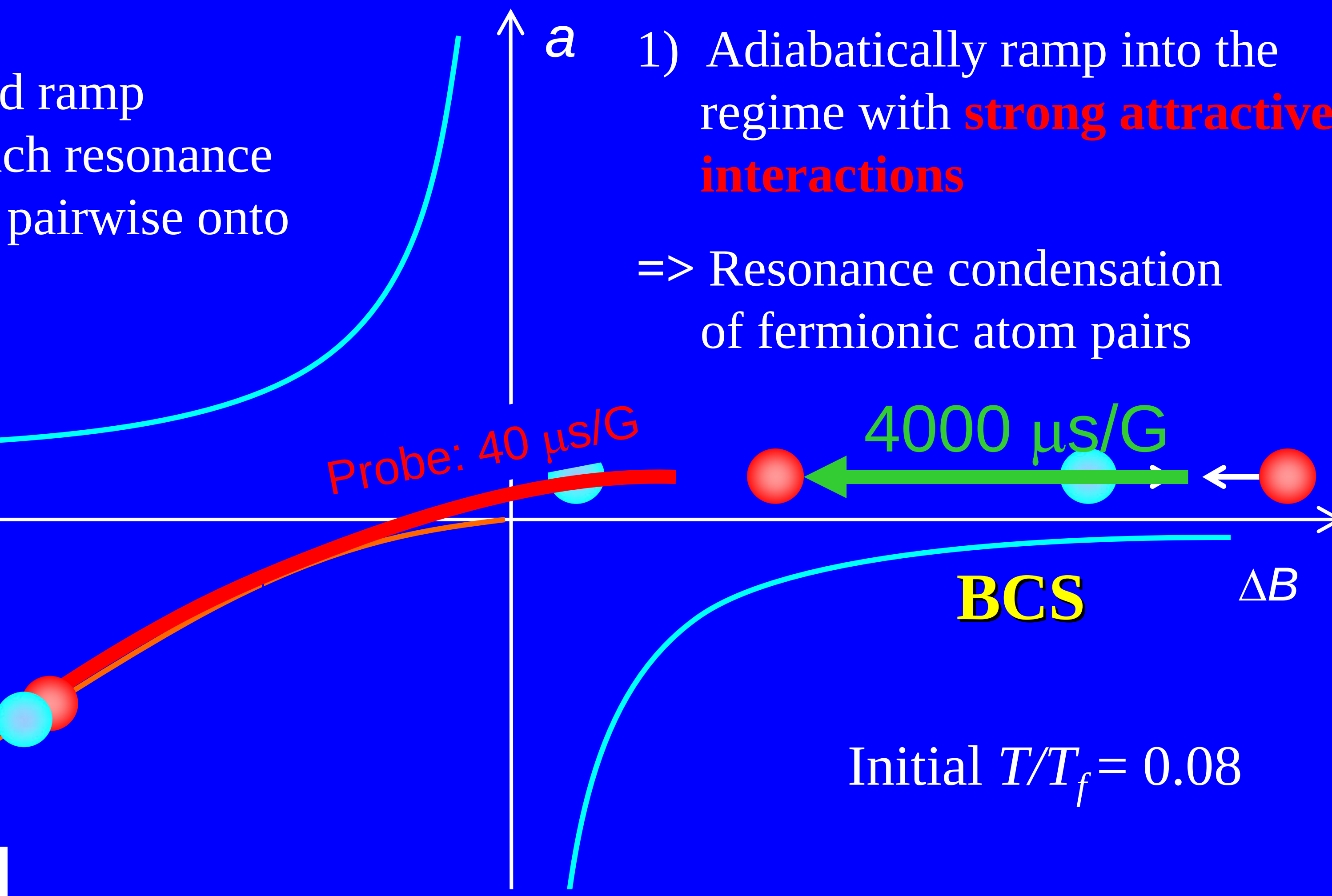
^{40}K **pairing at JILA**

C. A. Regal, M. Greiner, and D.S. Jin, Phys. Rev. Lett. 92, 040403 (2004)

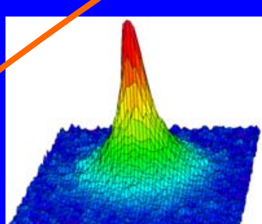
1) Adiabatically ramp into the regime with **strong attractive interactions**

=> Resonance condensation of fermionic atom pairs

2) Probe: rapid ramp across Feshbach resonance project atoms pairwise onto molecules



Initial $T/T_f = 0.08$



3) Molecule momentum distribution => Fermi condensate in initial state

Vortices in trapped Fermi gases: marker of superfluidity

M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, and W. Ketterle,
Nature 435, 1047 (2005)

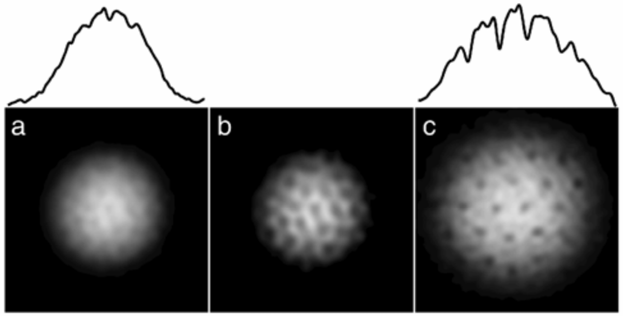


Fig. 1: Observation of a vortex lattice in a molecular condensate. (a) Fixed field. Stirring for 800 ms, followed by 400 ms of equilibration, and imaging after 12 ms time-of-flight all took place at 766 G. The vortex core depletion of the integrated density profile is barely 10%, as indicated by the 5-μm -wide cut on top. (b) Fourier-filter applied to (a) to accentuate the vortex contrast.

^{6}Li

Resonance at ~ 834G

B<834G = BEC
B>834G = BCS

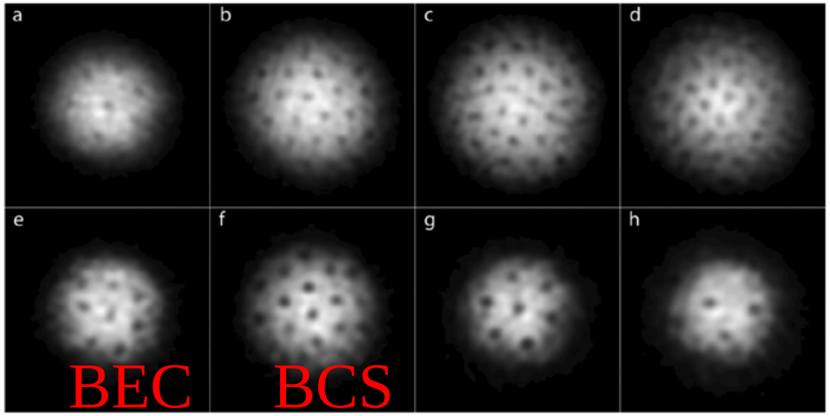
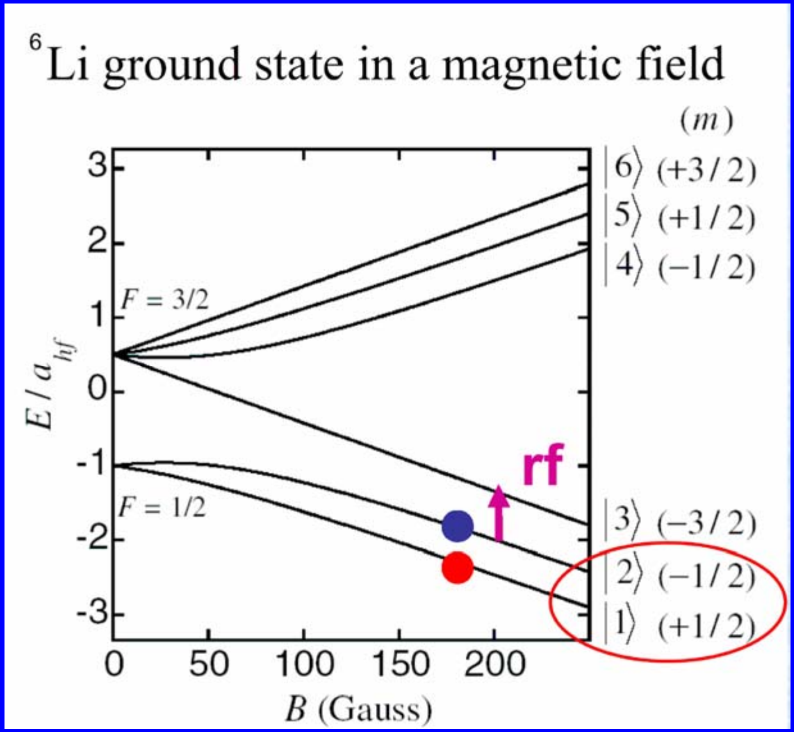
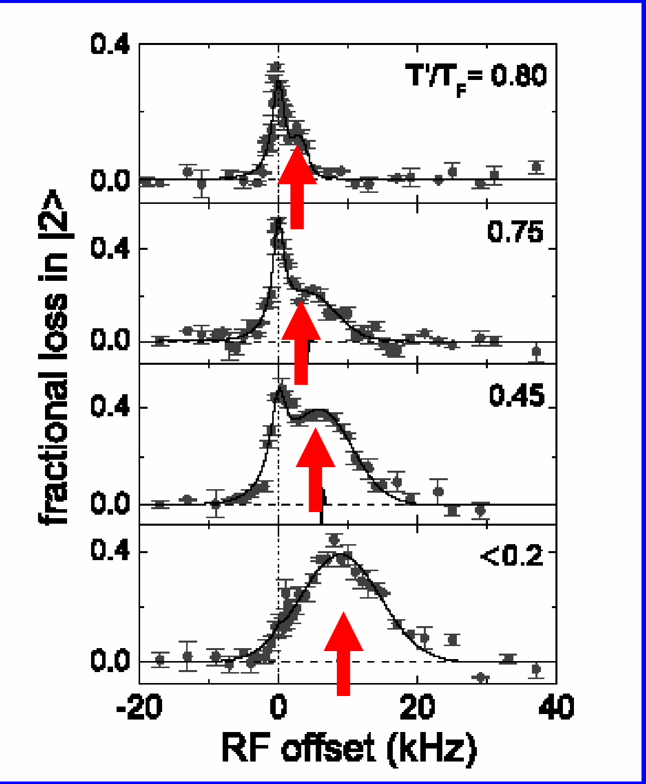


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu m \times 880\ \mu m$.

Detection of gap by breaking pairs via rf excitation

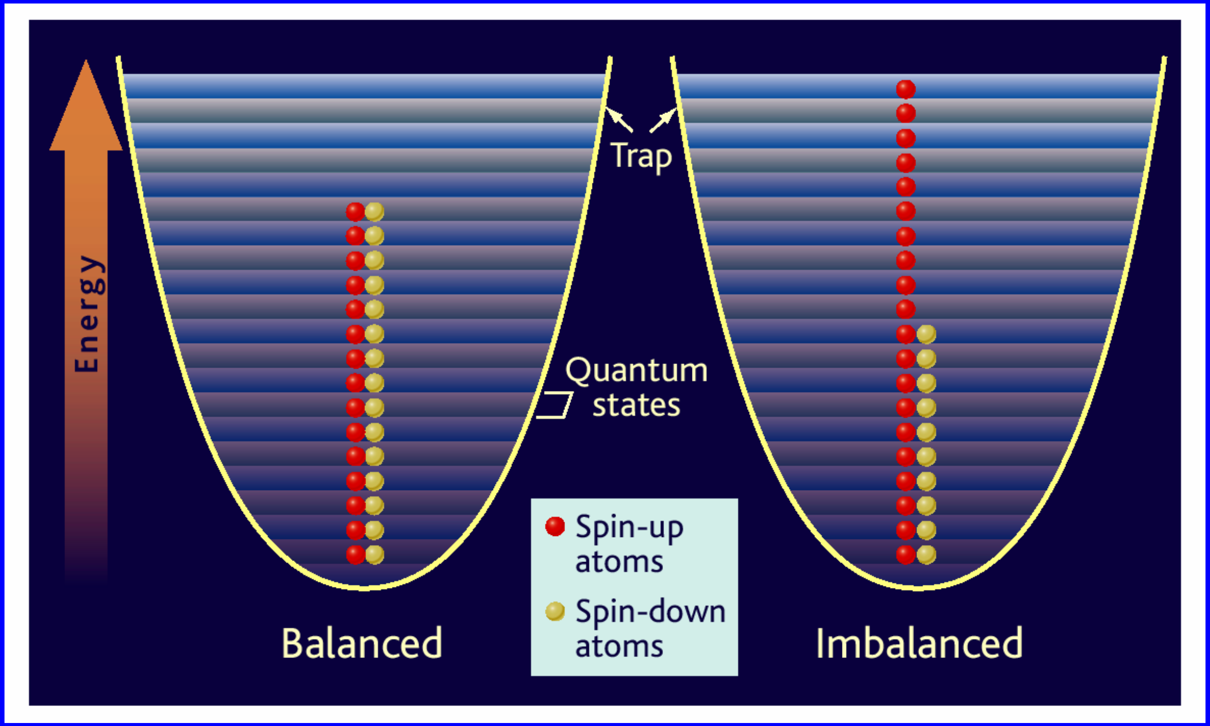
C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J.Hecker Denschlag, and R. Grimm, Science 305, 1128 (2004).





In strong magnetic field, ~800G, with T_f = 2.5 μK, pair atoms in nuclear spin states m_I = -1 and 0.

Superfluidity and pairing for unbalanced systems

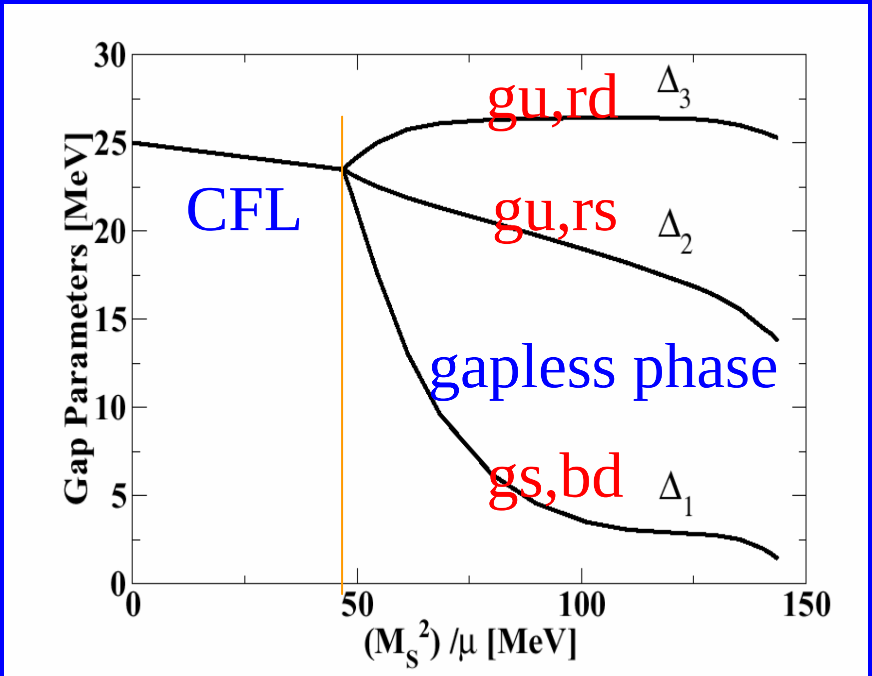


Trapped atoms: change relative populations of two states by hand

QGP: balance of strange (s) quarks to light (u,d) depends on

ratio of strange quark mass m_s to chemical potential μ (>0)

Color superconductor with $m_{strange} \neq m_{light}$



Decreasing pairing of strange quarks with increasing m_s
Alford, Kovaris & Rajagopal, hep-ph/0311286

In gapless phase for unbalanced color superconductors, Meissner screening length can be imaginary (superfluid mass density < 0)
M. Huang; M. Alford; and collaborators

Proposed resolutions

*Phase separation. *(Cf. neutron-rich nuclei with neutron skin)*

*FFLO state with spatial ordering

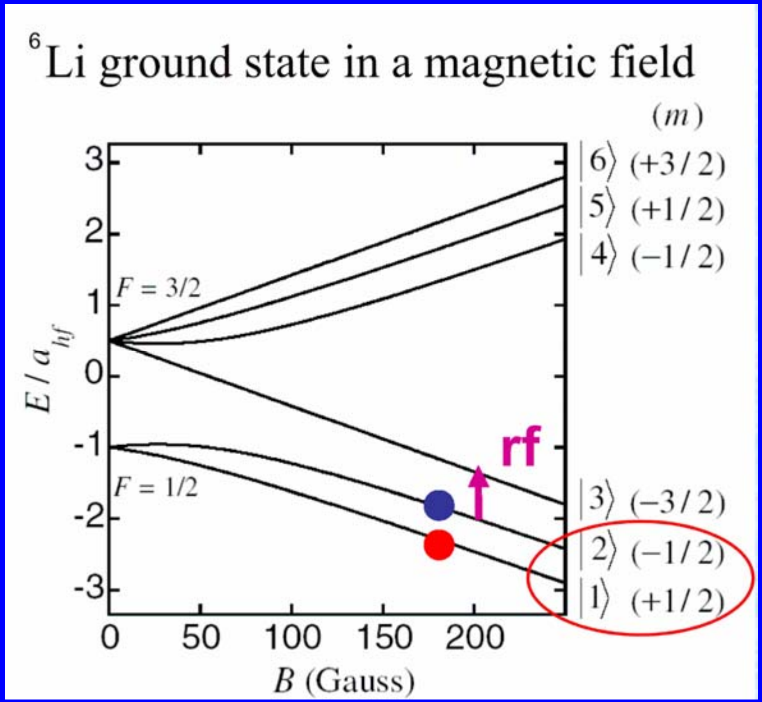
*Gluon condensate

…

Experiments on ^{6}Li with imbalanced populations of two hyperfine states, $|1\rangle$ and $|2\rangle$

MIT: Zwierlein et al., Science 311, 492 (2006); Nature 442, 54 (2006).

Rice: Partridge et al., Science 311, 503 (2006) cond-mat/0605581

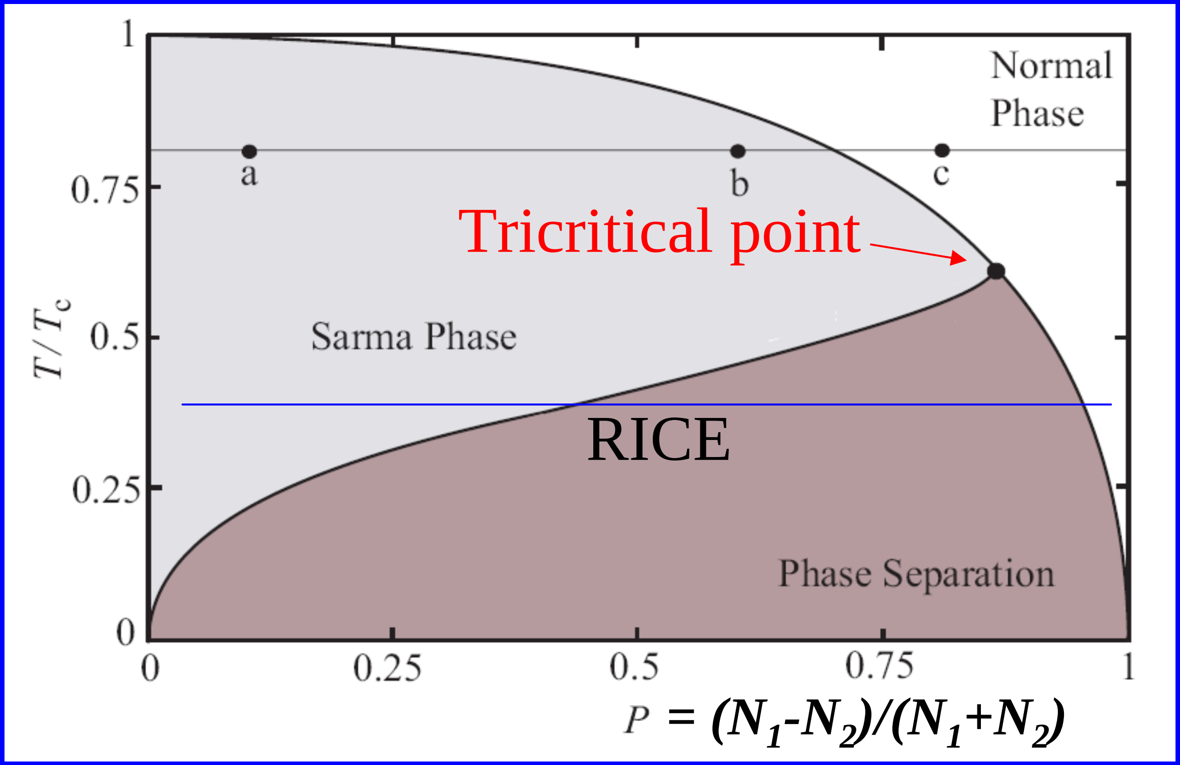


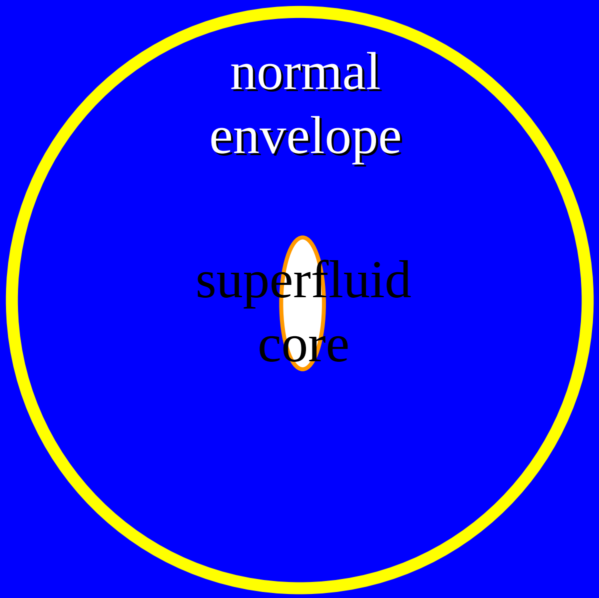
Fill trap with n_1 $|1\rangle$ atoms, and n_2 $|2\rangle$ atoms, with $n_1 > n_2$.

Study spatial distribution, and existence of superfluidity for varying n_1:n_2.

Phase diagram of trapped imbalanced Fermi gases

K. B. Gubbels, M. W. J. Romans, and H. T. C. Stoof, cond-mat/0606330





Trap geometry

Sarma: second order transition to normal phase with increasing radius with gapless superfluid near boundary

Phase separation: first order transition

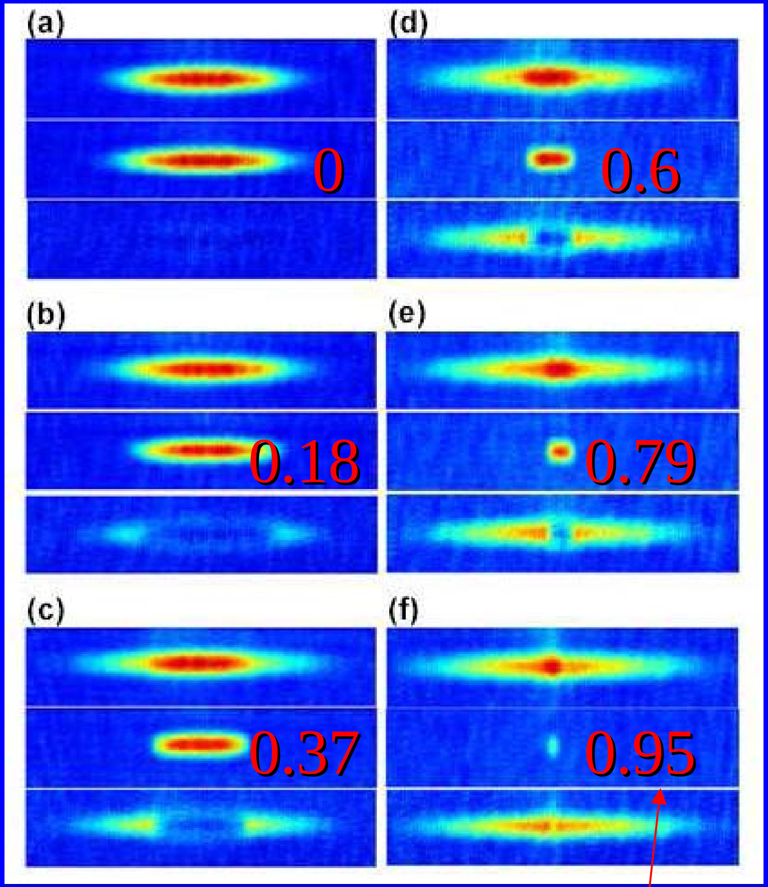
Spatial separation vs. polarization

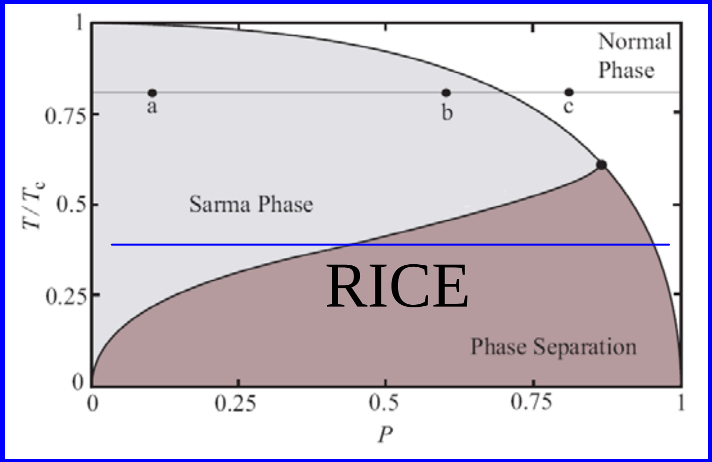
Partridge, Li, Liao, Hulet, Haque & Stoof, cond/mat 0608455

N_1

N_2

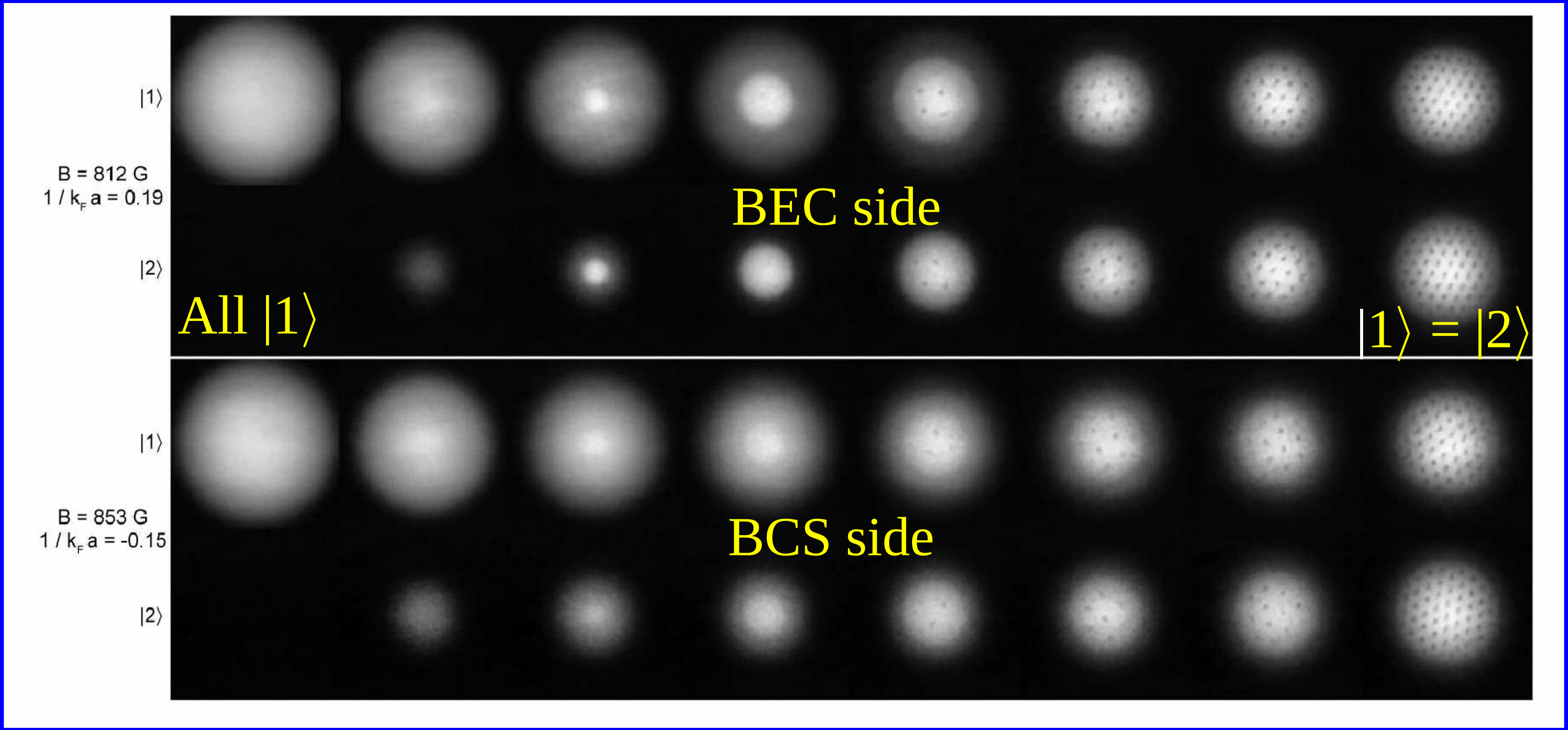
N_1-N_2

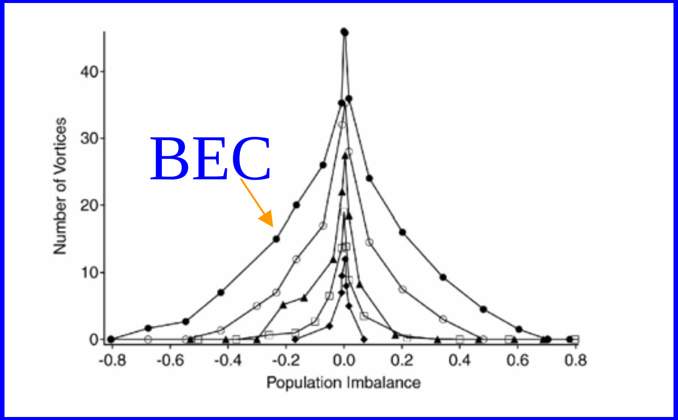




$$P = (N_1 - N_2)/(N_1 + N_2)$$

Vortices (MIT)





No. of vortices vs. population imbalance

John Bardeen – the Super Conductor

Bob & Anne Schrieffer

with his students, for his 60th birthday, 1968.